Ing. Alejandro Vera Lázaro

EDITORIAL MACRO®

CÁLCULO DIFERENCIAL

con MATLAB

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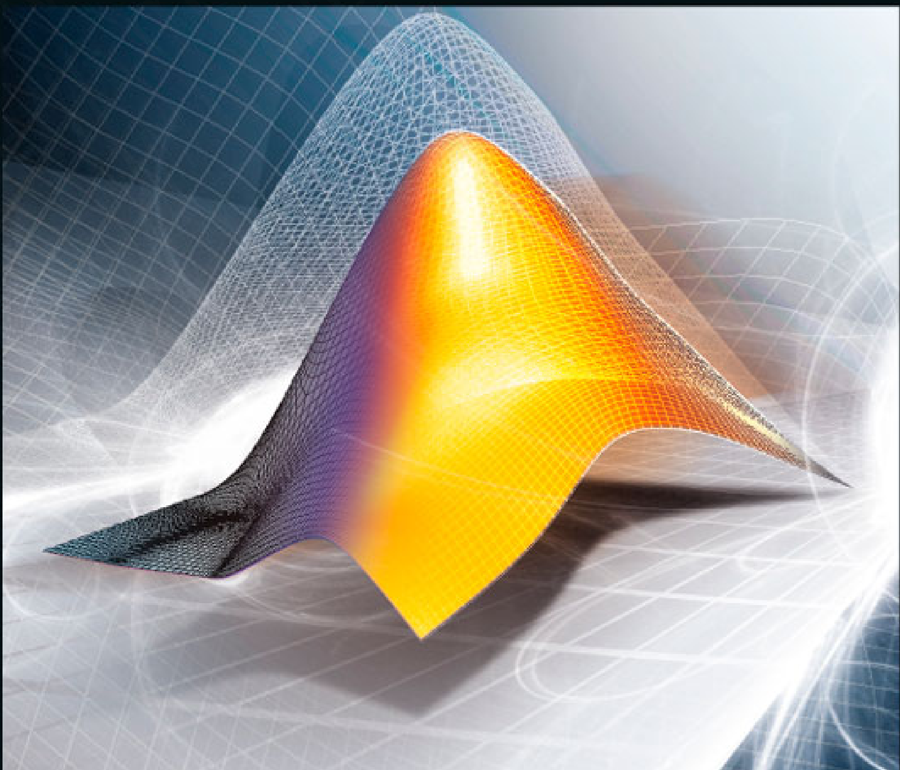

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Ingeniero mecánico egresado de la Universidad Nacional de Trujillo. Diplomado en *Computer Adding Design and Computer Adding Engineering* CAD-CAE-UCV, especialización en Análisis Vibracional en Máquinas y Estructuras Mecánicas con Modelamiento en Elementos Finitos en Diseño Mecánico. Cuenta con Maestría en Ingeniería Mecánica Eléctrica con mención en Energía (Convenio UNPRG-CARELEC). Docente universitario a tiempo completo en la Universidad Católica Santo Toribio de Mogrovejo, Escuela de Ingeniería Mecánica Eléctrica. Consultor del Área de Diseño de la empresa CAD-CAE Ingenieros y consultor en Eficiencia Energética bajo la Norma ISO 50001. Instructor de MATLAB y *Solidworks Simulation* en estudio estático, frecuencial y térmico; así como en *Computational Fluids Dynamics* (CFD).

Líneas de investigación

- Aplicación del método de los elementos finitos a la Ingeniería en Diseño de máquinas.
- Proyectos, planificación, gestión en energías renovables.
- Diseño y dimensionamiento de sistemas eólicos *onshore* y *offshore*.

Agradecimientos

A mi gran amigo, Ciro Bazán, un excelente profesional por permitirme hacer uso de sus apuntes de clase.

A Julia Maturana, una gran amiga, por los infinitos consejos brindados en los momentos exactos.

Dedicatoria

A mi madre, Violeta Lázaro Soriano, por haberme dado la vida, por cuidarme de niño, por apoyarme en la adolescencia y formar en mí esa fortaleza moral para soportar los malos momentos que la vida nos pone a prueba; solo lo puedo resumir en una frase: «Te amo mamá».

Introducción

El presente libro titulado Cálculo diferencial (con MATLAB) constituye un aporte para aproximar al estudiante al uso de uno de los software más empleados en la actualidad por las mejores universidades a nivel mundial; así como por la mayoría de empresas para optimizar sus procesos y ahorrar tiempo para diseño, manufactura o ventas de sus productos. Tiene como objetivo lograr que el estudiante pueda tener una opción adicional para comprobar los ejercicios que resuelva y comprometerse al uso del software más usado.

El texto está dividido en cuatro capítulos: Cálculo de límites, cálculo de derivadas, aplicaciones de los límites y aplicaciones de las derivadas. Respecto a la metodología, se presenta la resolución de los problemas realizados en forma convencional y luego está desarrollado sobre la base del uso de los comandos que MATLAB brinda. Se ha tratado de abarcar la mayor cantidad de ejercicios con diversos grados de dificultad.

Por otro lado, como se sabe, la matemática es uno de los pilares principales de todas las carreras de ciencia e ingeniería. A través del tiempo, muchas técnicas se aplican para resolver problemas referentes a límites y derivadas en base a las leyes y principios matemáticos los cuales le permitan al estudiante tener una base sólida para afrontar los demás cursos en donde las integrales y las ecuaciones diferenciales van a ser una poderosa arma para entender e interpretar las diferentes situaciones o fenómenos relacionados con los cursos de su carrera profesional..

El autor

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CAPÍTULO

1

CÁLCULO DE LÍMITES

1.1. LÍMITES POR SUSTITUCIÓN DIRECTA

EJEMPLO 1

$$\lim_{x\to 2} \frac{x}{4}+2=\frac{2}{4}+2=\frac{10}{4}=\frac{5}{2}$$

Con MATLAB

```
>> syms x
>> limit(x/4+2,x,2)

ans =

5/2

>> pretty(ans)

  5
  -
  2
```

EJEMPLO 2

$$\lim_{x\to -2} \frac{x^2-4}{x^2+4}=\frac{4-4}{4+4}=\frac{0}{8}=0$$

Con MATLAB

```
>> syms x
>> limit((x^2-4)/(x^2-4),x,2)

ans =

0
```

EJEMPLO 3

$$\lim_{x\to -2} (5x+7)^4=\left[5(-2)+7\right]^4=81$$

Con MATLAB

```
>> syms x
>> limit((5*x+7)^4,x,-2)

ans =

81
```

EJEMPLO 4

$$\lim_{x\to 4} \sqrt{25-x^2} = \sqrt{25-(4)^2} = \sqrt{25-16} = \sqrt{9} = 3$$

Con MATLAB

```
>> syms x
>> limit(sqrt(25-x^2),x,4)

ans =

3
```

EJEMPLO 5

$$\lim_{x\to 4} \sqrt[3]{\frac{x}{-7x+1}} = \sqrt[3]{\frac{4}{-7(4)+1}} = \sqrt[3]{-\frac{4}{27}} = -\frac{\sqrt[3]{4}}{3}$$

Con MATLAB

```
>> syms x
>> limit((x/(-7*x+1))^(1/3),x,4)

ans =

((-1)^(1/3)*4^(1/3))/3

>> pretty(ans)|
      1  1
      -  -
      3  3
  (-1)  4
  --------
     3
```

EJEMPLO 6

$$\lim_{x\to -c} \sqrt{x^2+c^2} = \sqrt{(-c)^2+c^2} = \sqrt{2c^2} = \sqrt{2}c$$

Con MATLAB

```
>> syms x c
>> limit(sqrt(x^2+c^2),x,-c)

ans =

(2*c^2)^(1/2)

>> pretty(ans)

      2 1/2
  (2 c )
```

Ejemplo 7

$$\lim_{x\to 2}\sqrt{\frac{x^3+2x+3}{x^2+5}}=\sqrt{\frac{2^3+2\,(2)+3}{2^2+5}}=\sqrt{\frac{15}{9}}=\frac{\sqrt{15}}{3}$$

Con MATLAB

```
>> syms x
>> limit(sqrt(x^3+2*x+3)/(x^2+5,x,2)

ans =

15^(1/2)/9

>> pretty(ans)

    1/2
  15
  -----
    9
```

Ejemplo 8

$$\lim_{r\to 1}\sqrt{\frac{8r+1}{r+5}}=\sqrt{\frac{8\,(\,1)\;+1}{1+5}}=\sqrt{\frac{9}{6}}=\frac{3}{\sqrt{6}}=\frac{\sqrt{6}}{2}$$

Con MATLAB

```
>> syms x
>> limit(sqrt((8*r+1)/(r+5)),r,1)

ans =

(2^(1/2)*3^(1/2))/2

>> pretty(ans)

   1/2  1/2
  2    3
  ---------
      2
```

1.2. LÍMITES APLICANDO FACTORIZACIÓN Y SIMPLIFICACIÓN

Ejemplo 1

$$\lim_{x\to -2}\frac{x^2-x-6}{x^2-4}=\lim_{x\to -2}\frac{(x+2)(x-3)}{(x+2)(x-2)}=\lim_{x\to -2}\frac{(x-3)}{(x-2)}=\frac{-2-3}{-2-2}=\frac{5}{4}$$

Con MATLAB

```
>> syms x
>> limit((x^2-x-6/(x^2-4,x,-2)

ans =

5/4
```

Ejemplo 2

$$\lim_{x\to 4}\frac{x^2-x-12}{x-4}=\lim_{x\to 4}\frac{(x-4)(x+3)}{(x-4)}=\lim_{x\to 4}(x+3)=4+3=7$$

Con MATLAB

```
>> syms x
>> limit((x^2-x-12)/(x-4),x,4)

ans =

7
```

Ejemplo 3

$$\lim_{x\to 1}\frac{x^2-3x+2}{x^2-4x+3}=\lim_{x\to 1}\frac{(x-1)(x-2)}{(x-1)(x-3)}=\lim_{x\to 1}\frac{x-2}{x-3}=\frac{-1}{-2}=0{,}5$$

Con MATLAB

```
>> syms x
>> limit((x^2-3*x+2)/(x^2-4*x+3),x,1)

ans =

1/2
```

EJEMPLO 4

$$\lim_{x\to -3} \frac{x^3+27}{x+3} = \lim_{x\to -3} \frac{\left(x^2-3x+9\right)\left(x+3\right)}{\left(x+3\right)} = \lim_{x\to -3} \left(x^2-3x+9\right) = 27$$

Con MATLAB

```
>> syms x
>> limit((x^3+27)/(x+3),x,-3)

ans =

27
```

EJEMPLO 5

$$\lim_{x\to -2} \frac{x^3-x^2-x+10}{x^2+3x+2} = \lim_{x\to -2} \frac{\left(x+2\right)\left(x^2-3x+5\right)}{\left(x+2\right)\left(x+1\right)} = \lim_{x\to -2} \frac{\left(x^2-3x+5\right)}{\left(x+1\right)} = \frac{\left(-2\right)^2-3\left(-2\right)+5}{\left(-2+1\right)} = \frac{15}{-1} = -15$$

Con MATLAB

```
>> syms x
>> limit((x^3-x^2-x+10)/(x^2+3*x+2),x,-2)

ans =

-15
```

EJEMPLO 6

$$\lim_{x\to 0} \sqrt{\frac{6x^3-x^2-2x}{2x^5+x^4-2x^2-x}} = \lim_{x\to 0} \sqrt{\frac{\left(2x+1\right)\left(3x-2\right)x}{x\left(x^3-1\right)\left(2x+1\right)}} = \lim_{x\to 0} \sqrt{\frac{\left(3x-2\right)}{\left(x^3-1\right)}} = \sqrt{\frac{-2}{-1}} = \sqrt{2}$$

Con MATLAB

```
>> syms x
>> limit(sqrt((6*x^3-x^2-2*x)/2*x^5+x^4-2*x^2-x)),x,0)

ans =

2^(1/2)

>> pretty(ans)

   1/2
  2
```

EJEMPLO 7

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}=\lim_{x\to 4}\frac{\left(\sqrt{x}-2\right)}{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)}=\lim_{x\to 4}\frac{1}{\left(\sqrt{x}+2\right)}=\frac{1}{\sqrt{4}+2}=\frac{1}{4}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x)-2/(x-4,x,4)

ans =

1/4

>> pretty(ans)

  1
  -
  4
```

EJEMPLO 8

$$\lim_{f\to -1}\left[\sqrt[3]{\frac{2f^2-f-3}{f^3+2f^2+6f+5}}\right]^{-1}=\lim_{f\to -1}\left[\sqrt[3]{\frac{(f+1)\ (2f-3)}{(f+1)\ (f^2+f+5)}}\right]^{-1}=\lim_{f\to -1}\sqrt[3]{\frac{(f^2+f+5)}{(2f-3)}}=\sqrt[3]{\frac{1-1+5}{-2-3}}=\sqrt[3]{-1}=-1$$

Con MATLAB

```
>> syms f
>> limit((((2*f^2-f-3)/(f^3+2*f^2+6*f+5))^1/3)^-1),f,-1

ans =

-5

f =

f

ans =

    -1
```

EJEMPLO 9

$$\lim_{x\to1}\frac{(x^n-1)}{(x-1)}=\lim_{x\to1}\frac{(x-1)\left(x^{n-1}+x^{n-2}+x^{n-3}+.....+x^3+x^2+x+1\right)}{(x-1)}$$

$$=\lim_{x\to1}\left(\underbrace{x^{n-1}+x^{n-2}+x^{n-3}+....+x^3+x^2+x}_{\text{"n–1" veces}}+1\right)=\underbrace{(1+1+1+....+1+1+1+1)}_{\text{"n" veces}}$$

$$=n(1)=n \qquad \text{donde: } n\in\mathbb{N}$$

Con MATLAB

```
>> syms x n
>> limit((x^n-1)/(x-1),x,1)

ans =

n
```

EJEMPLO 10

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to1}\frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}=\lim_{x\to1}\frac{1}{(\sqrt{x}+1)}=\frac{1}{(\sqrt{1}+1)}=\frac{1}{2}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x)-1/(x-1),x,1)

ans =

1/2

>> pretty(ans)

  1
  -
  2
```

EJEMPLO 11

$$\lim_{x\to9}\frac{3-\sqrt{x}}{9-x}=\lim_{x\to9}\frac{(3-\sqrt{x})}{(3+\sqrt{x})\,(3-\sqrt{x})}=\lim_{x\to9}\frac{1}{(3+\sqrt{x})}=\frac{1}{(3+\sqrt{9})}=\frac{1}{6}$$

Con MATLAB

```
>> syms x
>> limit((3-sqrt(x))/(9-x),x,9)

ans =

1/6
```

EJEMPLO 12

$$\lim_{x\to 2} \frac{x-2}{2|x-2|}$$

Sabemos que: $|x-2| = \begin{cases} (x-2) & , \text{ si } x-2 \geq 0 \Rightarrow x \geq 2 \\ -(x-2) & , \text{ si } x-2 < 0 \Rightarrow x < 2 \end{cases}$

$$\lim_{x\to 2^+} \left[\frac{(x-2)}{2\,(x-2)}\right] = \lim_{x\to 2^+}\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$\lim_{x\to 2^-} \left\{\frac{(x-2)}{2\left[-\,(x-2)\right]}\right\} = \lim_{x\to 2^-}\left(\frac{1}{-2}\right) = -\frac{1}{2}$$

Con MATLAB

```
>> syms x
>> limit((x-2)/(2*abs(x-2)),x,2)

ans =

NaN
```

Nota

En MATLAB la expresión "NaN" significa no existe, en nuestro caso implica que el límite no existe puesto que los valores obtenidos con los límites laterales no coinciden como se aprecia en la resolución "manual".

EJEMPLO 13

$$\lim_{x\to 1} \frac{x^6-1}{x^3-1} = \lim_{x\to 1} \frac{(x-1)(x^5+x^4+x^3+x^2+x+1)}{(x-1)(x^2+x+1)}$$

$$= \lim_{x\to 1} \frac{(x^5+x^4+x^3+x^2+x+1)}{(x^2+x+1)} = \lim_{x\to 1} \frac{(1^5+1^4+1^3+1^2+1+1)}{(1^2+1+1)} = \frac{6}{3} = 2$$

Con MATLAB

```
>> syms x
>> limit((x^6-1)/(x^3-1),x,1)

ans =

2
```

1.3. LÍMITES APLICANDO RACIONALIZACIÓN

Ejemplo 1

$$\lim_{x\to4}\frac{\sqrt{x}-2}{x-4}=\lim_{x\to4}\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}=\lim_{x\to4}\frac{(x-4)}{(x-4)\left(\sqrt{x}+2\right)}=\lim_{x\to4}\frac{1}{\left(\sqrt{x}+2\right)}=\frac{1}{\left(\sqrt{4}+2\right)}=\frac{1}{2+2}=\frac{1}{4}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x)-2/(x-4),x,4)

ans =

1/4
```

Ejemplo 2

$$\lim_{x\to1}\frac{(1-x)}{\sqrt{5-x^2}-2}=\lim_{x\to1}\frac{(1-x)}{\sqrt{5-x^2}-2}\cdot\frac{\sqrt{5-x^2}+2}{\sqrt{5-x^2}+2}$$

$$=\lim_{x\to1}\frac{(1-x)\left(\sqrt{5-x^2}+2\right)}{\left(5-x^2-4\right)}=\lim_{x\to1}\frac{(1-x)\left(\sqrt{5-x^2}+2\right)}{\left(1-x^2\right)}$$

$$=\lim_{x\to1}\frac{(1-x)\left(\sqrt{5-x^2}+2\right)}{(1+x)(1-x)}=\lim_{x\to1}\frac{\left(\sqrt{5-x^2}+2\right)}{(1+x)}=\frac{4}{2}=2$$

Con MATLAB

```
>> syms x
>> limit((1-x)/((sqrt(5-x^2)-2)),x,1)

ans =

2
```

Ejemplo 3

$$\lim_{x\to 1}\frac{(x-1)}{\sqrt{x^2+3}-2}=\lim_{x\to 1}\frac{(x-1)}{\sqrt{x^2+3}-2}\cdot\frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}$$

$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2+3-4\right)}=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2-1\right)}$$

$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{(x+1)(x-1)}=\lim_{x\to 1}\frac{\sqrt{x^2+3}+2}{x+1}=\frac{\sqrt{4}+2}{2}=\frac{2+2}{2}=2$$

Con MATLAB

```
>> syms x
>> limit((1-x)/(sqrt(x^2+3)-2),x,1)

ans =

2
```

Ejemplo 4

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}=\lim_{x\to 0}\left(\frac{\sqrt{x+1}-1}{x}\right)\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)=\lim_{x\to 0}\frac{x+1-1}{x\left(\sqrt{x+1}+1\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\left(\sqrt{x+1}+1\right)}=\lim_{x\to 0}\frac{1}{\left(\sqrt{x+1}+1\right)}=\frac{1}{2}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x+1)-1)/x,x,0)

ans =

1/2
```

Ejemplo 5

$$\lim_{h\to a}\frac{\sqrt{h}-\sqrt{a}}{h-a}=\lim_{h\to a}\frac{\sqrt{h}-\sqrt{a}}{h-a}\cdot\frac{\sqrt{h}+\sqrt{a}}{\sqrt{h}+\sqrt{a}}$$

$$=\lim_{h\to a}\frac{(h-a)}{(h-a)\left(\sqrt{h}+\sqrt{a}\right)}=\lim_{h\to a}\frac{1}{\left(\sqrt{h}+\sqrt{a}\right)}=\frac{1}{2\sqrt{a}}$$

Con MATLAB

```
>> syms h a
>> limit((sqrt(h)-sqrt(a))/(h-a),h,a)

ans =

1/(2*a^(1/2))

>> pretty(ans)

     1
  ---------
      1/2
  2 a
```

Ejemplo 6

$$\lim_{x\to 2}\frac{\left(4-x^2\right)}{3-\sqrt{x^2+5}}=\lim_{x\to 2}\frac{4-x^2}{\left(3-\sqrt{x^2+5}\right)}\frac{\left(3+\sqrt{x^2+5}\right)}{\left(3+\sqrt{x^2+5}\right)}$$

$$=\lim_{x\to 2}\frac{\left(4-x^2\right)\left(3+\sqrt{x^2+5}\right)}{\left(9-x^2-5\right)}=\lim_{x\to 2}\frac{\left(4-x^2\right)\left(3+\sqrt{x^2+5}\right)}{\left(4-x^2\right)}$$

$$=\lim_{x\to 2}\left(3+\sqrt{x^2+5}\right)=3+\sqrt{9}=6$$

Con MATLAB

```
>> syms x
>> limit((4-x^2)/(3-(sqrt(x^2+5))),x,2)

ans =

6
```

Ejemplo 7

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}=\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\left(\sqrt{x+1}+2\right)}{\left(\sqrt{x+1}+2\right)}=\lim_{x\to 3}\frac{(x+1-4)}{(x-3)\left(\sqrt{x+1}+2\right)}$$

$$=\lim_{x\to 3}\frac{(x-3)}{(x-3)\left(\sqrt{x+1}+2\right)}=\lim_{x\to 3}\frac{1}{\left(\sqrt{x+1}+2\right)}=\frac{1}{\sqrt{4}+2}=\frac{1}{4}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x+1)-2)/(x-3),x,3)

ans =

1/4
```

Ejemplo 8

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}=\lim_{x\to 9}\frac{(x-9)}{\left(\sqrt{x}-3\right)}\cdot\frac{\left(\sqrt{x}+3\right)}{\left(\sqrt{x}+3\right)}=\lim_{x\to 9}\frac{(x-9)\left(\sqrt{x}+3\right)}{(x-9)}=\lim_{x\to 9}\left(\sqrt{x}+3\right)=6$$

Con MATLAB

```
>> syms x
>> limit((x-9)/(sqrt(x)-3),x,9)

ans =

6
```

Ejemplo 9

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}=\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}\cdot\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}=\lim_{x\to 0}\frac{(x+4-4)}{x\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\left(\sqrt{x+4}+2\right)}=\lim_{x\to 0}\frac{1}{\sqrt{x+4}+2}=\frac{1}{4}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x+4)-2),x,x,0)

ans =

1/4
```

EJEMPLO 10

$$\lim_{x\to 3}\frac{\sqrt{2x+3}-x}{x-3}=\lim_{x\to 3}\frac{\sqrt{2x+3}-x}{x-3}\cdot\frac{\sqrt{2x+3}+x}{\sqrt{2x+3}+x}$$

$$=\lim_{x\to 3}\frac{\left(2x+3-x^2\right)}{(x-3)\left(\sqrt{2x+3}+x\right)}=\lim_{x\to 3}\frac{(x-3)(-x-1)}{(x-3)\left(\sqrt{2x+3}+x\right)}$$

$$=\lim_{x\to 3}\frac{(-x-1)}{\left(\sqrt{2x+3}+x\right)}=\frac{-4}{6}=-\frac{2}{3}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(2*x+3)-x)/(x-3),x,3)

ans =

-2/3
```

EJEMPLO 11

$$\lim_{x\to -1}\frac{\sqrt{x^2+3}-2}{x+1}=\lim_{x\to -1}\frac{\sqrt{x^2+3}-2}{x+1}\cdot\frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}$$

$$=\lim_{x\to -1}\frac{\left(x^2+3-4\right)}{(x+1)\left(\sqrt{x^2+3}+2\right)}=\lim_{x\to -1}\frac{\left(x^2-1\right)}{(x+1)\left(\sqrt{x^2+3}+2\right)}$$

$$=\lim_{x\to -1}\frac{(x+1)\,(x-1)}{(x+1)\left(\sqrt{x^2+3}+2\right)}=\lim_{x\to -1}\frac{(x-1)}{\left(\sqrt{x^2+3}+2\right)}=\frac{-2}{4}=-\frac{1}{2}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x^2+3)-2)/(x+1),x,-1)

ans =

-1/2
```

Ejemplo 12

$$\lim_{x\to -3} \frac{(x+3)}{\sqrt{x^2+7}-4} = \lim_{x\to -3}\left(\frac{x+3}{\sqrt{x^2+7}-4}\right)\left(\frac{\sqrt{x^2+7}+4}{\sqrt{x^2+7}+4}\right)$$

$$= \lim_{x\to -3} \frac{(x+3)\left(\sqrt{x^2+7}+4\right)}{\left(x^2+7-16\right)} = \lim_{x\to -3} \frac{(x+3)\left(\sqrt{x^2+7}+4\right)}{\left(x^2-9\right)}$$

$$= \lim_{x\to -3} \frac{(x+3)\left(\sqrt{x^2+7}+4\right)}{(x-3)(x+3)} = \lim_{x\to -3} \frac{\left(\sqrt{x^2+7}+4\right)}{(x-3)} = \frac{8}{-6} = -\frac{4}{3}$$

Con MATLAB

```
>> syms x
>> limit((x+3)/(sqrt(x^2+7)-4),x,-3)

ans =

-4/3
```

Ejemplo 13

$$\lim_{x\to 4} \frac{x-4}{3\left(\sqrt{x}-2\right)} = \lim_{x\to 4}\left(\frac{x-4}{3\left(\sqrt{x}-2\right)}\right)\left(\frac{\sqrt{x}+2}{\sqrt{x}+2}\right) = \lim_{x\to 4} \frac{(x-4)\left(\sqrt{x}+2\right)}{3(x-4)} = \lim_{x\to 4} \frac{\sqrt{x}+2}{3} = \frac{4}{3}$$

Con MATLAB

```
>> syms x
>> limit((x-4)/3*(sqrt(x)-2)),x,4)

ans =

4/3
```

Ejemplo 14

$$\lim_{x\to 5}\frac{\sqrt{x-4}-\sqrt{3x-14}}{x-5}=\lim_{x\to 5}\left(\frac{\sqrt{x-4}-\sqrt{3x-14}}{x-5}\right)\left(\frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x-4}+\sqrt{3x-14}}\right)$$

$$=\lim_{x\to 5}\frac{x-4-3x+14}{(x-5)\left(\sqrt{x-4}+\sqrt{3x-14}\right)}=\lim_{x\to 5}\frac{10-2x}{(x-5)\left(\sqrt{x-4}+\sqrt{3x-14}\right)}$$

$$=\lim_{x\to 5}\frac{-2(x-5)}{(x-5)\left(\sqrt{x-4}+\sqrt{3x-14}\right)}=\lim_{x\to 5}\frac{-2}{\left(\sqrt{x-4}+\sqrt{3x-14}\right)}=\frac{-2}{\sqrt{1}+\sqrt{1}}=-1$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x-4)-sqrt(3*x-14))/(x-5),x,5)

ans =

-1
```

Ejemplo 15

$$\lim_{x\to 3}\left(\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}\right)$$

$$=\lim_{x\to 3}\frac{\left(\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}\right)}{\left(x^2-4x+3\right)}\cdot\frac{\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}{\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$=\lim_{x\to 3}\frac{\left(x^2-2x+6\right)-\left(x^2+2x-6\right)}{(x-3)(x-1)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$=\lim_{x\to 3}\frac{-4x+12}{(x-1)\,(x-3)\,\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$=\lim_{x\to 3}\frac{-4(x-3)}{(x-1)\,(x-3)\,\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$=\lim_{x\to 3}\frac{-4}{(x-1)\,\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}=\frac{-4}{(3-1)\,\left(\sqrt{9}+\sqrt{9}\right)}=-\frac{1}{3}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x^2-2*x+6)-sqrt(x^2+2*x-6))/(x^2-4*x+3),x,3)

ans =

-1/3
```

Ejemplo 16

$$\lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)=\lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$=\lim_{h\to 0}\frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

Con MATLAB

```
>> syms x h
>> limit((sqrt(x+h)-(sqrt(x)))/(h,h,0)

ans =

1/(2*x^(1/2))

>> pretty(ans)

     1
   ------
      1/2
   2 x
```

Ejemplo 17

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{\sqrt{x^2-x-2}-2}=\lim_{x\to 3}\left(\frac{\sqrt{x+1}-2}{\sqrt{x^2-x-2}-2}\right)\left(\frac{\sqrt{x^2-x-2}+2}{\sqrt{x^2-x-2}+2}\right)$$

$$=\lim_{x\to 3}\frac{\left(\sqrt{x+1}-2\right)\left(\sqrt{x^2-x-2}+2\right)}{x^2-x-2-4}=\lim_{x\to 3}\frac{\left(\sqrt{x+1}-2\right)\left(\sqrt{x^2-x-2}+2\right)}{x^2-x-6}\cdot\frac{\left(\sqrt{x+1}+2\right)}{\left(\sqrt{x+1}+2\right)}$$

$$=\lim_{x\to 3}\frac{(x+1-4)\left(\sqrt{x^2-x-2}+2\right)}{\left(x^2-x-6\right)\left(\sqrt{x+1}+2\right)}=\lim_{x\to 3}\frac{(x-3)\left(\sqrt{x^2-x-2}+2\right)}{(x-3)(x+2)\left(\sqrt{x+1}+2\right)}$$

$$=\lim_{x\to 3}\frac{\left(\sqrt{x^2-x-2}+2\right)}{(x+2)\left(\sqrt{x+1}+2\right)}=\frac{\sqrt{3^2-3-2}+2}{(3+2)\left(\sqrt{3+1}+2\right)}=\frac{\sqrt{4}+2}{5\left(\sqrt{4}+2\right)}=\frac{4}{20}=\frac{1}{5}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x+1)-2)/(sqrt(x^2-x-2)-2),x,3)

ans =

1/5
```

Ejemplo 18

$$\lim_{x\to 0}\left(\frac{\sqrt{x+1}-1}{\sqrt{4+x}-2}\right)=\lim_{x\to 0}\left[\left(\frac{\sqrt{x+1}-1}{\sqrt{4+x}-2}\right)\left(\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right)\right]$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{x+1}-1\right)\left(\sqrt{4+x}+2\right)}{4+x-4}=\lim_{x\to 0}\frac{\left(\sqrt{x+1}-1\right)\left(\sqrt{4+x}+2\right)}{x}\cdot\frac{\left(\sqrt{x+1}+1\right)}{\left(\sqrt{x+1}+1\right)}$$

$$=\lim_{x\to 0}\frac{(x+1-1)\left(\sqrt{4+x}+2\right)}{x\left(\sqrt{x+1}+1\right)}=\lim_{x\to 0}\frac{x\left(\sqrt{4+x}+2\right)}{x\left(\sqrt{x+1}+1\right)}=\lim_{x\to 0}\frac{\left(\sqrt{4+x}+2\right)}{\left(\sqrt{x+1}+1\right)}=\frac{\sqrt{4}+2}{\sqrt{1}+1}=2$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x+1)-1)/(sqrt(4+x)-2),x,0)

ans =

2
```

1.4. LÍMITES APLICANDO FACTOR RACIONALIANTE (FR)

Debido a que en el cálculo de límites, en muchos casos, se debe racionalizar, es bueno tener presente que:

a) $\left(A^n-b^n\right)=\left(A-b\right)\underbrace{\left(A^{n-1}+A^{n-2}b+A^{n-3}b^2+\ldots\ldots\ldots+b^{n-1}\right)}_{FR(n\ \text{sumandos})}$

b) $\left(A^n+b^n\right)=\left(A+b\right)\underbrace{\left(A^{n-1}-A^{n-2}b+A^{n-3}b^2-\ldots\ldots\ldots+b^{n-1}\right)}_{FR(n\ \text{sumandos})}$

Para el caso b) debe verificarse que “n” sea impar.

Ejemplo 1

$$\lim_{x\to 2}\left(\frac{x^2-2x}{\sqrt[3]{x^2+2x}-2}\right)$$

Vamos a racionalizar el denominador, multiplicando y dividiendo toda la expresión a la cuál le vamos a calcular el límite por el factor racionalizante “FR”: de $\sqrt[3]{x^2+2x}-2$.

Para calcular dicho “FR” haremos:

$$\left.\begin{cases} A=\sqrt[3]{x^2+2x} & \Rightarrow\quad A^3=\left(x^2+2x\right)=A^n \\ b=2 & \Rightarrow\quad b^3=2^3=8=b^n \end{cases}\right\}\quad \therefore n=3$$

Por lo tanto, el “FR” tendrá “3” sumandos:

$$FR = A^{3-1} + A^{3-2}b + A^{3-3}b^2$$

$$FR = A^2 + Ab + A^0 b^2$$

$$FR = A^2 + Ab + b^2$$

Reemplazando “A” y “b” en “FR”:

$$FR = \left(\sqrt[3]{x^2+2x}\right)^2 + \left(\sqrt[3]{x^2+2x}\right)(2) + (2)^2$$

$$FR = \sqrt[3]{\left(x^2+2x\right)^2} + 2\sqrt[3]{x^2+2x} + 4$$

Multiplicando y dividiendo al límite por “FR”, tenemos:

$$= \lim_{x\to 2} \frac{\left(x^2-2x\right)}{\left(\sqrt[3]{x^2+2x}-2\right)} \cdot \frac{\left(\sqrt[3]{\left(x^2+2x\right)^2} + 2\sqrt[3]{x^2+2x}+4\right)}{\left(\sqrt[3]{\left(x^2+2x\right)^2} + 2\sqrt[3]{x^2+2x}+4\right)}$$

Además sabemos que:

$$A^n - b^n = (A-b)(FR)$$

Por lo que si reemplazamos en la expresión anterior “An”, “bn”, “A”, “b”, y “FR” tenemos:

$$\left(x^2+2x\right) - 8 = \left(\sqrt[3]{x^2+2x}-2\right)\left(\sqrt[3]{\left(x^2+2x\right)^2} + 2\sqrt[3]{x^2+2x}+4\right)$$

Reemplazamos esta última expresión en el denominador del límite tenemos:

$$= \lim_{x\to 2} \frac{x(x-2)\left(\sqrt[3]{\left(x^2+2x\right)^2} + 2\sqrt[3]{x^2+2x}+4\right)}{x^2+2x-8}$$

$$= \lim_{x\to 2} \frac{x\,(x-2)\left(\sqrt[3]{\left(x^2+2x\right)^2} + 2\sqrt[3]{x^2+2x}+4\right)}{(x-2)\,(x+4)}$$

$$= \lim_{x\to 2} \frac{x\left(\sqrt[3]{(x^2+2x)^2} + 2\sqrt[3]{x^2+2x}+4\right)}{(x+4)} = \frac{2\left(\sqrt[3]{64}+2\sqrt[3]{8}+4\right)}{6} = \frac{2\,(12)}{6} = 4$$

Con MATLAB

```
>> syms x
>> limit((x^2-2*x)/((x^2+2*x)^(1/3)-2),x,2)

ans =

4
```

Ejemplo 2

$$\lim_{x\to 2}\left(\frac{\sqrt[4]{3x^2+4}-2}{\sqrt[5]{4x^2+5x+6}-2}\right)$$

Sean "FR1" y "FR2" los factores racionalizantes:

$$FR1=\sqrt[4]{\left(3x^2+4\right)^3}+2\sqrt[4]{\left(3x^2+4\right)^2}+4\sqrt[4]{\left(3x^2+4\right)^1}+8$$

$$FR2=\left(\sqrt[5]{\left(4x^2+5x+6\right)^4}+2\sqrt[5]{\left(4x^2+5x+6\right)^3}+4\sqrt[5]{\left(4x^2+5x+6\right)^2}+8\sqrt[5]{\left(4x^2+5x+6\right)^1}+16\right)$$

Además sabemos que:

$$\begin{cases}\left(\sqrt[4]{3x^2+4}-2\right)(FR1)=\left(3x^2+4\right)-2^4=3\,(x+2)(x-2)\\ \left(\sqrt[5]{4x^2+5x+6}-2\right)(FR2)=\left(4x^2+5x+6\right)-2^5=(x-2)(4x+13)\end{cases}$$

Por tanto, multiplicando y dividiendo al límite tanto por "FR1" como por "FR2" tenemos:

$$\lim_{x\to 2}\frac{\sqrt[4]{3x^2+4}-2}{\sqrt[5]{4x^2+5x+6}-2}=\lim_{x\to 2}\left(\frac{\sqrt[4]{3x^2+4}-2}{\sqrt[5]{x^2+2x}-2}\right)\left(\frac{FR1}{FR1}\right)\left(\frac{FR2}{FR2}\right)$$

$$=\lim_{x\to 2}\frac{3(x+2)\,(x-2)\,(FR2)}{(x-2)\,(4x+13)\,(FR1)}$$

$$=\lim_{x\to 2}\frac{3(x+2)(FR2)}{(4x+13)(FR1)}=\lim_{x\to 2}\left[\frac{3(x+2)}{(4x+13)}\right]\left[\frac{\lim_{x\to 2}(FR2)}{\lim_{x\to 2}(FR1)}\right]$$

$$=\left[\frac{3(4)}{21}\right]\left[\frac{\lim_{x\to 2}(FR2)}{\lim_{x\to 2}(FR1)}\right]$$

Por otro lado:

$$\begin{cases}\lim_{x\to 2}(FR1)=\sqrt[4]{(16)^3}+2\sqrt[4]{(16)^2}+4\sqrt[4]{(16)}+8=32\\ \lim_{x\to 2}(FR2)=\sqrt[5]{(32)^4}+2\sqrt[5]{(32)^3}+4\sqrt[5]{(32)^2}+8\sqrt[5]{(32)^1}+16=80\end{cases}=\left[\frac{12}{21}\right]\left(\frac{80}{32}\right)=\frac{10}{7}$$

Con MATLAB

```
>> syms x
>> limit(((3*2^2+4)^(1/4)-2)/((4*x^2+5*x+6)^(1/5)-2),x,2)

ans =

10/7
```

1.5. LÍMITES AL INFINITO

EJEMPLO 1

$$\lim_{x\to+\infty}\frac{4x+1}{\sqrt{x^2+1}}=\lim_{x\to+\infty}\frac{4x+1}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}=\lim_{x\to+\infty}\frac{4x+1}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to+\infty}\frac{4x+1}{|x|\sqrt{1+\frac{1}{x^2}}}$$

Ahora como $x\to+\infty$ es una cantidad positiva $\Rightarrow|x|=x$

$$=\lim_{x\to+\infty}\frac{4x+1}{x\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to+\infty}\frac{\frac{4x}{x}+\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to+\infty}\frac{4+\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}}$$

$$=\frac{4+\left(\frac{1}{+\infty}\right)}{\sqrt{1+\left(\frac{1}{+\infty}\right)}}=\frac{4+\left(0^+\right)}{\sqrt{1+\left(0^+\right)}}=\frac{4^+}{\sqrt{1^+}}=\frac{4^+}{1^+}=4^+$$

Con MATLAB

```
>> syms x
>> limit((4*x+1)/(sqrt(x^2+1)),x,inf)

ans =

4
```

EJEMPLO 2

$$\lim_{x\to-\infty}\left(10^x\right)=10^{-\infty}=\frac{1}{10^{+\infty}}=\left(\frac{1}{+\infty}\right)=0^+$$

Con MATLAB

```
>> syms x
>> limit((4*x+1)/(sqrt(x^2+1)),x,inf)

ans =

4

>> syms x
>> limit((10^x),x,-inf)

ans =

0
```

EJEMPLO 3

$$\lim_{x\to+\infty}\left(2x^3-x^2+x-1\right)=\lim_{x\to+\infty}\left[\left(x^2\right)\left(2x-1+\frac{1}{x}-\frac{1}{x^2}\right)\right]$$

$$=\lim_{x\to+\infty}\left\{\left(x^2\right)\left[2x-1+\left(\frac{1}{x}\right)\left(1-\frac{1}{x}\right)\right]\right\}=(+\infty)^2\left\{2\,(+\infty)-1+\left(\frac{1}{+\infty}\right)\left[1-\left(\frac{1}{+\infty}\right)\right]\right\}$$

$$=(+\infty)\left\{(+\infty)+\left(0^+\right)\left[1-\left(0^+\right)\right]\right\}=(+\infty)\left[(+\infty)+\left(0^+\right)\left(1^-\right)\right]$$

$$=(+\infty)\left[(+\infty)+\left(0^+\right)\right]=(+\infty)\,(+\infty)=+\infty$$

Con MATLAB

```
>> syms x
>> limit((4*x+1)/(sqrt(x^2+1)),x,inf)

ans =

4

>> syms x
>> limit((10^x),x,-inf)

ans =

0

>> syms x
>> limit((2*x^3-x^2+x-1),x,inf)

ans =

Inf
```

EJEMPLO 4

$$\lim_{x\to+\infty}\left(\frac{2x^2-x+5}{4x^3-1}\right)=\lim_{x\to+\infty}\frac{\frac{(2x^2-x+5)}{x^3}}{\frac{(4x^3-1)}{x^3}}=\lim_{x\to+\infty}\frac{\frac{2}{x}-\frac{1}{x^2}+\frac{5}{x^3}}{4-\frac{1}{x^3}}$$

$$=\lim_{x\to+\infty}\left[\frac{\left(\frac{1}{x}\right)\left(2-\frac{1}{x}+\frac{5}{x^2}\right)}{4-\left(\frac{1}{x}\right)^3}\right]=\lim_{x\to+\infty}\left[\frac{\left(\frac{1}{x}\right)\left\{2-\left(\frac{1}{x}\right)\left[1-\left(\frac{5}{x}\right)\right]\right\}}{4-\left(\frac{1}{x}\right)^3}\right]$$

$$=\frac{\left(\frac{1}{+\infty}\right)\left\{2-\left(\frac{1}{+\infty}\right)\left[1-\left(\frac{5}{+\infty}\right)\right]\right\}}{4-\left(\frac{1}{+\infty}\right)^3}=\frac{\left(0^+\right)\left\{2-\left(0^+\right)\left[1-\left(0^+\right)\right]\right\}}{4-\left(\frac{1}{+\infty}\right)}$$

$$=\frac{\left(0^+\right)\left[2-\left(0^+\right)\left(1^-\right)\right]}{4-\left(0^+\right)}=\frac{\left(0^+\right)\left(2-0^+\right)}{4^-}=\frac{\left(0^+\right)\left(2^-\right)}{4^-}=\frac{0^+}{4^-}=0^+$$

Con MATLAB

```
>> syms x
>> limit((2*x^2-x+5)/(4*x^3-1),x,inf)

ans =

0
```

Ejemplo 5

$$\lim_{x\to+\infty}\left(\sqrt{x^2+x+5}-x\right)=\lim_{x\to+\infty}\left(\sqrt{x^2+x+5}-x\right)\cdot\frac{\left(\sqrt{x^2+x+5}+x\right)}{\left(\sqrt{x^2+x+5}+x\right)}$$

$$=\lim_{x\to+\infty}\frac{\left(x^2+x+5-x^2\right)}{\left(\sqrt{x^2+x+5}+x\right)}=\lim_{x\to+\infty}\frac{\frac{(x+5)}{x}}{\frac{\sqrt{x^2+x+5}+x}{x}}$$

$$=\lim_{x\to+\infty}\frac{1+\frac{5}{x}}{\sqrt{1+\frac{1}{x}+\frac{5}{x^2}}+1}=\frac{1+\left(\frac{5}{+\infty}\right)}{\sqrt{1+\left(\frac{1}{+\infty}\right)+\frac{5}{(+\infty)^2}}+1}=\frac{1+\left(0^+\right)}{\sqrt{1+\left(0^+\right)+\frac{5}{(+\infty)}}+1}$$

$$=\frac{1^+}{\sqrt{1+\left(0^+\right)+\left(0^+\right)}+1}=\frac{1^+}{\sqrt{1^+}+1}=\frac{1^+}{1^++1}=\frac{1^+}{2^+}=\left(\frac{1}{2}\right)^+=(0,5)^+$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x^2+x+5)-x),x,inf)

ans =

1/2
```

Ejemplo 6

$$\lim_{x\to+\infty}\left(\frac{2x-x^2}{3x+5}\right)=\lim_{x\to+\infty}\frac{\frac{2x-x^2}{x^2}}{\frac{3x+5}{x^2}}=\lim_{x\to+\infty}\frac{\frac{2}{x}-1}{\frac{3}{x}+\frac{5}{x^2}}=\frac{\left(\frac{2}{+\infty}\right)-1}{\left(\frac{3}{+\infty}\right)+\frac{5}{(+\infty)^2}}$$

$$=\frac{\left(0^+\right)-1}{\left(0^+\right)+\frac{5}{(+\infty)}}=\frac{(-1)^+}{\left(0^+\right)+\left(0^+\right)}=\frac{(-1)^+}{\left(0^+\right)}=-\infty$$

Con MATLAB

```
>> syms x
>> limit((2*x-x^2)/(3*x+5),x,inf)

ans =

-Inf
```

EJEMPLO 7

$$\lim_{x\to+\infty}\left(\sqrt{x+a}-\sqrt{x}\right)=\lim_{x\to+\infty}\left(\sqrt{x+a}-\sqrt{x}\right)\cdot\frac{\left(\sqrt{x+a}+\sqrt{x}\right)}{\left(\sqrt{x+a}+\sqrt{x}\right)}$$

$$=\lim_{x\to+\infty}\frac{(x+a-x)}{\left(\sqrt{x+a}+\sqrt{x}\right)}=\lim_{x\to+\infty}\frac{a}{\left(\sqrt{x+a}+\sqrt{x}\right)}=\frac{a}{\left(\sqrt{+\infty+a}+\sqrt{+\infty}\right)}$$

$$=\frac{a}{\left(\sqrt{+\infty}+\sqrt{+\infty}\right)}=\frac{a}{(+\infty+\infty)}=\frac{a}{+\infty}=\begin{cases}0^{+} & \text{si } a>0\\ 0 & \text{si } a=0\\ 0^{-} & \text{si } a<0\end{cases}$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x+a)-sqrt(x)),x,inf)

ans =

0
```

Dependiendo del signo de “a” tal conforme se aprecia en la solución “manual”.

EJEMPLO 8

$$\lim_{a\to+\infty}\left(\sqrt{a^2x^2+\frac{a-b}{2}}-ax\right)$$

$$=\lim_{a\to+\infty}\left(\sqrt{a^2x^2+\frac{a-b}{2}}-ax\right)\cdot\frac{\left(\sqrt{a^2x^2+\frac{a-b}{2}}+ax\right)}{\left(\sqrt{a^2x^2+\frac{a-b}{2}}+ax\right)}$$

$$= \lim_{a\to+\infty} \frac{\left(a^2x^2 + \frac{a-b}{2} - a^2x^2\right)}{\left(\sqrt{a^2x^2 + \frac{a-b}{2}} + ax\right)} = \lim_{a\to+\infty} \frac{\left(\frac{a-b}{2}\right)}{\left(\sqrt{a^2x^2 + \frac{a-b}{2}} + ax\right)}$$

$$= \lim_{a\to+\infty} \frac{\left(\frac{a-b}{2}\right)}{\left(\sqrt{a^2\left(x^2 + \frac{1/a - b/a^2}{2}\right)} + ax\right)} = \lim_{a\to+\infty} \frac{\left(\frac{a-b}{2}\right)}{\left(\sqrt{a^2}\sqrt{x^2 + \frac{1/a - b/a^2}{2}} + ax\right)}$$

Recordemos que: $\sqrt{a^2} = |\,a\,|$, además como $a \to +\infty > 0 \Rightarrow |\,a\,| = a$

$$= \lim_{a\to+\infty} \frac{\left(\frac{a-b}{2}\right)}{\left(|a|\sqrt{x^2 + \frac{1/a - b/a^2}{2}} + ax\right)} = \lim_{a\to+\infty} \frac{\left(\frac{a-b}{2}\right)}{\left(a\sqrt{x^2 + \frac{1/a - b/a^2}{2}} + ax\right)}$$

$$= \lim_{a\to+\infty} \frac{\left(\frac{1-b/a}{2}\right)}{\left(\sqrt{x^2 + \frac{1/a - b/a^2}{2}} + x\right)} = \frac{1/2}{\sqrt{x^2}+x} = \frac{1/2}{|x|+x}$$

$$\begin{cases} \text{si } x \geq 0 \Rightarrow |x| = x \\ \text{si } x < 0 \Rightarrow |x| = -x \end{cases} \Rightarrow \lim_{a\to+\infty}\left(\sqrt{a^2x^2 + \frac{a-b}{2}} - ax\right) = \begin{cases} \frac{1/2}{x+x} = \frac{1/2}{2x} = \frac{1}{4x} \\ \frac{1/2}{-x+x} = \frac{1/2}{0} = \nexists \end{cases}$$

Con MATLAB

```
>> syms a x b
>> limit(((sqrt(a^2*x^2+((a-b)/2)))-a*x),a,inf)

ans =

limit((a^2*x^2 + a/2 - b/2)^(1/2) - a*x, a = Inf)

>> pretty(ans)

     / /  2  2   a   b \1/2               \
limit| | a  x  + - - - |    - a x, a = Inf |
     \ \         2   2 /                  /
```

Ejemplo 9

$$\lim_{x\to+\infty}\left(\frac{x^3+9x^2+20x}{x^2+x-12}\right)=\lim_{x\to+\infty}\frac{x\left(x^2+9x+20\right)}{(x-3)(x+4)}=\lim_{x\to+\infty}\frac{x(x+4)(x+5)}{(x-3)(x+4)}$$

$$=\lim_{x\to+\infty}\frac{x(x+5)}{(x-3)}=\lim_{x\to+\infty}\left(\frac{x^2+5x}{x-3}\right)=\lim_{x\to+\infty}\left(\frac{x^2+5x}{x-3}\right)\cdot\left(\frac{1/x^2}{1/x^2}\right)$$

$$=\lim_{x\to+\infty}\left(\frac{\frac{x^2}{x^2}+\frac{5x}{x^2}}{\frac{x}{x^2}-\frac{3}{x^2}}\right)=\lim_{x\to+\infty}\left(\frac{1+\frac{5}{x}}{\frac{1}{x}-\frac{3}{x^2}}\right)=\lim_{x\to+\infty}\frac{1+\left(\frac{5}{x}\right)}{\left(\frac{1}{x}\right)\left(1-\frac{3}{x}\right)}$$

$$=\frac{1+\left(\frac{5}{+\infty}\right)}{\left(\frac{1}{+\infty}\right)\left[1-\left(\frac{3}{+\infty}\right)\right]}=\frac{1+\left(0^+\right)}{\left(0^+\right)\left[1-\left(0^+\right)\right]}=\frac{1^+}{\left(0^+\right)\left(1^-\right)}=\frac{1^+}{\left(0^+\right)}=+\infty$$

Con MATLAB

```
>> syms x
>> limit((x^3+9*x^2+20*x)/(x^2+x-12),x,inf)

ans =

Inf
```

Ejemplo 10

$$\lim_{x\to+\infty}\left(\frac{5x^3+4x^2-5}{2x^3+2x+5}\right)=\lim_{x\to+\infty}\frac{\left(5x^3+4x^2-5\right)/x^3}{\left(2x^3+2x+5\right)/x^3}$$

$$=\lim_{x\to+\infty}\left(\frac{5+4/x-5/x^3}{2+2/x^2+5/x^3}\right)=\lim_{x\to+\infty}\frac{5+(1/x)\left(4-5/x^2\right)}{2+\left(1/x^2\right)(2+5/x)}$$

$$=\frac{5+(1/+\infty)\left[4-5/(+\infty)^2\right]}{2+\left[1/(+\infty)^2\right]\left[2+(5/+\infty)\right]}=\frac{5+\left(0^+\right)\left[4-5/(+\infty)\right]}{2+\left[1/(+\infty)\right]\left[2+\left(0^+\right)\right]}$$

$$=\frac{5+\left(0^+\right)\left[4-\left(0^+\right)\right]}{2+\left(0^+\right)\left(2^+\right)}=\frac{5+\left(0^+\right)\left(4^-\right)}{2+\left(0^+\right)}=\frac{5+\left(0^+\right)}{2^+}=\frac{5^+}{2^+}=\left(\frac{5}{2}\right)^+=(2{,}5)^+$$

Con MATLAB

```
>> syms x
>> limit((x^3+9*x^2+20*x)/(x^2+x-12),x,inf)

ans =

Inf

>> syms x
>> limit((5*x^3+4*x^2-5)/(2*x^3+2*x+5),x,inf)

ans =

5/2
```

EJEMPLO 11

$$\lim_{x\to+\infty}\left(\frac{3x^3+6x^2-5}{5x^4-2x^3+x+1}\right)=\lim_{x\to+\infty}\frac{\dfrac{\left(3x^3+6x^2-5\right)}{x^4}}{\dfrac{\left(5x^4-2x^3+x+1\right)}{x^4}}$$

$$=\lim_{x\to+\infty}\left(\frac{3/x+6/x^2-5/x^4}{5-2/x+1/x^3+1/x^4}\right)=\lim_{x\to+\infty}\left[\frac{(1/x)\left(3+6/x-5/x^3\right)}{5-(1/x)\left(2-1/x^2-1/x^3\right)}\right]$$

$$=\lim_{x\to+\infty}\frac{(1/x)\left[3+(1/x)\left(6-5/x^2\right)\right]}{5-(1/x)\left[2-\left(1/x^2\right)\left(1+1/x\right)\right]}=\frac{(1/+\infty)\left[3+(1/+\infty)\left\{6-\left[5/(+\infty)^2\right]\right\}\right]}{5-(1/+\infty)\left\{2-\left[1/(+\infty)^2\right]\left[1+(1/+\infty)\right]\right\}}$$

$$=\frac{\left(0^+\right)\left[3+\left(0^+\right)\left\{6-\left[5/(+\infty)\right]\right\}\right]}{5-\left(0^+\right)\left\{2-\left[1/(+\infty)\right]\left[1+\left(0^+\right)\right]\right\}}=\frac{\left(0^+\right)\left\{3+\left(0^+\right)\left[6-\left(0^+\right)\right]\right\}}{5-\left(0^+\right)\left[2-\left(0^+\right)\left(1^+\right)\right]}$$

$$=\frac{\left(0^+\right)\left[3+\left(0^+\right)\left(6^-\right)\right]}{5-\left(0^+\right)\left[2-\left(0^+\right)\right]}=\frac{\left(0^+\right)\left[3+\left(0^+\right)\right]}{5-\left(0^+\right)\left(2^-\right)}=\frac{\left(0^+\right)\left(3^+\right)}{5-\left(0^+\right)}=\frac{\left(0^+\right)}{\left(5^-\right)}=0^+$$

Con MATLAB

```
>> syms x
>> limit((3*x^3+6*x^2-5)/(5*x^4-2*x^3+x+1),x,inf)

ans =

0
```

EJEMPLO 12

$$\lim_{x\to+\infty}\frac{\sqrt{16x^2-3}+3x}{2x}=\lim_{x\to+\infty}\frac{\sqrt{x^2\left(16-3/x^2\right)}+3x}{2x}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x^2}\left(\sqrt{16-3/x^2}\right)+3x}{2x}=\lim_{x\to+\infty}\frac{|x|\sqrt{\left(16-3/x^2\right)}+3x}{2x}$$

Como $x\to+\infty>0\Rightarrow|x|=x$, por lo tanto:

$$=\lim_{x\to+\infty}\frac{x\sqrt{\left(16-3/x^2\right)}+3x}{2x}=\lim_{x\to+\infty}\frac{\sqrt{\left(16-3/x^2\right)}+3}{2}$$

$$\lim_{x\to+\infty}\frac{\sqrt{16-3/(x)^2}+3}{2}=\frac{\sqrt{16-3/(+\infty)^2}+3}{2}=\frac{\sqrt{16-3/(+\infty)}+3}{2}$$

$$=\frac{\sqrt{16-0^+}+3}{2}=\frac{\sqrt{16^-}+3}{2}=\frac{4^-+3}{2}=\frac{7^-}{2}=\left(3,5^-\right)$$

Con MATLAB

```
>> syms x
>> limit((sqrt(16*x^2-3)+3*x)/(2*x),x,inf)

ans =

7/2
```

EJEMPLO 13

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+3}}{4x}=\lim_{x\to-\infty}\frac{\sqrt{x^2\left(1+3/x^2\right)}}{4x}=\lim_{x\to-\infty}\frac{\sqrt{x^2}\sqrt{\left(1+3/x^2\right)}}{4x}=\lim_{x\to-\infty}\frac{|x|\sqrt{\left(1+3/x^2\right)}}{4x}$$

Como $x\to-\infty<0\Rightarrow|x|=-x$, por lo tanto:

$$=\lim_{x\to-\infty}\frac{-x\sqrt{\left(1+3/x^2\right)}}{4x}=\lim_{x\to-\infty}\frac{-\sqrt{\left(1+3/x^2\right)}}{4}$$

$$=\frac{-\sqrt{1+3/(-\infty)^2}}{4}=\frac{-\sqrt{1+3/(+\infty)}}{4}=\frac{-\sqrt{1+0^+}}{4}=\frac{-\sqrt{1^+}}{4}=\frac{-\left(1^+\right)}{4}=\left(-0,25\right)^-$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x^2+3)/(4*x),x,-inf)

ans =

-1/4
```

Ejemplo 14

$$\lim_{x\to-\infty}\left(\frac{4x+5}{3x+\sqrt[3]{x}}\right)=\lim_{x\to-\infty}\left(\frac{4x+5}{3x+\sqrt[3]{x}}\right)\frac{(1/x)}{(1/x)}=\lim_{x\to-\infty}\left(\frac{4+5/x}{3+\sqrt[3]{1/x^2}}\right)$$

$$=\frac{4+(5/-\infty)}{3+\sqrt[3]{1/(-\infty)^2}}=\frac{4+\left(0^-\right)}{3+\sqrt[3]{1/(+\infty)}}=\frac{4+\left(0^-\right)}{3+\sqrt[3]{0^+}}=\frac{4^-}{3+0^+}=\frac{4^-}{3^+}=\left(\frac{4}{3}\right)^-$$

Con MATLAB

```
>> syms x
>> limit((4*x+5)/(3*x+(x^(1/3))),x,-inf)

ans =

4/3
```

```
>> syms x
>> limit((4*x+5)/(3*x+(x^(1/3))),x,-inf,'left')

ans =

4/3
```

```
>> syms x
>> limit((4*x+5)/(3*x+(x^(1/3))),x,-inf,'right')

ans =

4/3
```

Se observa que las soluciones obtenidas con MATLAB coinciden con la solución “manual”.

Ejemplo 15

$$\lim_{x\to+\infty}\frac{\sqrt{4x^3+2x^2-5}-\sqrt[3]{x^4+2x}}{\sqrt[4]{x^6+2x^5+4}+\sqrt[5]{x^7+1}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x^3\left(4+2/x-5/x^3\right)}-\sqrt[3]{x^4\left(1+2/x^3\right)}}{\sqrt[4]{x^6\left(1+2/x+4/x^6\right)}+\sqrt[5]{x^7\left(1+1/x^7\right)}}$$

$$=\lim_{x\to+\infty}\frac{x^{3/2}\sqrt{\left(4+2/x-5/x^3\right)}-x^{4/3}\sqrt[3]{\left(1+2/x^3\right)}}{x^{3/2}\sqrt[4]{\left(1+2/x+4/x^6\right)}+x^{7/5}\sqrt[5]{\left(1+1/x^7\right)}}\cdot\frac{\left(1/x^{3/2}\right)}{\left(1/x^{3/2}\right)}$$

$$=\lim_{x\to+\infty}\left(\frac{\sqrt{\left(4+2/x-5/x^3\right)}-\left(1/x^{1/6}\right)\sqrt[3]{\left(1+2/x^3\right)}}{\sqrt[4]{\left(1+2/x+4/x^6\right)}+\left(1/x^{1/10}\right)\sqrt[5]{\left(1+1/x^7\right)}}\right)$$

$$= \lim_{x\to+\infty} \frac{\sqrt{4+(1/x)\left[2-5/(x)^2\right]}-(1/x^{1/6})\sqrt[3]{(1+2/x^3)}}{\sqrt[4]{(1+2/x+4/x^6)}+(1/x^{1/10})\sqrt[5]{(1+1/x^7)}}$$

$$= \frac{\sqrt{4+(1/+\infty)\left[2-5/(+\infty)^2\right]}-\left[1/(+\infty)\right]\sqrt[3]{1+2/(+\infty)}}{\sqrt[4]{1+(0^+)+4/(+\infty)}+\left[1/(+\infty)\right]\sqrt[5]{1+1/(+\infty)}}$$

$$= \frac{\sqrt{4+(0^+)\left[2-5/(+\infty)\right]}-\left[(0)^+\right]\sqrt[3]{1+(0)^+}}{\sqrt[4]{1+(0^+)+(0)^+}+\left[(0)^+\right]\sqrt[5]{1+(0)^+}} = \frac{\sqrt{4+(0^+)\left[2-(0)^+\right]}-\left[(0)^+\right]\sqrt[3]{1^+}}{\sqrt[4]{1^+}+\left[(0)^+\right]\sqrt[5]{1^+}}$$

$$= \frac{\sqrt{4+(0^+)(2^-)}-\left[(0)^+\right](1^+)}{(1^+)+\left[(0)^+\right](1^+)} = \frac{\sqrt{4+(0^+)}-(0^+)}{(1^+)+(0^+)} = \frac{\sqrt{4^+}-(0^+)}{(1^+)} = \frac{2^+-(0^+)}{(1^+)} = \frac{2^+-(0^+)}{(1^+)} = 2^-$$

Con MATLAB

```
A =

(4*x^3 + 2*x^2 – 5)^(1/2)

>> B=(x^4+2*x)^(1/3)

B =

(x^4 + 2*x)^(1/3)

>> C=(x^6+2*x^5+4)^(1/4)

C =

(x^6 + 2*x^5 + 4)^(1/4)

>> D=(x^7+1)^(1/5)

D =

(x^7 + 1)^(1/5)

>> limit((A–B)/(C+D),x,inf)

ans =

2
```

En este caso se ha preferido individualizar cada término del límite original debido a la complejidad de este en cuatro expresiones: A, B, C, D. De tal forma que se puedan evitar errores y hacerlo más sencillo para MATLAB.

Ejemplo 16

$$\lim_{x\to-\infty} \sqrt[4]{x^2-2x-x^4}$$

Primero calculamos el dominio de la función $\sqrt[4]{x^2-2x-x^4}$:

$$x^2-2x-x^4 \geq 0 \Rightarrow x\left(x-2-x^3\right) \geq 0 \Rightarrow -x\left(x^3-x+2\right) \geq 0$$

$$-x\,(x+1{,}5213797)\,\underbrace{\left(x^2-1{,}5213797\,x+1{,}314596213\right)}_{(+)} \geq 0$$

$$-x\,(x+1{,}5213797) \geq 0 \Rightarrow \text{Dominio:}\quad \{x/x\in\mathbb{R} \wedge 0 \geq x \geq -1{,}5213797\}$$

El límite no existe ya que $x\to-\infty$ y no está incluido en el dominio de la función.

Con MATLAB

```
>> syms x
>> limit(((x^2-2*x-x^4)^(1/4)),x,-inf)

ans =

Inf + i*Inf
```

La solución indica que el cálculo del límite no se encuentra en el campo real, incluso tiene un componente imaginario, razón por la cual coincide con la solución obtenida “manualmente”.

Ejemplo 17

$$\lim_{x\to+\infty} \frac{\sqrt{x\,(x+b)}-x}{x\left(\sqrt{x^2+5}-x\right)} = \lim_{x\to+\infty} \frac{\left(\sqrt{x\,(x+b)}-x\right)\left(\sqrt{x^2+5}+x\right)}{x\left(\sqrt{x^2+5}-x\right)\left(\sqrt{x^2+5}+x\right)}$$

$$= \lim_{x\to+\infty} \frac{\left(\sqrt{x(x+b)}-x\right)\left(\sqrt{x^2+5}+x\right)}{x\left(x^2+5-x^2\right)} = \lim_{x\to+\infty} \frac{\left(\sqrt{x\,(x+b)}-x\right)\left(\sqrt{x^2+5}+x\right)}{5x}$$

$$= \lim_{x\to+\infty} \frac{\left(\sqrt{x(x+b)}-x\right)\left(\sqrt{x^2+5}+x\right)\left(\sqrt{x\,(x+b)}+x\right)}{5\,x\left(\sqrt{x\,(x+b)}+x\right)}$$

$$= \lim_{x\to+\infty} \frac{\left(x^2+bx-x^2\right)\left(\sqrt{x^2+5}+x\right)}{5\,x\left(\sqrt{x(x+b)}+x\right)} = \lim_{x\to+\infty} \frac{b\,x\left(\sqrt{x^2+5}+x\right)}{5\,x\left(\sqrt{x(x+b)}+x\right)}$$

$$= \lim_{x\to+\infty} \frac{b\left(\sqrt{1+5/x^2}+1\right)}{5\left(\sqrt{1+b/x}+1\right)} = \frac{b\left(\sqrt{1+5/(+\infty)^2}+1\right)}{5\left(\sqrt{1+(b/+\infty)}+1\right)} = \frac{b\left(\sqrt{1+5/(+\infty)}+1\right)}{5\left(\sqrt{1+(0^+)}+1\right)}$$

$$= \frac{b\left(\sqrt{1+(0^+)}+1\right)}{5\left(\sqrt{1^+}+1\right)} = \frac{b\left(\sqrt{1^+}+1\right)}{5\left(1^++1\right)} = \frac{b\left(1^++1\right)}{5\left(2^+\right)} = \frac{b\left(2^+\right)}{5\left(2^+\right)} = \left(\frac{b}{5}\right)^+$$

Con MATLAB

```
>> syms x b
>> limit((sqrt(x*(x+b))-x)/(x*(sqrt(x^2+5)-x)),x,inf)

ans =

b/5
```

Ejemplo 18

Hallar el valor de "k" si: $\lim\limits_{x\to+\infty}\left[\dfrac{x^4+kx^3-2}{x^3-x-2}-\sqrt{x^2+3x+5}\right]=\dfrac{1}{2}$

En este caso debido a que el límite es indeterminado se emplea un artificio, que consiste en sumar y restar un valor apropiado de tal forma que la expresión inicial no se altere. Como $\dfrac{x^4+kx^3-2}{x^3-x-2}$ se comporta como "x", se le resta "x". De la misma, manera como $\sqrt{x^2+3x+5}$ se comporta como "x" también se le resta "x".

$$\lim_{x\to+\infty}\left[\left(\frac{x^4+kx^3-2}{x^3-x-2}-x\right)-\left(\sqrt{x^2+3x+5}-x\right)\right]=\frac{1}{2}$$

$$\lim_{x\to+\infty}\left[\left(\frac{x^4+kx^3-2-x^4+x^2+2x}{x^3-x-2}\right)-\frac{\left(\sqrt{x^2+3x+5}-x\right)\left(\sqrt{x^2+3x+5}+x\right)}{\left(\sqrt{x^2+3x+5}+x\right)}\right]=\frac{1}{2}$$

$$\lim_{x\to+\infty}\left[\left(\frac{kx^3-2+x^2+2x}{x^3-x-2}\right)-\frac{\left(x^2+3x+5-x^2\right)}{\left(\sqrt{x^2+3x+5}+x\right)}\right]=\frac{1}{2}$$

$$\lim_{x\to+\infty}\left[\left(\frac{k-\frac{2}{x^3}+\frac{1}{x}+\frac{2}{x^2}}{1-\frac{1}{x^2}-\frac{2}{x^3}}\right)-\frac{(3x+5)}{\left(\sqrt{x^2+3x+5}+x\right)}\right]=\frac{1}{2}$$

$$\lim_{x\to+\infty}\left[\left(\frac{k-\left(\frac{1}{x}\right)\left[\frac{2}{(x)^2}-1-\left(\frac{2}{x}\right)\right]}{1-\left(\frac{1}{x}\right)\left[\left(\frac{1}{x}\right)+\left(\frac{2}{x^2}\right)\right]}\right)-\frac{x\left(3+\frac{5}{x}\right)}{\left(\sqrt{\left(x^2\right)\left(1+\frac{3}{x}+\frac{5}{x^2}\right)}+x\right)}\right]=\frac{1}{2}$$

$$\lim_{x\to+\infty}\left[\left(\frac{k-\left(\frac{1}{x}\right)\left[\frac{2}{(x)^2}-1-\left(\frac{2}{x}\right)\right]}{1-\left(\frac{1}{x}\right)\left[\left(\frac{1}{x}\right)+\left(\frac{2}{x^2}\right)\right]}\right)-\frac{x\left(3+\frac{5}{x}\right)}{\left(\sqrt{x^2}\sqrt{\left[1+\left(\frac{1}{x}\right)\left(3+\frac{5}{x}\right)\right]}+x\right)}\right]=\frac{1}{2}$$

$$\lim_{x\to+\infty}\left[\left(\frac{k-\left(\frac{1}{x}\right)\left[\frac{2}{(x)^2}-1-\left(\frac{2}{x}\right)\right]}{1-\left(\frac{1}{x}\right)\left[\left(\frac{1}{x}\right)+\left(\frac{2}{x^2}\right)\right]}\right)-\frac{x\left(3+\frac{5}{x}\right)}{\left(|x|\sqrt{\left[1+\left(\frac{1}{x}\right)\left(3+\frac{5}{x}\right)\right]}+x\right)}\right]=\frac{1}{2}$$

Como $x\to+\infty>0\Rightarrow|x|=x$, entonces:

$$\lim_{x\to+\infty}\left[\left(\frac{k-\left(\frac{1}{x}\right)\left[\frac{2}{(x)^2}-1-\left(\frac{2}{x}\right)\right]}{1-\left(\frac{1}{x}\right)\left[\left(\frac{1}{x}\right)+\left(\frac{2}{x^2}\right)\right]}\right)-\frac{x\left(3+\frac{5}{x}\right)}{\left(x\sqrt{\left[1+\left(\frac{1}{x}\right)\left(3+\frac{5}{x}\right)\right]}+x\right)}\right]=\frac{1}{2}$$

$$\lim_{x\to+\infty}\left[\left(\frac{k-\left(\frac{1}{x}\right)\left[\left(\frac{2}{x}\right)\left(\frac{1}{x}-1\right)-1\right]}{1-\left(\frac{1}{x}\right)\left[\left(\frac{1}{x}\right)+\left(\frac{2}{x^2}\right)\right]}\right)-\frac{\left(3+\frac{5}{x}\right)}{\left(\sqrt{\left[1+\left(\frac{1}{x}\right)\left(3+\frac{5}{x}\right)\right]}+1\right)}\right]=\frac{1}{2}$$

$$\left(\frac{k-\left(\frac{1}{+\infty}\right)\left[\left(\frac{2}{+\infty}\right)\left(\frac{1}{+\infty}-1\right)-1\right]}{1-\left(\frac{1}{+\infty}\right)\left[\left(\frac{1}{+\infty}\right)+\left(\frac{2}{+\infty^2}\right)\right]}\right)-\frac{\left[3+\left(\frac{5}{+\infty}\right)\right]}{\left(\sqrt{\left[1+\left(\frac{1}{+\infty}\right)\left[3+\left(\frac{5}{+\infty}\right)\right]\right]}+1\right)}=\frac{1}{2}$$

$$\left(\frac{k-(0^+)\left[(0^+)(0^+-1)-1\right]}{1-(0^+)\left[(0^+)+0^+\right]}\right)-\frac{\left[3+(0^+)\right]}{\left(\sqrt{\left\{1+(0^+)\left[3+(0^+)\right]\right\}}+1\right)}=\frac{1}{2}$$

$$\left(\frac{k-(0^+)\left[(0^+)(-1)^+-1\right]}{1-(0^+)\left[(0)^+\right]}\right)-\frac{(3^+)}{\left(\sqrt{1+(0^+)(3^+)}+1\right)}=\frac{1}{2}$$

$$\left(\frac{k-(0^+)\left[(0^-)-1\right]}{1-(0^+)}\right)-\frac{(3^+)}{\left(\sqrt{1+(0^+)}+1\right)}=\frac{1}{2}$$

$$\left(\frac{k-\left(0^{+}\right)\left(-1\right)^{-}}{1^{-}}\right)-\frac{\left(3^{+}\right)}{\left(1^{+}+1\right)}=\frac{1}{2}$$

$$\left(\frac{k-\left(0^{-}\right)}{1^{-}}\right)-\frac{\left(3^{+}\right)}{\left(2^{+}\right)}=\frac{1}{2}$$

$$\left(\frac{k-0^{-}}{1^{-}}\right)-\frac{\left(3^{+}\right)}{\left(2^{+}\right)}=\frac{1}{2}\Rightarrow\left(\frac{k-0^{-}}{1^{-}}\right)=\frac{1}{2}+\frac{\left(3^{+}\right)}{\left(2^{+}\right)}$$

$$\left(\frac{k-0^{-}}{1^{-}}\right)=\frac{\left(2^{+}\right)+2\left(3^{+}\right)}{2\left(2^{+}\right)}$$

$$\left(\frac{k-0^{-}}{1^{-}}\right)=\frac{\left(2^{+}\right)+\left(6^{+}\right)}{\left(4^{+}\right)}=\frac{\left(8^{+}\right)}{\left(4^{+}\right)}$$

$$k-0^{-}=\frac{\left(8^{+}\right)\left(1^{-}\right)}{\left(4^{+}\right)}\Rightarrow k=0^{-}+\frac{8^{-}}{4^{+}}=0^{-}+2^{-}$$

$$k=2^{-}$$

Con MATLAB

```
>> syms x k
>> A=(x^4+k*x^3-2)

A =

x^4 + k*x^3 - 2

>> B=(x^3-x-2)

B =

x^3 - x - 2

>> C=(sqrt(x^2+3*x+5)

C =

(x^2 + 3*x + 5)^(1/2)

>> limit(((A/B)-C),x,inf)

ans =

k = 3/2

>> solve('k-3/2=1/2','k')

ans =

2
```

EJEMPLO 19

Calcular si existen los valores de "t" y "c" que satisfacen la siguiente expresión:

$$\lim_{x\to+\infty}\left(tx+c-\frac{x^3+1}{x^2+1}\right)=0.\ \lim_{x\to+\infty}\left(\frac{tx^3+tx+cx^2+c-x^3-1}{x^2+1}\right)=\lim_{x\to+\infty}\left(\frac{(t-1)x^3+cx^2+tx+(c-1)}{x^2+1}\right)=0$$

Para que el límite converja a cero el grado del numerador debe ser menor al grado del denominador, por lo tanto los coeficientes de x3 y de x2 deberán ser nulos.

$$t-1=0\Rightarrow t=1\ \text{ y }\ c=0$$

$$\therefore\ t=1\ \text{ y }\ c=0.$$

Con MATLAB

```
>> A=(t*x+c-((x^3+1)/(x^2+1)))

A =

c - (x^3 + 1)/(x^2 + 1) + t*x

>> pretty(A)

     3
    x  + 1
c - ------ + t x
     2
    x  + 1

>> collect(A)

ans =

((t - 1)*x^3 + c*x^2 + t*x + c - 1)/(x^2 + 1)

>> pretty(ans)

           3      2
  (t - 1) x  + c x  + t x + c - 1
  -------------------------------
               2
              x  + 1
```

Para que el límite converja a cero el grado del numerador debe ser menor al grado del denominador, por lo tanto los coeficientes de x^3 y de x^2 deberán ser nulos.

$$t-1=0\Rightarrow t=1\ \text{ y }\ c=0$$

$$\therefore\ t=1\ \text{ y }\ c=0.$$

EJEMPLO 20

$$\lim_{x\to+\infty}\frac{x^2-3x}{4x^2+5}=\lim_{x\to+\infty}\frac{\left(x^2-3x\right)}{\left(4x^2+5\right)}\cdot\frac{\left(1/x^2\right)}{\left(1/x^2\right)}=\lim_{x\to+\infty}\frac{\left(x^2/x^2-3x/x^2\right)}{\left(4x^2/x^2+5/x^2\right)}$$

$$=\lim_{x\to+\infty}\left(\frac{1-3/x}{4+5/x^2}\right)=\frac{1-\left(3/+\infty\right)}{4+5/\left(+\infty\right)^2}=\frac{1-\left(0^+\right)}{4+5/\left(+\infty\right)}=\frac{1^-}{4+\left(0^+\right)}=\frac{1^-}{4^+}=\left(\frac{1}{4}\right)^-=\left(0{,}25\right)^-$$

Con MATLAB

```
>> syms x
>> limit((x^2-3*x)/(4*x^2+5),x,inf)

ans =

1/4
```

EJEMPLO 21

$$\lim_{x\to+\infty}\left(5-\frac{2}{x^4}\right)=\lim_{x\to+\infty}(5)-2\lim_{x\to+\infty}\left(\frac{1}{x^4}\right)=5-2\left[\frac{1}{\left(+\infty\right)^4}\right]$$

$$=5-2\frac{1}{\left(+\infty\right)}=5-2\left(0^+\right)=5-\left(0^+\right)=5^-$$

Con MATLAB

```
>> syms x
>> limit((5-(2/(x^4))),x,inf)

ans =

5
```

EJEMPLO 22

$$\lim_{x\to+\infty}\left(\frac{x^2}{x^{-2}+1}\right)=\lim_{x\to+\infty}\left(\frac{x^2}{1/x^2+1}\right)=\lim_{x\to+\infty}\left[\frac{x^2}{\left(1+x^2\right)/x^2}\right]=\lim_{x\to+\infty}\left(\frac{x^4}{1+x^2}\right)$$

En el término $1 + x^2$, el 1 no afecta al valor de x^2 ya que $x \to +\infty$; entonces $(1 + x^2) \approx x^2$; por lo que:

$$\lim_{x\to+\infty}\frac{x^4}{x^2}=\lim_{x\to+\infty}x^2=+\infty$$

Con MATLAB

```
>> syms x
>> limit((x^2)/(x^-2+1),x,inf)

ans =

Inf
```

Ejemplo 23

$$\lim_{x\to+\infty}\frac{8-\sqrt{x}}{1+4\sqrt{x}}=\lim_{x\to+\infty}\frac{\left(8-\sqrt{x}\right)}{\left(1+4\sqrt{x}\right)}\cdot\frac{\left(8+\sqrt{x}\right)}{\left(8+\sqrt{x}\right)}=\lim_{x\to+\infty}\frac{64-x}{8+33\sqrt{x}+4x}$$

$$=\lim_{x\to+\infty}\frac{(64-x)}{\left(8+33\sqrt{x}+4x\right)}\cdot\frac{(1/x)}{(1/x)}=\lim_{x\to+\infty}\left(\frac{64/x-x/x}{8/x+33\sqrt{x}/x+4x/x}\right)$$

$$=\lim_{x\to+\infty}\left(\frac{64/x-1}{8/x+33\sqrt{x/x^2}+4}\right)=\frac{(64/+\infty)-1}{(8/+\infty)+33\sqrt{(1/+\infty)}+4}$$

$$=\frac{\left(0^+\right)-1}{\left(0^+\right)+33\sqrt{\left(0^+\right)}+4}=\frac{(-1)^+}{\left(0^+\right)+33\left(0^+\right)+4}=\frac{(-1)^+}{\left(0^+\right)+4}=\frac{(-1)^+}{\left(4^+\right)}=\left(-\frac{1}{4}\right)^-$$

Con MATLAB

```
>> syms x
>> limit((8-(sqrt(x)))/(1+4*(sqrt(x))),x,inf)

ans =

-1/4
```

Ejemplo 24

$$\lim_{x\to+\infty}\left(\frac{3x}{x+2}-\frac{x-1}{2x+6}\right)=\lim_{x\to+\infty}\left[\frac{3x}{x+2}-\frac{x-1}{2(x+3)}\right]$$

$$=\lim_{x\to+\infty}\left[\frac{6x(x+3)-(x-1)(x+2)}{2(x+3)(x+2)}\right]=\lim_{x\to+\infty}\frac{6x(x+3)-(x-1)(x+2)}{2(x+3)(x+2)}\cdot\frac{\left(1/x^2\right)}{\left(1/x^2\right)}$$

$$=\lim_{x\to+\infty}\left[\frac{\dfrac{6x(x+3)-(x-1)\ (x+2)}{x^2}}{\dfrac{2(x+3)\ (x+2)}{x^2}}\right]=\lim_{x\to+\infty}\left[\frac{\dfrac{6\,x(x+3)}{x^2}-\dfrac{(x-1)\ (x+2)}{x^2}}{2\left(\dfrac{x+3}{x}\right)\left(\dfrac{x+2}{x}\right)}\right]$$

$$=\lim_{x\to+\infty}\left[\frac{6\left(\dfrac{x}{x}\right)\left(\dfrac{x+3}{x}\right)-\left(\dfrac{x-1}{x}\right)\left(\dfrac{x+2}{x}\right)}{2\left(\dfrac{x}{x}+\dfrac{3}{x}\right)\left(\dfrac{x}{x}+\dfrac{2}{x}\right)}\right]$$

$$=\lim_{x\to+\infty}\left[\frac{6\left(1+\frac{3}{x}\right)-\left(1-\frac{1}{x}\right)\left(1+\frac{2}{x}\right)}{2\left(1+\frac{3}{x}\right)\left(1+\frac{2}{x}\right)}\right]=\lim_{x\to+\infty}\left[\frac{3}{\left(1+\frac{2}{x}\right)}-\frac{\left(1-\frac{1}{x}\right)}{2\left(1+\frac{3}{x}\right)}\right]$$

$$=\frac{3}{\left(1+0^{+}\right)}-\frac{\left(1-0^{+}\right)}{2\left(1+0^{+}\right)}=\frac{3}{1^{+}}-\frac{1^{-}}{2\left(1^{+}\right)}=3^{-}-\frac{1^{-}}{2}=\frac{6^{-}-1^{-}}{2}=\frac{5^{-}}{2}=\left(\frac{5}{2}\right)^{-}=\left(2,5\right)^{-}$$

Con MATLAB

```
>> syms x
>> limit((3*x)/(x+2)-((x-1)/(2*x+6)),x,inf)

ans =

5/2
```

EJEMPLO 25

$$\lim_{x\to+\infty}\left[\left(\frac{x}{3x+1}\right)\cdot\left(\frac{4x^2+1}{2x^2+x}\right)^3\right]=\lim_{x\to+\infty}\frac{x\left(4x^2+1\right)^3}{\left(3x+1\right)\left(2x^2+x\right)^3}\cdot\frac{\left(\frac{1}{x^7}\right)}{\left(\frac{1}{x^7}\right)}$$

$$=\lim_{x\to+\infty}\frac{\frac{x}{x}\frac{\left(4x^2+1\right)^3}{x^6}}{\left(\frac{3x+1}{x}\right)\frac{\left(2x^2+x\right)^3}{x^6}}=\lim_{x\to+\infty}\frac{\left(\frac{4x^2+1}{x^2}\right)^3}{\left(3\frac{x}{x}+\frac{1}{x}\right)\left(\frac{2x^2+x}{x^2}\right)^3}$$

$$=\lim_{x\to+\infty}\frac{\left(4\frac{x^2}{x^2}+\frac{1}{x^2}\right)^3}{\left(3+\frac{1}{x}\right)\left(2\frac{x^2}{x^2}+\frac{x}{x^2}\right)^3}=\lim_{x\to+\infty}\frac{\left(4+\frac{1}{x^2}\right)^3}{\left(3+\frac{1}{x}\right)\left(2+\frac{1}{x}\right)^3}$$

$$=\lim_{x\to+\infty}\frac{\left[4+\frac{1}{(x)^2}\right]^3}{\left[3+\left(\frac{1}{x}\right)\right]\left[2+\left(\frac{1}{x}\right)\right]^3}=\frac{\left[4+\frac{1}{(+\infty)^2}\right]^3}{\left[3+\left(\frac{1}{+\infty}\right)\right]\left[2+\left(\frac{1}{+\infty}\right)\right]^3}$$

$$=\frac{\left[4+\frac{1}{(+\infty)}\right]^3}{\left[3+\left(0^{+}\right)\right]\left[2+\left(0^{+}\right)\right]^3}=\frac{\left[4+\left(0^{+}\right)\right]^3}{\left(3^{+}\right)\left(2^{+}\right)^3}=\frac{\left(4^{+}\right)^3}{\left(3^{+}\right)\left(8^{+}\right)}=\frac{\left(64^{+}\right)}{\left(24^{+}\right)}=\left(\frac{8}{3}\right)^{-}$$

Con MATLAB

```
A =

x/(3*x + 1)

>> B=4*x^2+1

B =

4*x^2 + 1

>> C=2*x^2+x

C =

2*x^2 + x

>> limit((A*(B/C)^3),x,inf)

ans =

8/2
```

EJEMPLO 26

$$\lim_{x\to+\infty}\left(\frac{1+7\sqrt[3]{x}}{2\sqrt[3]{x}}\right)=\lim_{x\to+\infty}\left[\frac{\left(1+7\sqrt[3]{x}\right)}{2\sqrt[3]{x}}\cdot\frac{\left(\frac{1}{\sqrt[3]{x}}\right)}{\left(\frac{1}{\sqrt[3]{x}}\right)}\right]=\lim_{x\to+\infty}\left[\frac{\left(\frac{1}{\sqrt[3]{x}}+\frac{7\sqrt[3]{x}}{\sqrt[3]{x}}\right)}{\frac{2\sqrt[3]{x}}{\sqrt[3]{x}}}\right]$$

$$=\lim_{x\to+\infty}\frac{\left(\frac{1}{\sqrt[3]{x}}+7\right)}{2}=\frac{\left(\frac{1}{\sqrt[3]{+\infty}}+7\right)}{2}=\frac{\left[\left(\frac{1}{+\infty}\right)+7\right]}{2}=\frac{\left[\left(0^+\right)+7\right]}{2}=\frac{7^+}{2}=\left(\frac{7}{2}\right)^+$$

Con MATLAB

```
>> syms x
>> limit((1+7*(x^(1/3)))/(2*(x^(1/3))),x,inf)

ans =

7/2
```

EJEMPLO 27

$$\lim_{x\to+\infty}\left(\frac{\sqrt{3x+2}}{\sqrt{6x-8}}\right)=\lim_{x\to+\infty}\left[\frac{\left(\sqrt{3x+2}\right)}{\left(\sqrt{6x-8}\right)}\cdot\frac{\left(\frac{1}{\sqrt{x}}\right)}{\left(\frac{1}{\sqrt{x}}\right)}\right]=\lim_{x\to+\infty}\left(\frac{\sqrt{\frac{3x+2}{x}}}{\sqrt{\frac{6x-8}{x}}}\right)$$

$$=\lim_{x\to+\infty}\left(\frac{\sqrt{3+\frac{2}{x}}}{\sqrt{6-\frac{8}{x}}}\right)=\frac{\sqrt{3+\left(\frac{2}{+\infty}\right)}}{\sqrt{6-\left(\frac{8}{+\infty}\right)}}=\frac{\sqrt{3+\left(0^+\right)}}{\sqrt{6-\left(0^+\right)}}=\frac{\sqrt{3^+}}{\sqrt{6^-}}=\sqrt{\frac{3^+}{6^-}}=\sqrt{\left(\frac{1}{2}\right)^+}=\left(\sqrt{\frac{1}{2}}\right)^+$$

Con MATLAB

```
>> syms x
>> A=(3*x+2)^(1/2)

A =

(3*x + 2)^(1/2)

>> B=(6*x-8)^(1/2)

B =

(6*x - 8)^(1/2)

>> limit((A/B),x,inf)

ans =

(3^(1/2)*6^(1/2))/6

>> pretty(ans)

    1/2  1/2
   3    6
   ---------
       6
```

EJEMPLO 28

$$\lim_{x\to+\infty}\left(\frac{\sqrt{9x^2+6}}{5x+1}\right)=\lim_{x\to+\infty}\left[\frac{\sqrt{9x^2+6}}{(5x+1)}\cdot\frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}\right]=\lim_{x\to+\infty}\left(\frac{\frac{\sqrt{9x^2+6}}{x}}{\frac{5x+1}{x}}\right)$$

$$=\lim_{x\to+\infty}\left(\frac{\sqrt{\frac{9x^2+6}{x^2}}}{5\frac{x}{x}+\frac{1}{x}}\right)=\lim_{x\to+\infty}\left(\frac{\sqrt{9+\frac{6}{x^2}}}{5+\frac{1}{x}}\right)=\frac{\sqrt{9+\frac{6}{(+\infty)^2}}}{5+\left(\frac{1}{+\infty}\right)}$$

$$=\frac{\sqrt{9+\frac{6}{(+\infty)}}}{5+\left(0^+\right)}=\frac{\sqrt{9+\left(0^+\right)}}{5+\left(0^+\right)}=\frac{\sqrt{\left(9^+\right)}}{\left(5^+\right)}=\frac{3^+}{5^+}=\left(\frac{3}{5}\right)^-=(0{,}6)^-$$

Con MATLAB

```
>> syms x
>> A=(9*x^2+6)^(1/2)

A =

(9*x^2 + 6)^(1/2)

>> B=(5*x+1)

B =

5*x + 1

>> limit((A/B),x,inf)

ans =

3/5
```

EJEMPLO 29

$$\lim_{x\to+\infty}\left(x-\sqrt{x^2+1}\right)=\lim_{x\to+\infty}\left[\left(x-\sqrt{x^2+1}\right)\frac{\left(x+\sqrt{x^2+1}\right)}{\left(x+\sqrt{x^2+1}\right)}\right]$$

$$=\lim_{x\to+\infty}\left[\frac{x^2-\left(\sqrt{x^2+1}\right)^2}{x+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}\right]=\lim_{x\to+\infty}\left[\frac{x^2-\left(x^2+1\right)}{\left(x+\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}\right)}\right]=\lim_{x\to+\infty}\left[\frac{x^2-x^2-1}{\left(x+|x|\sqrt{1+\frac{1}{x^2}}\right)}\right]$$

$$\text{como } x\to+\infty \text{ es } >0 \Rightarrow |x|=x$$

$$=\lim_{x\to+\infty}\left(\frac{-1}{x+x\sqrt{1+\frac{1}{x^2}}}\right)=\lim_{x\to+\infty}\left[\frac{-1}{x\left(1+\sqrt{1+\frac{1}{x^2}}\right)}\right]=\frac{-1}{(+\infty)\left(1+1^+\right)}=\frac{-1}{(+\infty)\left(2^+\right)}=\frac{-1}{(+\infty)}=0^-$$

Con MATLAB

```
>> syms x
>> limit((x-(sqrt(x^2+1))),x,inf)

ans =

0
```

Ejemplo 30

$$\lim_{x\to+\infty}\left(\sqrt{x^2+5x}-x\right)=\lim_{x\to+\infty}\left(\sqrt{x^2+5x}-x\right)\cdot\frac{\left(\sqrt{x^2+5x}+x\right)}{\left(\sqrt{x^2+5x}+x\right)}$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+5x}\right)^2-x^2}{\sqrt{x^2+5x}+x}=\lim_{x\to+\infty}\frac{x^2+5x-x^2}{\sqrt{x^2+5x}+x}$$

$$=\lim_{x\to+\infty}\frac{(5x)(1/x)}{\left(\sqrt{x^2+5x}+x\right)(1/x)}=\lim_{x\to+\infty}\frac{5(x/x)}{\sqrt{x^2+5x}\,/\,x+(x/x)}$$

$$=\lim_{x\to+\infty}\frac{5}{\sqrt{\frac{x^2+5x}{x^2}}+1}=\lim_{x\to+\infty}\frac{5}{\sqrt{\frac{x^2}{x^2}+5\frac{x}{x^2}}+1}=\lim_{x\to+\infty}\frac{5}{\sqrt{1+\frac{5}{x}}+1}=\frac{5}{\sqrt{1+\left(\frac{5}{+\infty}\right)}+1}$$

$$=\frac{5}{\sqrt{1+\left(0^+\right)}+1}=\frac{5}{\sqrt{\left(1^+\right)}+1}=\frac{5}{\left(1^+\right)+1}=\frac{5}{\left(2^+\right)}=\left(\frac{5}{2}\right)^-=(2,5)^-$$

Con MATLAB

```
>> syms x
>> limit((sqrt(x^2+5*x)-x),x,inf)

ans =

5/2
```

1.6. LÍMITES INFINITOS

Ejemplo 1

$$\lim_{x\to0^+}\left(\frac{1}{x^2}\right)=\frac{1}{\left(0^+\right)^2}=\frac{1}{0^+}=+\infty$$

Con MATLAB

```
>> syms x
>> limit((1/(x^2)),x,0)

ans =

Inf
```

Ejemplo 2

$$\lim_{x\to 0^-}\left(\frac{1}{x^3}\right)=\frac{1}{\left(0^-\right)^3}=\frac{1}{0^-}=-\infty$$

Con MATLAB

```
>> syms x
>> limit((1/(x^3)),x,0)

ans =

NaN
```

Ejemplo 3

$$\lim_{x\to 0^+}\left(\frac{2}{\sqrt{x}}\right)=\lim_{x\to 0^+}\left(\frac{2}{x^{1/2}}\right)=\left[\frac{2}{\left(0^+\right)^{1/2}}\right]=\left(\frac{2}{0^+}\right)=+\infty$$

Con MATLAB

```
>> syms x
>> limit((2/(x^(1/2))),x,0)

ans =

NaN
```

Ejemplo 4

$$\lim_{x\to 2^+}\left(\frac{x^3-2x^2+5}{x^3+2x^2-12}\right)=\lim_{x\to 2^+}\left[\frac{x^3-2x^2+5}{(x-2)\left(x^2+2x+6\right)}\right]$$

$$=\frac{\left[\left(2^+\right)^3-2\left(2^+\right)^2+5\right]}{\left[\left(2^+\right)-2\right]\left[\left(2^+\right)^2+2\left(2^+\right)+6\right]}=\frac{\left(8^+-8^++5\right)}{\left(0^+\right)\left(4^++4^++6\right)}=\frac{5^+}{\left(0^+\right)\left(14^+\right)}=\frac{5^+}{0^+}=+\infty$$

EJEMPLO 5

$$\lim_{x\to 2^-}\left(\frac{x^3-2x^2+5}{x^3+2x-12}\right)=\lim_{x\to 2^-}\frac{x^3-2x^2+5}{(x-2)(x^2+2x+6)}$$

$$=\lim_{x\to 2^-}\frac{\left[\left(x^2\right)\ (x-2)+5\right]}{(x-2)\ \left[x\ (x+2)+6\ \right]}=\frac{\left[\left(2^-\right)^2\left(2^- -2\ \right)+5\right]}{\left(2^- -2\right)\ \left[\ 2^-\left(2^- +2\right)+6\ \right]}$$

$$=\frac{\left[\ \left(4^-\right)\ \left(0^-\right)+5\ \right]}{\left(0^-\right)\ \left[2^-\left(0^-\right)+6\ \right]}=\frac{\left(0^- +5\ \right)}{\left(0^-\right)\ \left(0^- +6\ \right)}=\frac{5^-}{\left(0^-\right)\left(6^-\right)}=\frac{5^-}{0^-}=-\infty$$

Con MATLAB

```
>> syms x
>> A=(x^3-2*x^2+5)
A =
x^3 - 2*x^2 + 5
>> B=x^3+2*x-12
B =
x^3 + 2*x - 12
>> limit((A/B),x,2,'right')
ans =
Inf
```

EJEMPLO 6

$$\lim_{x\to 2^+}\left(\frac{\sqrt{x^2-4}}{x-2}\right)=\lim_{x\to 2^+}\left(\frac{\sqrt{(x+2)(x-2)}}{(x-2)}\right)=\lim_{x\to 2^+}\left(\sqrt{\frac{(x+2)}{(x-2)}}\right)$$

$$=\sqrt{\frac{\left(2^+ +2\right)}{\left(2^+ -2\right)}}=\sqrt{\frac{4^+}{0^+}}=\sqrt{+\infty}=+\infty$$

EJEMPLO 7

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x^2}\right)=\lim_{x\to 0}\left(\frac{x^2-x}{x^3}\right)=\lim_{x\to 0}\left[\frac{x(x-1)}{x^3}\right]=\lim_{x\to 0}\left(\frac{x-1}{x^2}\right)=\frac{0-1}{0^2}=\frac{-1}{0}=-\infty$$

Con MATLAB

```
>> syms x
>> limit((1/x)-(1/(x^2)),x,0)

ans =

-Inf
```

CAPÍTULO

2

CÁLCULO DE DERIVADAS

1.1. DERIVADAS DE PRODUCTO Y/O COCIENTE

Calcule la derivada de primer orden de las siguientes funciones:

EJEMPLO 1

$$y=\frac{x^2+3x+5}{x^{-2}+3}$$

$$y'=\frac{(2x+3)\cdot(x^{-2}+3)-(x^2+3x+5)\cdot(-2x^{-3})}{(x^{-2}+3)^2}$$

Operando, eliminando los exponentes negativos y simplificando tenemos que:

$$y'=\frac{6x+9x^{-2}+4x^{-1}+9+10x^{-3}}{(x^{-2}+3)^2}=\frac{x(6x^4+9x^3+4x^2+9x+10)}{(3x^2+1)^2}$$

Con MATLAB

```
>> syms x
>> diff((x^2+3*x+5)/((x^(-2))+3),x)

ans =

(2*x + 3)/(1/x^2 + 3) + (2*(x^2 + 3*x + 5))/(x^3*(1/x^2 + 3)^2)

>> pretty(ans)

                2
2 x + 3   2 (x  + 3 x + 5)
------- + ----------------
 1          3 /  1     \2
-- + 3     x  | -- + 3 |
 2            |  2     |
x             \ x      /
>> collect(ans)

ans =

(6*x^5 + 9*x^4 + 4*x^3 + 9*x^2 + 10*x)/(9*x^4 + 6*x^2 + 1)

>> pretty(ans)

   5      4      3      2
6 x  + 9 x  + 4 x  + 9 x  + 10 x
--------------------------------
           4      2
        9 x  + 6 x  + 1
```

Ejemplo 2

$$y = \left(x^5 + x^{-5} + 1\right)\cdot\left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)$$

$$y = \left(x^5 + x^{-5} + 1\right)\cdot\left(x^{\frac{-1}{2}} + x^{\frac{1}{2}}\right)$$

$$y' = \left(5x^4 - 5x^{-6}\right)\cdot\left(x^{\frac{-1}{2}} + x^{\frac{1}{2}}\right) + \left(x^5 + x^{-5} + 1\right)\cdot\left(-\frac{1}{2}x^{\frac{-3}{2}} + \frac{1}{2}x^{\frac{-1}{2}}\right)$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{9}{2}\sqrt{x^7} + \frac{11}{2}\sqrt{x^9} - \frac{11}{2\sqrt{x^{13}}} - \frac{9}{2\sqrt{x^{11}}} - \frac{1}{2\sqrt{x^3}} + \frac{1}{2\sqrt{x}}$$

Con MATLAB

```
>> syms x
>> A=x^5+x^(-5)+1

A =

1/x^5 + x^5 + 1

>> B=(1/(x^(1/2))+x^(1/2))

B =

1/x^(1/2) + x^(1/2)

>> diff((A*B),x)
|
ans =

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

>> pretty(ans)

/    1         1   \ /  1    5   \   /   1     1/2 \ /    4    5 \
| ------ - ------ | | -- + x  1 | + | ---- + x    | | 5 x  - -- |
|    1/2      3/2 | |  5        |   |  1/2        | |         6 |
\ 2 x      2 x    / \ x         /   \ x           / \        x  /

>> simple(ans)

simplify:

-(x^5/2 - (9*x^10)/2 - x^6/2 - (11*x^11)/2 + (9*x)/2 + 11/2)/x^(13/2)

radsimp:

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

simplify(100):

-(x^5 - 9*x^10 - x^6 - 11*x^11 + 9*x + 11)/(2*x^(13/2))
```

```
combine(sincos):

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

combine(sinhcosh):

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

combine(ln):

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

factor:

((x^(1/2) - 1)*(x^(1/2) + 1)*(11*x^10 + 20*x^9 + 20*x^8 + 20*x^7 + 20*x^6 + 21*x^5
+ 20*x^4 + 20*x^3 + 20*x^2 + 20*x + 11))/(2*x^(13/2))

expand:

1/(2*x^(1/2)) - 1/(2*x^(3/2)) + (9*x^(7/2))/2 + (11*x^(9/2))/2 - 9/(2*x^(11/2))
- 11/(2*x^(13/2))

combine:

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

rewrite(exp):

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

rewrite(sincos):

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

rewrite(sinhcosh):

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

rewrite(tan):

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

collect(x):

-(x^5 - 9*x^10 - x^6 - 11*x^11 + 9*x + 11)/(2*x^(13/2))

mwcos2sin:

(1/(2*x^(1/2)) - 1/(2*x^(3/2)))*(1/x^5 + x^5 + 1) + (1/x^(1/2) + x^(1/2))*(5*x^4 - 5/x^6)

ans =

-(x^5 - 9*x^10 - x^6 - 11*x^11 + 9*x + 11)/(2*x^(13/2)

>> pretty(ans)

        11       10     6    5
  - 11 x    - 9 x    - x  + x  + 9 x + 11
- -------------------------------------
                    13/2
                 2 x
```

EJEMPLO 3

$$y=\frac{\left(x^2+\sqrt[3]{x^2}\right)}{x}$$

$$y=\frac{\left(x^2+x^{\frac{2}{3}}\right)}{x}$$

$$y'=\frac{\left(2x+\frac{2}{3}x^{\frac{-1}{3}}\right)\cdot x-\left(x^2+x^{\frac{2}{3}}\right)}{x^2}=\frac{2x^2+\frac{2}{3}x^{\frac{2}{3}}-x^2-x^{\frac{2}{3}}}{x^2}=\frac{x^2-\frac{x^{\frac{2}{3}}}{3}}{x^2}$$

$$y'=1-\frac{x^{\frac{-4}{3}}}{3}=1-\frac{1}{3\sqrt[3]{x^4}}$$

Con MATLAB

```
>> syms x
>> A=x^2

A =

x^2

>> B=x^(2/3)

B =

x^(2/3)

>> C=x

C =

x

>> diff(((A+B)/C),x)

ans =

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

>> pretty(ans)

            2
  2 x + ------
           1/3     2     2/3
        3 x       x   + x
  ------------ - ---------
        x              2
                      x
```

```
>> simple(ans)

simplify:

(x^(4/3) - 1/3)/x^(4/3)

radsimp:

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

simplify(100):

1 - 1/(3*x^(4/3))

combine(sincos):

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

combine(sinhcosh):

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

combine(ln):

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

factor:

(3*x^(4/3) - 1)/(3*x^(4/3))

expand:

1 - 1/(3*x^(4/3))

combine:

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

rewrite(exp):

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

rewrite(sincos):

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

rewrite(sinhcosh):

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

rewrite(tan):

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2

collect(c):

(3*x^(4/3) - 1)/(3*x^(4/3))

mwcos2sin:

(2*x + 2/(3*x^(1/3)))/x - (x^2 + x^(2/3))/x^2
```

```
ans =

1 - 1/(3*x^(4/3))

>> collect(ans)

ans =

1 - 1/(3*x^(4/3))

>> pretty(ans)

         1
  1 - ------
         4/3
      3 x
```

EJEMPLO 4

$$y = \frac{x^2 + 3x + 5}{\sqrt{x}}$$

$$y = \frac{x^2 + 3x + 5}{x^{\frac{1}{2}}}$$

$$y' = \frac{(2x+3)\left(x^{\frac{1}{2}}\right) - \left(x^2 + 3x + 5\right)\left(\frac{1}{2}\right)\left(x^{\frac{-1}{2}}\right)}{x}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{3}{2}\sqrt{x} + \frac{3}{2\sqrt{x}} - \frac{5}{2\sqrt{x^3}}$$

Con MATLAB

```
>> syms x
>> diff((x^2+3*x+5)/(sqrt(x)),x)

ans =

(2*x + 3)/x^(1/2) - (x^2 + 3*x + 5)/(2*x^(3/2))

>> pretty(ans)

              2
  2 x + 3    x  + 3 x + 5
  ------- - ------------
     1/2           3/2
    x           2 x
```

```
>> collect(ans)

ans =

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

>> pretty(ans)

                                1/2
    /   2       3   \      3     x         5
  x | ---- - ------ | + ---- - ---- - ------
    |  1/2      3/2 |    1/2     2       3/2
    \ x      2 x    /   x             2 x
>> simple(ans)

simplify:

((3*x^2)/2 + (3*x)/2 - 5/2)/x^(3/2)

radsimp:

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

simplify(100):

(3*x^2 + 3*x - 5)/(2*x^(3/2))

combine(sincos):

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

combine(sinhcosh):

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

combine(ln):

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

factor:

(3*x^2 + 3*x - 5)/(2*x^(3/2))

expand:

3/(2*x^(1/2)) + (3*x^(1/2))/2 - 5/(2*x^(3/2))

combine:

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

rewrite(exp):

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

rewrite(sincos):

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

rewrite(sinhcosh):

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))
```

```
rewrite(tan):

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

collect(x):

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

mwcos2sin:

x*(2/x^(1/2) - 3/(2*x^(3/2))) + 3/x^(1/2) - x^(1/2)/2 - 5/(2*x^(3/2))

ans =

(3*x^2 + 3*x - 5)/(2*x^(3/2))

>> pretty(ans)

     2
  3 x  + 3 x - 5
  --------------
         3/2
      2 x
```

EJEMPLO 5

$$y = \frac{\sqrt{x} + x^{\frac{1}{3}} + x^3}{\sqrt[5]{x^2}}$$

$$y = \frac{x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^3}{x^{\frac{2}{5}}}$$

$$y' = \frac{\left(\frac{1}{2}x^{\frac{-1}{2}} + \frac{1}{3}x^{\frac{-2}{3}} + 3x^2\right)\left(x^{\frac{2}{5}}\right) - \left(x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^3\right)\left(\frac{2}{5}x^{\frac{-3}{5}}\right)}{x^{\frac{4}{5}}}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{13\sqrt[5]{x^8}}{5} + \frac{1}{10\sqrt[10]{x^9}} - \frac{1}{15\sqrt[15]{x^{16}}}$$

Con MATLAB

```
>> syms x
>> A=x^(1/2)

A =

x^(1/2)
```

```
>> B=x^(1/3)

B =

x^(1/3)

>> C=x^3

C =

x^3

>> D=x^(2/5)

D =

x^(2/5)

>> diff(((A+B+C)/D),x)

ans =

(3*x^2 + 1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^3 + x^(1/3)))/(5*x^(7/5))

>> pretty(ans)

     2      1         1
  3 x  + ------ + ------
           1/2       2/3        1/2     3     1/3
        2 x       3 x       2 (x    + x  + x    )
  --------------------- - --------------------
           2/5                      7/5

>> collect(ans)

ans =

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

>> pretty(ans)

     1        1
  ------ + ------
     1/2      2/3       1/2    1/3         8/5
  2 x      3 x      2 (x    + x    )   13 x
  --------------- - --------------- + -------
        2/5                  7/5          5
       x                  5 x
>> simple(ans)

simplify:

(x^(1/6)/10 + (13*x^(8/3))/5 - 1/15)/x^(16/15)

radsimp:

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

simplify(100):

(3*x^(1/6) + 78*x^(8/3) - 3)/(30*x^(16/15))
```

```
combine(sincos):

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

combine(sinhcosh):

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

combine(ln):

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

factor:

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

expand:

(13*x^(8/5))/5 + 1/(10*x^(9/10)) - 1/(15*x^(16/15))

combine:

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - ((2*x^(1/2))/5 + (2*x^(1/3))/5)/x^(7/5)
+ (13*x^(8/5))/5

rewrite(exp):

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

rewrite(sincos):

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

rewrite(sinhcosh):

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

rewrite(tan):

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*x^(8/5))/5

collect(x):

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*(x^(1/2) + x^(1/3)))/(5*x^(7/5)) + (13*X^(8/5))/5

mwcos2sin:

(1/(2*x^(1/2)) + 1/(3*x^(2/3)))/x^(2/5) - (2*x^(1/2) + 2*x^(1/3))/(5*x^(7/5)) + (13*x^(8/5))/5

ans =

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

>> pretty(ans)

     1/6       8/3
  3 x    + 78 x    - 2
  --------------------
            16/15
        30 x
```

Manualmente:

```
>> E=(13*x^(8/5))/5

E =

(13*x^(8/5))/5

>> F=(1/(10*x^(9/10)))

F =

1/(10*x^(9/10))

>> G=(-1/(15*x^(16/15)))

G =

-1/(15*x^(16/15))

>> collect(ans)

ans =

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

>> pretty(ans)

     1/6       8/3
  3 x    + 78 x    - 2
  --------------------
             16/15
        30 x

>> simple(ans)

simplify:

(x^(1/6)/10 + (13*x^(8/3))/5 - 1/15)/x^(16/15)

radsimp:

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

simplify(100):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

combine(sincos):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

combine(sinhcosh):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

combine(ln):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))
```

```
factor:

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

expand:

(13*x^(8/5))/5 + 1/(10*x^(9/10)) - 1/(15*x^(16/15))

combine:

(x^(1/6)/10 + (13*x^(8/3))/5 - 1/15)/x^(16/15)

rewrite(exp):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

rewrite(sincos):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

rewrite(sinhcosh):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

rewrite(tan):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

collect(x):

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

mwcos2sin:

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

ans =

(3*x^(1/6) + 78*x^(8/3) - 2)/(30*x^(16/15))

>> pretty(ans)

     1/6       8/3
  3 x    + 78 x    - 2
  --------------------
              16/15
        30 x
```

EJEMPLO 6

$$y = \frac{1}{x^2 + x + \frac{3}{x}}$$

$$y = \left(x^2 + x + \frac{3}{x}\right)^{-1}$$

$$y' = -1\left(x^2 + x + \frac{3}{x}\right)^{-2}\left(2x + 1 - \frac{3}{x^2}\right) = -\frac{2x + 1 - \frac{3}{x^2}}{\left(x^2 + x + \frac{3}{x}\right)^2} = -\frac{\frac{2x^3 + x^2 - 3}{x^2}}{\frac{\left(x^3 + x^2 + 3\right)^2}{x^2}}$$

$$y' = -\frac{2x^3 + x^2 - 3}{\left(x^3 + x^2 + 3\right)^2}$$

Con MATLAB

```
>> syms x
>> A=1

A =

      1

>> B=x^2+x+3/x

B =

x + 3/x + x^2

>> diff((A/B),x)

ans =

-(2*x - 3/x^2 + 1)/(x + 3/x + x^2)^2

>> pretty(ans)

             3
       2 x - -- + 1
              2
             x
  - --------------
    /     3    2 \2
    | x + - + x  |
    \     x      /

>> collect(and)

ans =

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)
```

```
>> pretty(ans)

                    3     2
                 2 x   + x   - 3
  - ------------------------------
      6      5     4      3      2
     x  + 2 x  + x  + 6 x  + 6 x  + 9
>> simple(ans)

simplify:

-(2*x^3 + x^2 - 3)/(x^3 + x^2 + 3)^2

radsimp:

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

simplify(100):

-(2*x^3 + x^2 - 3)/(x^3 + x^2 + 3)^2

combine(sincos):

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

combine(sinhcosh):

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

combine(ln):

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

factor:

-((x - 1)*(2*x^2 + 3*x + 3))/(x^3 + x^2 + 3)^2

expand:

3/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9) - x^2/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)
- (2*x^3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

combine:

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

rewrite(exp):

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

rewrite(sincos):

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

rewrite(sinhcosh):

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

rewrite(tan):

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)
```

```
collect(x):

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

mwcos2sin:

-(2*x^3 + x^2 - 3)/(x^6 + 2*x^5 + x^4 + 6*x^3 + 6*x^2 + 9)

ans=

-(2*x^3 + x^2 - 3)/(x^3 + x^2 + 3)^2

>> pretty(ans)

        3    2
     2 x  + x  - 3
  - ---------------
       3    2     2
    (x  + x  + 3)
```

Ejemplo 7

$$y=\left(x^6+5x^3+\frac{3}{x}\right)\left(x^{-1}+x+2x^{\frac{2}{3}}\right)$$

$$y'=\left(6x^5+15x^2-3x^{-2}\right)\left(x^{-1}+x+2x^{\frac{2}{3}}\right)+\left(x^6+5x^3+3x^{-1}\right)\left(-x^{-2}+1+\frac{4}{3}x^{\frac{-1}{3}}\right)$$

Operando, eliminando exponentes negativos y simplificando:

$$y'=7x^6+5x^4+20x^3+10x+\frac{40}{3}\sqrt[3]{x^{17}}+\frac{110}{3}\sqrt[3]{x^8}-\frac{2}{\sqrt[3]{x^4}}-\frac{6}{x^3}$$

Con MATLAB

```
>> syms x
>> A=(x^6+5*x^3+(3/x))

A =

3/x + 5*x^3 + x^6

>> B=(x^(-1)+x+2*x^(2/3))

B =

x + 1/x + 2*x^(2/3)

>> diff((A*B),x)

ans =

(15*x^2 - 3/x^2 + 6*x^5)*(x + 1/x + 2*x^(2/3)) + (3/x + 5*x^3 + x^6)*(4/(3*x^(1/3))
- 1/x^2 + 1)
```

```
>> pretty(ans):

  /     2    3      5 \ /     1      2/3 \   / 3      3    6 \ /    4      1     \
  | 15 x  - -- + 6 x  | | x + - + 2 x    | + | - + 5 x  + x  | | ------ - -- + 1 |
  |         2         | \     x          /   \ x             / |   1/3    2      |
  \        x          /                                        \ 3 x      x      /
>> collect(ans)

ans =

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

>> pretty(ans)

      4        6        7       5/3        9         17/3        26/3
  30 x  + 60 x  + 15 x  - 6 x    + 21 x  + 110 x     + 40 x     - 18
  -------------------------------------------------------------------
                                        3
                                     3 x

>> simple(ans)

simplify:

(10*x^4 + 20*x^6 + 5*x^7 - 2*x^(5/3) + 7*x^9 + (110*x^(17/3))/3 + (40*x^(26/3))/3 - 6)/x^3

radsimp:

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

simplify(100):

10*x - 6/x^3 + 20*x^3 + 5*x^4 + 7*x^6 - 2/x^(4/3) + (110*x^(8/3))/3 + (40*x^(17/3))/3

combine(sincos):

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

combine(sinhcosh):

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

combine(ln):

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

factor:

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

expand:

10*x - 6/x^3 + 20*x^3 + 5*x^4 + 7*x^6 - 2/x^(4/3) + (110*x^(8/3))/3 + (40*x^(17/3))/3

combine:

(10*x^4 + 20*x^6 + 5*x^7 - 2*x^(5/3) + 7*x^9 + (110*x^(17/3))/3 + (40*x^(26/3))/3 - 6)/x^3

rewrite(exp):

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)
```

```
rewrite(sincos):

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

rewrite(sinhcosh):

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

rewrite(tan):

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

collect(x):

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

mwcos2sin:

(30*x^4 + 60*x^6 + 15*x^7 - 6*x^(5/3) + 21*x^9 + 110*x^(17/3) + 40*x^(26/3) - 18)/(3*x^3)

ans =

10*x - 6/x^3 + 20*x^3 + 5*x^4 + 7*x^6 - 2/x^(4/3) + (110*x^(8/3))/3 + (40*x^(17/3))/3

>> pretty(ans)

                                                8/3        17/3
          6       3      4      6     2    110 x       40 x
  10 x - -- + 20 x  + 5 x  + 7 x  - ---- + -------- + --------
          3                          4/3       3           3
         x                          x
```

Ejemplo 8

$$y = \frac{x + \dfrac{1}{\sqrt{x}} + \sqrt[5]{x^3}}{x^2 + x^{\frac{2}{5}}}$$

$$y = \frac{x + x^{\frac{-1}{2}} + x^{\frac{3}{5}}}{x^2 + x^{\frac{2}{5}}}$$

$$y' = \frac{\left(1 - \dfrac{1}{2}x^{\frac{-3}{2}} + \dfrac{3}{5}x^{\frac{-2}{5}}\right)\left(x^2 + x^{\frac{2}{5}}\right) - \left(x + x^{\frac{-1}{2}} + x^{\frac{3}{5}}\right)\left(2x + \dfrac{2}{5}x^{\frac{-3}{5}}\right)}{\left(x^2 + x^{\frac{2}{5}}\right)^2}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{-x^2 + \dfrac{3}{5}\sqrt[5]{x^2} - \dfrac{7}{5}\sqrt[5]{x^8} - \dfrac{5}{2}\sqrt{x} - \dfrac{9}{10\sqrt[10]{x^{11}}} + \dfrac{1}{5}}{x^4 + 2\sqrt[5]{x^{12}} + \sqrt[5]{x^4}}$$

Con MATLAB

```
>> syms x
>> A=(x+(1/(x^(1/2)))+x^(3/5))

A =

x + 1/x^(1/2) + x^(3/5)

>> B=x^2+(x^(2/5))

B =

x^2 + x^(2/5)

>> diff((A/B),x)

ans =

(3/(5*x^(2/5)) - 1/(2*x^(3/2)) + 1)/(x^2 + x^(2/5)) - ((2*x + 2/(5*x^(3/5)))*(x +
1/x^(1/2) + x^(3/5)))/(x^2 + x^(2/5))^2

>> pretty(ans)

     3        1         /          2   \ /       1     3/5 \
 ------- -------- + 1   | 2 x + ------ | | x + ---- + x    |
     2/5      3/2       |          3/5 | |      1/2        |
  5 x      2 x          \       5 x    / \     x           /
 ------------------- - --------------------------------------
        2    2/5                      2    2/5 2
       x  + x                       (x  + x   )

>> collect(ans)

ans =

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))


>> pretty(ans)

        8/5      3/2      11/10       27/10       31/10
    25 x    - 6 x    - 2 x      + 14 x      + 10 x      + 9
  - -------------------------------------------------------
                    7/2       19/10       51/10
                20 x    + 10 x      + 10 x

>> simple(ans)

simplify:

-((5*x^(8/5))/2 - (3*x^(3/2))/5 - x^(11/10)/5 + (7*x^(27/10))/5 + x^(31/10) +
9/10)/(x^(19/10)*(x^(8/5) + 1)^2)

radsimp:

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))
```

```
simplify(100):

-((5*x^(8/5))/2 - (3*x^(3/2))/5 - x^(11/10)/5 + (7*x^(27/10))/5 + x^(31/10) +
9/10)/(x^(19/10)*(x^(8/5) + 1)^2)

combine(sincos):

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

combine(sinhcosh):

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

combine(ln):

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

factor:

(6*x^(3/2)-25*x^(8/5)+2*x^(11/10)-14*x^(27/10)-10*x^(31/10)-9)/
(10*x^(19/10)*(x^(8/5)+1)^2)

expand:

(6*x^(3/2))/(20*x^(7/2)+10*x^(19/10)+10*x^(51/10))-9/(20*x^(7/2)+10*x^(19/10)+1
0*x^(51/10))-(25*x^(8/5))/(20*x^(7/2)+10*x^(19/10)+10*x^(51/10))+(2*x^(11/10))/
(20*x^(7/2)+10*x^(19/10)+10*x^(51/10))-(14*x^(27/10))/(20*x^(7/2)+10*x^(19/10)+1
0*x^(51/10))-(10*x^(31/10))/(20*x^(7/2)+10*x^(19/10)+10*x^(51/10))

combine:

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

rewrite(exp):

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

rewrite(sincos):

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

rewrite(sinhcosh):

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

rewrite(tan):

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

collect(x):

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))
```

```
mwcos2sin:

-(25*x^(8/5)  -  6*x^(3/2)  -  2*x^(11/10)  +  14*x^(27/10)  +  10*x^(31/10)  +  9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

ans =

(6*x^(3/2)  -  25*x^(8/5)  +  2*x^(11/10)  -  14*x^(27/10)  -  10*x^(31/10)  -  9)/
(10*x^(19/10)*(x^(8/5) + 1)^2)

>> pretty(ans)

     3/2       8/5       11/10       27/10       31/10
  6 x    - 25 x    + 2 x      - 14 x      - 10 x      - 9
 --------------------------------------------------------
                     1/10 19    8/5     2
                10 (x    )   (x    + 1)
```

Manualmente:

```
>> C=(-x^2+(3/5*x^(2/5))-(7/5*x^(8/5))-(5/2*x^(1/2))-(9/(10*x^(11/10)))+(1/5))

C =

(3*x^(2/5))/5 - (5*x^(1/2))/2 - x^2 - (7*x^(8/5))/5 - 9/(10*x^(11/10)) + 1/5

>> D=x^4+(2*x^(12/5))+(x^(4/5))

D =

x^4 + x^(4/5) + 2*x^(12/5)

>> collect((C/D),x)

ans =

-(25*x^(8/5)  -  6*x^(3/2)  -  2*x^(11/10)  +  14*x^(27/10)  +  10*x^(31/10)  +  9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

>> pretty(ans)

        8/5      3/2      11/10       27/10       31/10
    25 x    - 6 x    - 2 x      + 14 x      + 10 x      + 9
  - --------------------------------------------------------
                   7/2       19/10       51/10
               20 x    + 10 x      + 10 x
>> simple(ans)

simplify:

-((5*x^(8/5))/2  -  (3*x^(3/2))/5  -  x^(11/10)/5  +  (7*x^(27/10))/5  +  x^(31/10)  +
9/10)/(x^(19/10)*(x^(8/5) + 1)^2)

radsimp:

-(25*x^(8/5)  -  6*x^(3/2)  -  2*x^(11/10)  +  14*x^(27/10)  +  10*x^(31/10)  +  9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))
```

```
simplify(100):

-((5*x^(8/5))/2-(3*x^(3/2))/5-x^(11/10)/5+(7*x^(27/10))/5+x^(31/10)+9/10)/
(x^(19/10)*(x^(8/5)+1)^2)

combine(sincos):

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

combine(sinhcosh)

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

combine(ln):

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

factor:

(6*x^(3/2)-25*x^(8/5)+2*x^(11/10)-14*x^(27/10)-10*x^(31/10)-9)/
(10*x^(19/10)*(x^(8/5)+1)^2)

expand:

(6*x^(3/2))/(20*x^(7/2)+10*x^(19/10)+10*x^(51/10))-9/(20*x^(7/2)+10*x^(19/10)+1
0*x^(51/10))-(25*x^(8/5))/(20*x^(7/2)+10*x^(19/10)+10*x^(51/10))+(2*x^(11/10))/
(20*x^(7/2)+10*x^(19/10)+10*x^(51/10))-(14*x^(27/10))/(20*x^(7/2)+10*x^(19/10)+1
0*x^(51/10))-(10*x^(31/10))/(20*x^(7/2)+10*x^(19/10)+10*x^(51/10))

combine:

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

rewrite(exp):

-(25*x^(8/5)-6*x^(3/2)-2*x^(11/10)+14*x^(27/10)+10*x^(31/10)+9)/(20*x^(7/2)+10*x
^(19/10)+10*x^(51/10))

rewrite(sincos):

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

rewrite(sinhcosh):

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

rewrite(tan):

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

collect(x):

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))
```

```
mwcos2sin:

-(25*x^(8/5) - 6*x^(3/2) - 2*x^(11/10) + 14*x^(27/10) + 10*x^(31/10) + 9)/
(20*x^(7/2) + 10*x^(19/10) + 10*x^(51/10))

ans =

(6*x^(3/2) - 25*x^(8/5) + 2*x^(11/10) - 14*x^(27/10) - 10*x^(31/10) - 9)/
(10*x^(19/10)*(x^(8/5) + 1)^2)

>> pretty(ans)

     3/2       8/5       11/10       27/10       31/10
  6 x    - 25 x    + 2 x      - 14 x      - 10 x      - 9
 -------------------------------------------------------
                      1/10 19    8/5     2
                10 (x     )   (x    + 1)
```

EJEMPLO 9

$$y = \frac{x^2 + x + x^{-2}}{x^2 + x + \frac{3}{x}}$$

$$y' = \frac{\left(2x + 1 - 2x^{-3}\right)\left(x^2 + x + 3x^{-1}\right) - \left(x^2 + x + x^{-2}\right)\left(2x + 1 - 3x^{-2}\right)}{\left(x^2 + x + 3x^{-1}\right)^2}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{9\,x^4 + 2\,x^3 - 3\,x^2 - 3}{x^2\left(x^3 + x^2 + 3\right)^2}$$

Con MATLAB

```
>> syms x
>> A=x^2+x+(x^(-2))

A =

x + 1/x^2 + x^2

>> B=(x^2+x+(3/x))

B =

x + 3/x + x^2

>> diff((A/B),x)

ans =

(2*x - 2/x^3 + 1)/(x + 3/x + x^2) - ((2*x - 3/x^2 + 1)*(x + 1/x^2 + x^2))/(x + 3/x + x^2)^2
```

```
>> pretty(ans)

         2       /        3     \ /      1    2 \
  2 x - -- + 1   | 2 x - -- + 1 | | x + -- + x  |
         3       |        2     | |      2      |
        x        \       x      / \     x       /
  ------------ - ---------------------------------
       3    2             /     3    2 \2
   x + - + x              | x + - + x  |
       x                  \     x      /
>> collect(ans)

ans =

-(3*x^2 - 2*x^3 - 9*x^4 + 3)/(x^8 + 2*x^7 + x^6 + 6*x^5 + 6*x^4 + 9*x^2)

>> pretty(ans)

               4      3      2
          - 9 x  - 2 x  + 3 x  + 3
  - -------------------------------------
     8      7    6      5      4      2
    x  + 2 x  + x  + 6 x  + 6 x  + 9 x
>> simple(ans)

simplify:

-(3*x^2 - 2*x^3 - 9*x^4 + 3)/(x^2*(x^3 + x^2 + 3)^2)

radsimp:

-(3*x^2 - 2*x^3 - 9*x^4 + 3)/(x^8 + 2*x^7 + x^6 + 6*x^5 + 6*x^4 + 9*x^2)

simplify(100):

-(3*x^2 - 2*x^3 - 9*x^4 + 3)/(x^2*(x^3 + x^2 + 3)^2)

combine(sincos):

-(3*x^2 - 2*x^3 - 9*x^4 + 3)/(x^8 + 2*x^7 + x^6 + 6*x^5 + 6*x^4 + 9*x^2)

combine(sinhcosh):

-(3*x^2 - 2*x^3 - 9*x^4 + 3)/(x^8 + 2*x^7 + x^6 + 6*x^5 + 6*x^4 + 9*x^2)

combine(ln):

-(3*x^2 - 2*x^3 - 9*x^4 + 3)/(x^8 + 2*x^7 + x^6 + 6*x^5 + 6*x^4 + 9*x^2)

factor:

(9*x^4+2*x^3-3*x^2-3)/(x^2*(x^3+x^2+3)^2)

expand:

(2*x^3)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)-(3*x^2)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9
*x^2)-3/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)+(9*x^4)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)

combine:

-(3*x^2-2*x^3-9*x^4+3)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)
```

```
rewrite(exp):

-(3*x^2-2*x^3-9*x^4+3)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)

rewrite(sincos):

-(3*x^2-2*x^3-9*x^4+3)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)

rewrite(sinhcosh):

-(3*x^2-2*x^3-9*x^4+3)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)

rewrite(tan):

-(3*x^2-2*x^3-9*x^4+3)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)

collect(x):

-(3*x^2-2*x^3-9*x^4+3)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)

mwcos2sin:

-(3*x^2-2*x^3-9*x^4+3)/(x^8+2*x^7+x^6+6*x^5+6*x^4+9*x^2)

ans =

(9*x^4+2*x^3-3*x^2-3)/(x^2*(x^3+x^2+3)^2)

>> pretty(ans)

      4       3       2
   9 x  + 2 x  - 3 x  - 3
  ----------------------
     2   3     2      2
    x  (x  + x  + 3)
```

Ejemplo 10

$$y = \frac{x^{-5} + \dfrac{1}{\sqrt{x^3}} + \sqrt[8]{x^3}}{x^5 + x^{\frac{3}{7}}}$$

$$y = \frac{x^{-5} + x^{\frac{-3}{2}} + x^{\frac{3}{8}}}{x^5 + x^{\frac{3}{7}}}$$

$$y' = \frac{\left(-5x^{-6} - \dfrac{3}{2}x^{\frac{-5}{2}} + \dfrac{3}{8}x^{\frac{-5}{8}}\right)\left(x^5 + x^{\frac{3}{7}}\right) - \left(x^{-5} + x^{\frac{-3}{2}} + x^{\frac{3}{8}}\right)\left(5x^4 + \dfrac{3}{7}x^{\frac{-4}{7}}\right)}{\left(x^5 + x^{\frac{3}{7}}\right)^2}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{-\frac{10}{x} - \frac{38}{7\sqrt[7]{x^{39}}} - \frac{13}{2}\sqrt{x^5} - \frac{27}{14\sqrt[14]{x^{29}}} - \frac{37\sqrt[8]{x^{35}}}{8} - \frac{3}{56\sqrt[56]{x^{11}}}}{x^{10} + 2\sqrt[7]{x^{38}} + \sqrt[7]{x^6}}$$

Con MATLAB

```
>> syms x
>> A=(x^(-5))+(1/(x^(3/2)))+(x^(3/8))

A =

1/x^5 + 1/x^(3/2) + x^(3/8)

>> B=x^5+(x^(3/7))

B =

x^5 + x^(3/7)

>> diff((A/B),x)

ans =

- (5/x^6 + 3/(2*x^(5/2)) - 3/(8*x^(5/8)))/(x^5 + x^(3/7)) - ((5*x^4 + 3/
(7*x^(4/7)))*(1/x^5 + 1/x^(3/2) + x^(3/8)))/(x^5 + x^(3/7))^2

>> pretty(ans)

    5       3         3       /    4     3    \ /  1    1     3/8 \
    -- + ------ - ------      | 5 x  + ------ | | -- + --- + x    |
     6      5/2       5/8     |           4/7 | |  5   3/2        |
    x    2 x      8 x         \        7 x    / \ x   x           /
  - ------------------- - -----------------------------------
            #1                             2
                                         #1

  where

          5    3/7
     #1 == x  + x

>> simple(ans)

simplify:

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x(113/14) + 259*x^(557/56) + 304)/
(56*x^(45/7)*(x^(32/7) + 1)^2)

radsimp:

- (5/x^6 + 3/(2*x^(5/2)) - 3/(8*x^(5/8)))/(x^5 + x^(3/7)) - ((5*x^4 + 3/
(7*x^(4/7)))*(1/x^5 + 1/x^(3/2) + x^(3/8)))/(x^5 + x^(3/7))^2

simplify(100):

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(56*x^(45/7)*(x^(32/7) + 1)^2)
```

```
combine(sincos):

- (5/x^6 + 3/(2*x^(5/2)) - 3/(8*x^(5/8)))/(x^5 + x^(3/7)) - ((5*x^4 + 3/
(7*x^(4/7)))*(1/x^5 + 1/x^(3/2) + x^(3/8)))/(x^5 + x^(3/7))^2

combine(sinhcosh):

- (5/x^6 + 3/(2*x^(5/2)) - 3/(8*x^(5/8)))/(x^5 + x^(3/7)) - ((5*x^4 + 3/
(7*x^(4/7)))*(1/x^5 + 1/x^(3/2) + x^(3/8)))/(x^5 + x^(3/7))^2

combine(ln):

-(5/x^6+3/(2*x^(5/2))-3/(8*x^(5/8)))/(x^5+x^(3/7))-((5*x^4+3/(7*x^(4/7)))*(1/
x^5+1/x^(3/2)+x^(3/8)))/(x^5+x^(3/7))^2

factor:

-(108*x^(7/2)+560*x^(32/7)+3*x^(43/8)+364*x^(113/14)+259*x^(557/56)+304)/
(56*x^(45/7)*(x^(32/7)+1)^2)

expand:

3/(8*x^(5/8)*(x^5+x^(3/7)))-3/(2*x^(5/2)*(x^5+x^(3/7)))-5/(x^11+x^(45/7))-5/
(x^11+x^(13/7)+2*x^(45/7))-(5*x^(5/2))/(x^10+x^(6/7)+2*x^(38/7))-3/(7*x^(29/14)*
(x^10+x^(6/7)+2*x^(38/7)))-(5*x^(35/8))/(x^10+x^(6/7)+2*x^(38/7))-3/(7*x^(39/7)*
(x^10+x^(6/7)+2*x^(38/7)))-3/(7*x^(11/56)*(x^10+x^(6/7)+2*x^(38/7)))

combine:

-(5/x^6+3/(2*x^(5/2))-3/(8*x^(5/8)))/(x^5+x^(3/7))-((5*x^4+3/(7*x^(4/7)))*(1/
x^5+1/x^(3/2)+x^(3/8)))/(x^5+x^(3/7))^2

rewrite(exp):

-(5/x^6+3/(2*x^(5/2))-3/(8*x^(5/8)))/(x^5+x^(3/7))-((5*x^4+3/(7*x^(4/7)))*(1/
x^5+1/x^(3/2)+x^(3/8)))/(x^5+x^(3/7))^2

rewrite(sincos):

-(5/x^6+3/(2*x^(5/2))-3/(8*x^(5/8)))/(x^5+x^(3/7))-((5*x^4+3/(7*x^(4/7)))*(1/
x^5+1/x^(3/2)+x^(3/8)))/(x^5+x^(3/7))^2

rewrite(sinhcosh):

-(5/x^6+3/(2*x^(5/2))-3/(8*x^(5/8)))/(x^5+x^(3/7))-((5*x^4+3/(7*x^(4/7)))*(1/
x^5+1/x^(3/2)+x^(3/8)))/(x^5+x^(3/7))^2

rewrite(tan):

- (5/x^6 + 3/(2*x^(5/2)) - 3/(8*x^(5/8)))/(x^5 + x^(3/7)) - ((5*x^4 + 3/
(7*x^(4/7)))*(1/x^5 + 1/x^(3/2) + x^(3/8)))/(x^5 + x^(3/7))^2

mwcos2sin:

- (5/x^6 + 3/(2*x^(5/2)) - 3/(8*x^(5/8)))/(x^5 + x^(3/7)) - ((5*x^4 + 3/
(7*x^(4/7)))*(1/x^5 + 1/x^(3/2) + x^(3/8)))/(x^5 + x^(3/7))^2

collect(x):

(- 108*x^(7/2) - 560*x^(32/7) - 3*x^(43/8) - 364*x^(113/14) - 259*x^(557/56) -
304)/(112*x^11 + 56*x^(45/7) + 56*x^(109/7))
```

```
ans =

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(56*x^(45/7)*(x^(32/7) + 1)^2)

>> pretty(ans)

        7/2         32/7      43/8         113/14         557/56
   108 x    + 560 x      + 3 x      + 364 x         + 259 x         + 304
 - -----------------------------------------------------------------
                                    45/7   32/7      2
                                56 x      (x      + 1)

>> collect(ans)

ans =

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(56*x^(45/7)*(x^(32/7) + 1)^2)

>> pretty(ans)

        7/2         32/7      43/8         113/14         557/56
   108 x    + 560 x      + 3 x      + 364 x         + 259 x         + 304
 - -----------------------------------------------------------------
                                    45/7   32/7      2
                                56 x      (x      + 1)
```

Manualmente:

```
>> D=(-10/x)-(38/(7*x^(39/7)))-((13/2)*x^(5/2))-(27/(14*x^(29/14)))-
((37*x^(35/8))/8)-(3/(56*x^(11/56)))

D =
- 10/x - (13*x^(5/2))/2 - 27/(14*x^(29/14)) - (37*x^(35/8))/8 - 38/(7*x^(39/7))
- 3/(56*x^(11/56))

>> E=(x^10)+(2*x^(38/7))+(x^(6/7))

E =

x^10 + x^(6/7) + 2*x^(38/7)

>> collect((D/E),x)

ans =

(- 108*x^(7/2) - 560*x^(32/7) - 3*x^(43/8) - 364*x^(113/14) - 259*x^(557/56) -
304)/(112*x^11 + 56*x^(45/7) + 56*x^(109/7))

>> pretty(ans)

        7/2         32/7      43/8         113/14         557/56
 - 108 x    - 560 x      - 3 x      - 364 x         - 259 x         - 304
 -----------------------------------------------------------------
                                       11        45/7        109/7
                                112 x    + 56 x      + 56 x
```

```
>> simple(ans)
simplify:

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(56*x^(45/7)*(x^(32/7) + 1)^2)

radsimp:

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(112*x^11 + 56*x^(45/7) + 56*x^(109/7))

simplify(100):

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(56*x^(45/7)*(x^(32/7) + 1)^2)

combine(sincos):

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(112*x^11 + 56*x^(45/7) + 56*x^(109/7))

combine(sinhcosh):

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(112*x^11 + 56*x^(45/7) + 56*x^(109/7))

combine(ln):

(- 108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) - 259*x^(557/56) -
304)/(112*x^11 + 56*x^(45/7) + 56*x^(109/7))

factor:

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(56*x^(45/7)*(x^(32/7) + 1)^2)

expand:

-304/(112*x^11+56*x^(45/7)+56*x^(109/7))-(108*x^(7/2))/(112*x^11+56*x^(45/7)+5
6*x^(109/7))-(560*x^(32/7))/(112*x^11+56*x^(45/7)+56*x^(109/7))-(3*x^(43/8))/
(112*x^11+56*x^(45/7)+56*x^(109/7))-(364*x^(113/14))/(112*x^11+56*x^(45/7)+56
*x^(109/7))-(259*x^(557/56))/(112*x^11+56*x^(45/7)+56*x^(109/7))

combine:

-(108*x^(7/2)+560*x^(32/7)+3*x^(43/8)+364*x^(113/14)+259*x^(557/56)+304)/(112*x^
11+56*x^(45/7)+56*x^(109/7))

rewrite(exp):

-(108*x^(7/2)+560*x^(32/7)+3*x^(43/8)+364*x^(113/14)+259*x^(557/56)+304)/(112*x^
11+56*x^(45/7)+56*x^(109/7))

rewrite(sincos):

-(108*x^(7/2)+560*x^(32/7)+3*x^(43/8)+364*x^(113/14)+259*x^(557/56)+304)/(112*x^
11+56*x^(45/7)+56*x^(109/7))

rewrite(sinhcosh):

(-108*x^(7/2)+560*x^(32/7)+3*x^(43/8)+364*x^(113/14)-259*x^(557/56)+304)/(112*x^
11+56*x^(45/7)+56*x^(109/7))
```

```
rewrite(tan):

(-108*x^(7/2)+560*x^(32/7)+3*x^(43/8)+364*x^(113/14)-259*x^(557/56)+304)/(112*x^
11+56*x^(45/7)+56*x^(109/7))

mwcos2sin:

-(108*x^(7/2)+560*x^(32/7)+3*x^(43/8)+364*x^(113/14)+259*x^(557/56)+304)/(112*x^
11+56*x^(45/7)+56*x^(109/7))

collect(x):

(- 108*x^(7/2) - 560*x^(32/7) - 3*x^(43/8) - 364*x^(113/14) - 259*x^(557/56) -
304)/(112*x^11 + 56*x^(45/7) + 56*x^(109/7))

ans =

-(108*x^(7/2) + 560*x^(32/7) + 3*x^(43/8) + 364*x^(113/14) + 259*x^(557/56) +
304)/(56*x^(45/7)*(x^(32/7) + 1)^2)

>> pretty(ans)

        7/2        32/7      43/8         113/14         557/56
   108 x    + 560 x     + 3 x     + 364 x        + 259 x        + 304
 - ----------------------------------------------------------------
                                      45/7   32/7     2
                                  56 x     (x     + 1)
```

Ejemplo 11

$$y = x^{\frac{2}{9}} + x^{\frac{2}{7}} + \sqrt{x^5}$$

$$y = x^{\frac{2}{9}} + x^{\frac{2}{7}} + x^{\frac{5}{2}}$$

$$y' = \frac{2}{9}x^{\frac{-7}{9}} + \frac{2}{7}x^{\frac{-5}{7}} + \frac{5}{2}x^{\frac{3}{2}}$$

$$y' = \frac{2}{9\sqrt[9]{x^7}} + \frac{2}{7\sqrt[7]{x^5}} + \frac{5\sqrt{x^3}}{2}$$

Con MATLAB

```
>> syms x
>> diff((x^(2/9)+(x^(2/7))+((x^(5/2)))),x)

ans =

(5*x^(3/2))/2 + 2/(7*x^(5/7)) + 2/(9*x^(7/9))

>> pretty(ans)

    3/2
 5 x             2        2
 ------ + ------ + ------
   2         5/7      7/9
          7 x      9 x
```

Ejemplo 12

$$y=\frac{x^{\frac{2}{7}}+x^{\frac{3}{5}}+\sqrt[5]{x^7}}{x^2+3x+8+x^{-3}}$$

$$y=\frac{x^{\frac{2}{7}}+x^{\frac{3}{5}}+x^{\frac{7}{5}}}{x^2+3x+8+x^{-3}}$$

$$y'=\frac{\left(\frac{2}{7}x^{\frac{-5}{7}}+\frac{3}{5}x^{\frac{-2}{5}}+\frac{7}{5}x^{\frac{2}{5}}\right)\left(x^2+3x+8+x^{-3}\right)-\left(x^{\frac{2}{7}}+x^{\frac{3}{5}}+x^{\frac{7}{5}}\right)\left(2x+3-3x^{-4}\right)}{\left(x^2+3x+8+x^{-3}\right)^2}$$

Operando, eliminando exponentes negativos y simplificando:

$$y'=-\left[\frac{\sqrt[7]{x^{16}}\left(21\sqrt[35]{x^{214}}+49\sqrt[35]{x^{186}}-42\sqrt[35]{x^{179}}+60x^5+42\sqrt[35]{x^{151}}\right.}{35\left(x^5+3x^4+8x^3+1\right)^2}\right.$$

$$\left.\frac{\left.-392\sqrt[35]{x^{144}}+75x^4-168\sqrt[35]{x^{116}}-80x^3-154\sqrt[35]{x^{39}}-126\sqrt[35]{x^{11}}-115\right)}{35\left(x^5+3x^4+8x^3+1\right)^2}\right]$$

Con MATLAB

```
>>syms x

>>A=(x^(2/7))+(x^(3/5))+(x^(7/5))

A=

x^(3/5)+x^(2/7)+x^(7/5)

>>B=(x^(2))+(3*x)+(8)+(x^(-3))

B=

3*x+x^2+1/x^3+8

>>diff((A/B),x)

ans=

(3/(5*x^(2/5))+(7*x^(2/5))/5+2/(7*x^(5/7)))/(3*x+x^2+1/x^3+8)-((2*x-3/x^4+3)*(x^
(3/5)+x^(2/7)+x^(7/5)))/(3*x+x^2+1/x^3+8)^2

>> pretty(ans)

    16/7      3       4       5        11/35         39/35         116/35
  (x     (80 x  - 75 x  - 60 x  + 126 x      + 154 x       + 168 x       +

        144/35        151/35        179/35        186/35        214*35
    392 x       - 42 x        + 42 x        - 49 x        - 21 x       + 115)) /

          5      4       3      2
    (35 (x  + 3 x  + 8 x  + 1) )
```

```
>> pretty(ans)

              2/5
   3       7 x         2       /       3      \  3/5    2/7    7/5
 ------ + ------ + -------    | 2 x - -- + 3 | (x    + x    + x    )
    2/5     5          5/7    |        4     |
 5 x                7 x       \       x      /
 ------------------------- - ------------------------------------
            #1                                    2
                                                #1

 where

                   2    1
    #1 == 3 x + x  + -- + 8
                      3
                     x

>>collect(ans)

ans=

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

>> pretty(ans)

      13/5       17/5       16/7       28/5       32/5       33/5
 (126 x     + 154 x     + 115 x     + 168 x     + 392 x     - 42 x     +

       37/5       38/5       37/7       42/5       44/7       51/7
   42 x     - 49 x     + 80 x     - 21 x     - 75 x     - 60 x     ) /

        10        9        8         7         6       5        4        3
   (35 x   + 210 x  + 875 x  + 1680 x  + 2240 x  + 70 x  + 210 x  + 560 x  + 35)

>>simple(ans)

simplify:

(x^(16/7)*(80*x^3-75*x^4-60*x^5+126*x^(11/35)+154*x^(39/35)+168*x^(116/35)+3
92*x^(144/35)-42*x^(151/35)+42*x^(179/35)-49*x^(186/35)-21*x^(214/35)+115))/
(35*(x^5+3*x^4+8*x^3+1)^2)

radsimp:

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+395*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

simplify(100):

(x^(16/7)*(80*x^3-75*x^4-60*x^5+126*x^(11/35)+154*x^(39/35)+168*x^(116/35)+3
92*x^(144/35)-42*x^(151/35)+42*x^(179/35)-49*x^(186/35)-21*x^(214/35)+115))/
(35*(x^5+3*x^4+8*x^3+1)^2)

combine(sincos):

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+395*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)
```

```
combine(sinhcosh):

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

combine(ln):

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

factor:

-((x^(1/35))^80*(75*x^4-80*x^3+60*x^5-126*x^(11/35)-154*x^(39/35)-
168*x^(116/35)-392*x^(144/35)+42*x^(151/35)-42*x^(179/35)-
49*x^(179/35)+21*x^(214/35)-115))/(35*(x^5+3*x^4+8*x^3+1)^2)

expand:

(126*x^(13/5))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)+
(154*x^(17/5))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)+
(115*x^(16/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)+
(164*x^(28/5))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)+
(392*x^(32/5))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)-
(42*x^(33/5))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)+
(42*x^(37/5))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)-
(49*x^(38/5))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)+
(80*x^(37/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)-
(21*x^(42/5))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)-
(75*x^(44/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)-
(60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

combine:

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

rewrite(exp):

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

rewrite(sincos):

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

rewrite(sinhcosh):

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

rewrite(tan):

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)
```

```
mwcos2sin:

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

collect(x):

(126*x^(13/5)+154*x^(17/5)+115*x^(16/7)+168*x^(28/5)+392*x^(32/5)-
42*x^(33/5)+42*x^(37/5)-49*x^(38/5)+80*x^(37/7)-21*x^(42/5)-75*x^(44/7)-
60*x^(51/7))/(35*x^10+210*x^9+875*x^8+1680*x^7+2240*x^6+70*x^5+210*x^4+560*x^3+35)

ans=

(x^(16/7)*(80*x^3-75*x^4-60*x^5+126*x^(11/35)+154*x^(39/35)+168*x^(116/35)+3
92*x^(144/35)-42*x^(151/35)+42*x^(179/35)-49*x^(186/35)-21*x^(214/35)+115))/
(35*(x^5+3*x^4+8*x^3+1)^2)

>> pretty(ans)

    16/7      3       4       5        11/35        39/35        116/35
  (x     (80 x  - 75 x  - 60 x  + 126 x      + 154 x      + 168 x       +

          144/35       151/35       179/35       186/35       214/35
     392 x       - 42 x       + 42 x       - 49 x       - 21 x       + 115)) /

           5      4      3    2
     (35 (x  + 3 x  + 8 x  + 1) )
```

1.2. DERIVADAS APLICANDO LA REGLA DE LA CADENA

Calcule la derivada de primer orden de las siguientes funciones:

EJEMPLO 1

$$y = \frac{\sqrt[3]{x^3 + 3x}}{\sqrt{x^3 + 2x}}$$

$$y = \frac{\left(x^3 + 3x\right)^{\frac{1}{3}}}{\left(x^3 + 2x\right)^{\frac{1}{2}}}$$

$$y' = \frac{\left(\frac{1}{3}\right)\left(x^3 + 3x\right)^{\frac{-2}{3}}\left(3x^2 + 3\right)\left(x^3 + 2x\right)^{\frac{1}{2}} - \left(x^3 + 3x\right)^{\frac{1}{3}}\left(\frac{1}{2}\right)\left(x^3 + 2x\right)^{\frac{-1}{2}}\left(3x^2 + 2\right)}{x^3 + 2x}$$

Operando, eliminando los exponentes negativos y simplificando tenemos que:

$$y' = -\frac{x^4 + 5x^2 + 2}{2\sqrt[6]{x^7}\sqrt{\left(x^2 + 2\right)^3}\sqrt[3]{\left(x^2 + 3\right)^2}}$$

Con MATLAB

```
>> syms x
>> A=(x^3+3*x)^(1/3)

A =

(x^3 + 3*x)^(1/3)

>> B=(x^3+2*x)^(1/2)

B =

(x^3 + 2*x)^(1/2)

>> diff((A/B),x)

ans =

(3*x^2 + 3)/(3*(x^3 + 2*x)^(1/2)*(x^3 + 3*x)^(2/3)) - ((x^3 + 3*x)^(1/3)*(3*x^2 +
2))/(2*(x^3 + 2*x)^(3/2))

>> pretty(ans)

                2                    3        1/3      2
             3 x  + 3              (x  + 3 x)    (3 x  + 2)
  ------------------------------ - ------------------------
      3        1/2   3        2/3             3        3/2
  3 (x  + 2 x)    (x  + 3 x)           2 (x  + 2 x)

>> simple(ans)

simplify:

-(x^(x^4 + 5*x^2 + 2))/(2*(x*(x^2 + 2))^(3/2)*(x*(x^2 + 3))^(2/3))

radsimp:

(3*x^2 + 3)/(3*(x^3 + 2*x)^(1/2)*(x^3 + 3*x)^(2/3)) - ((x^3 + 3*x)^(1/3)*(3*x^2 +
2))/(2*(x^3 + 2*x)^(3/2))

simplify(100):

-(x*(x^4+5*x^2+2))/(2*(x^3+2*x)^(3/2)*(x^3+3*x)^(2/3))

combine(sincos):

(3*x^2+3)/(3*(x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*(3*x^2+2))/
(2*(x^3+2*x)^(3/2))

combine(sinhcosh):

(3*x^2+3)/(3*(x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*(3*x^2+2))/
(2*(x^3+2*x)^(3/2))

combine(ln):

(3*x^2+3)/(3*(x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*(3*x^2+2))/
(2*(x^3+2*x)^(3/2))
```

```
factor:

-(x*(x^4+5*x^2+2))/(2*(2*x+x^3)^(3/2)*(3*x+x^3)^(2/3))

expand:

1/((x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-(x^3+3*x)^(1/3)/(x^3+2*x)^(3/2)+x^2/((x^3+
2*x)^(1/2)*(x^3+3*x)^(2/3))-(3*x^2*(x^3+3*x)^(1/3))/(2*(x^3+2*x)^(3/2))

combine:

(x^2+1)/((x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*((3*x^2)/2+1))/(x^3+
2*x)^(3/2)

rewrite(exp):

(3*x^2+3)/(3*(x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*(3*x^2+2))/
(2*(x^3+2*x)^(3/2))

rewrite(sincos):

(3*x^2+3)/(3*(x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*(3*x^2+2))/
(2*(x^3+2*x)^(3/2))

rewrite(sinhcosh):

(3*x^2+3)/(3*(x^3+2*x)^(1/2)+(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*(3*x^2+2))/
(2*(x^3+2*x)^(3/2))

rewrite(tan):

(3*x^2+3)/(3*(x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*(3*x^2+2))/
(2*(x^3+2*x)^(3/2))

collect(x):

x^2*(1/((x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-(3*(x^3+3*x)^(1/3))/
(2*(x^3+2*x)^(3/2)))+1/((x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-(x^3+3*x)^(1/3)/
(x^3+2*x)^(3/2)

mwcos2sin:

(3*x^2+3)/(3*(x^3+2*x)^(1/2)*(x^3+3*x)^(2/3))-((x^3+3*x)^(1/3)*(3*x^2+2))/
(2*(x^3+2*x)^(3/2))

ans=

-(x*(x^4+5*x^2+2))/(2*(x^3+2*x)^(3/2)*(x^3+3*x)^(2/3))

>> pretty(ans)

              4      2
          x (x  + 5 x  + 2)
  - ---------------------------
         3       3/2   3      2/3
    2 (x  + 2 x)    (x  + 3 x)
```

Ejemplo 2

$$y=\frac{x^2+\sqrt{x}+x^{\frac{1}{3}}}{\sqrt{x+1}}$$

$$y=\frac{x^2+x^{\frac{1}{2}}+x^{\frac{1}{3}}}{(x+1)^{\frac{1}{2}}}$$

$$y'=\frac{\left(2x+\frac{1}{2}x^{\frac{-1}{2}}+\frac{1}{3}x^{\frac{-2}{3}}\right)(x+1)^{\frac{1}{2}}-\left(x^2+x^{\frac{1}{2}}+x^{\frac{1}{3}}\right)\left(\frac{1}{2}\right)(x+1)^{\frac{-1}{2}}}{x+1}$$

Operando, eliminando exponentes negativos y simplificando:

$$y'=\frac{9\sqrt[3]{x^8}+12\sqrt[3]{x^5}-x+3\sqrt[6]{x}+2}{6\sqrt[3]{x^2}\sqrt{(x+1)^3}}$$

Con MATLAB

```
>> syms x
>> A=x^2+(x^(1/2))+(x^(1/3))

A =

x^2 + x^(1/2) + x^(1/3)

>> B=(x+1)^(1/2)

B =

(x + 1)^(1/2)

>> diff((A/B),x)

ans =

(2*x + 1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - (x^2 + x^(1/2) + x^(1/3))/
(2*(x + 1)^(3/2))

>> pretty(ans)

           1        1
  2 x + ------ + -----
           1/2      2/3     2     1/2     1/3
        2 x      3 x       x   + x     + x
 --------------------- - ---------------
               1/2                   3/2
        (x + 1)            2 (x + 1)
>> collect(ans)

ans =

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) - (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))
```

```
>> pretty(ans)

                 1        1
              ------ + ------
                 1/2      2/3         2           1/2    1/3
     2 x      2 x      3 x           x           x    + x
 ---------- + -------------- - ------------ - -----------
        1/2             1/2             3/2            3/2
 (x + 1)         (x + 1)        2 (x + 1)      2 (x + 1)

>> simple(ans)

simplify:

(x^(1/6)/2 - x/6 + 2*x^(5/3) + (3*x^(8/3))/2 + 1/3)/(x^(2/3)*(x + 1)^(3/2))

radsimp:

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) - (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

simplify(100):

(12*x + 9*x^2 + 3/x^(1/2) - x^(/3) + 2/x^(2/3))/(6*(x + 1)^(3/2))

combine(sincos):

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) - (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

combines(sinhcosh):

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) - (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

combine(ln):

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) - (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

factor:

-(x - 3*x^(1/6) - 12*x^(5/3) - 9*x^(8/3) - 2)/(6*x^(2/3)*(x + 1)^(3/2))

expand:

(2*x)/(x + 1)^(1/2) + 1/(2*x^(1/2)*(x + 1)^(1/2)) - x^2/(2*(x + 1)^(3/2)) - x^(1/2)/
(2*(x + 1)^(3/2)) + 1/(3*x^(2/3)*(x + 1)^(1/2)) - x^(1/3)/(2*(x + 1)^(3/2))

combine:

(2*x)/(x + 1)^(1/2) - (x^(1/2)/2 + x^(1/3)/2)/(x + 1)^(3/2) + (1/(2*x^(1/2)) + 1/
(3*x^(2/3)))/(x + 1)^(1/2) + x^2/(2*(x + 1)^(3/2))

rewrite(exp):

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) + (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))
```

```
rewrite(sincos):

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) + (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

rewrite(sinhcosh):

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) + (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

rewrite(tan):

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) + (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

collect(x):

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) + (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

mwcos2sin:

(2*x)/(x + 1)^(1/2) + (1/(2*x^(1/2)) + 1/(3*x^(2/3)))/(x + 1)^(1/2) - x^2/(2*(x +
1)^(3/2)) - (x^(1/2) + x^(1/3))/(2*(x + 1)^(3/2))

ans =

(12*x + 9*x^2 + 3/x^(1/2) - x^(1/3) + 2/x^(2/3))/(6*(x + 1)^(3/2))

>> pretty(ans)

             2     3     1/3     2
  12 x + 9 x  + ---- - x    + ----
                 1/2           2/3
                x             x
  -------------------------------
                      3/2
            6 (x + 1)

>> A=9*(x^(8/3))+(12*x^(5/3))-(x)+(3*(x^(1/6)))+2

A =

3*x^(1/6) - x + 12*x^(5/3) + 9*x^(8/3) + 2

>> C=(6*x^(2/3))

C =

6*x^(2/3)

>> D=(x+1)^(3/2)

D =

(x + 1)^(3/2)

>> collect(((A)/(C,D)),x)
```

```
ans =

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

>> pretty(ans)

     1/6       5/3      8/3            1/3
  3 x    + 12 x    + 9 x    + 2       x
  ----------------------------- - -------------
           2/3        3/2                  3/2
        6 x    (x + 1)            6 (x + 1)

>> simple(ans)

simplify:

(x^(1/6)/2 - x/6 + 2*x^(5/3) + (3*x^(8/3))/2 + 1/3)/(x^(2/3)*(x + 1)^(3/2))

radsimp:

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

simplify(100):

(12*x + 9*x^2 + 3/x^(1/2) - x^(1/3) + 2/x^(2/3))/(6*(x + 1)^(3/2))

combine(sincos):

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

combine(sinhcosh):

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

combine(ln):

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

factor:

-(x - 3*x^(1/6) - 12*x^(5/3) - 9*x^(8/3) - 2)/(6*x^(2/3)*(x + 1)^(3/2))

expand:

(2*x)/(x + 1)^(3/2) + (3*x^2)/(2*(x + 1)^(3/2)) + 1/(2*x^(1/2)*(x + 1)^(3/2)) -
x^(1/3)/(6*(x + 1)^(3/2)) + 1/(3*x^(2/3)*(x + 1)^(3/2))

combine:

(x^(1/6)/2 + 2*x^(5/3) + (3*x^(8/3))/2 + 1/3)/(x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/
(6*(x + 1)^(3/2))

rewrite(exp):

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))
```

```
rewrite(sincos):

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

rewrite(sinhcosh):

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

rewrite(tan):

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

collect(x):

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

mwcos2sin:

(3*x^(1/6) + 12*x^(5/3) + 9*x^(8/3) + 2)/(6*x^(2/3)*(x + 1)^(3/2)) - x^(1/3)/(6*(x
+ 1)^(3/2))

ans =

(12*x + 9*x^2 + 3/x^(1/2) - x^(1/3) + 2/x^(2/3))/(6*(x + 1)^(3/2))

>> pretty(ans)

            2     3     1/3     2
  12 x + 9 x  + ---- - x    + ----
                 1/2           2/3
                x             x
  --------------------------------
                      3/2
            6 (x + 1)
```

EJEMPLO 3

$$y=\left(x^{\frac{1}{2}}+5\right)^{4}\left(x^{2}+3x\right)^{5}$$

$$y'=4\left(x^{\frac{1}{2}}+5\right)^{3}\left(\frac{1}{2}x^{\frac{-1}{2}}\right)\left(x^{2}+3x\right)^{5}+\left(x^{\frac{1}{2}}+5\right)^{4}(5)\left(x^{2}+3x\right)^{4}(2x+3)$$

Simplificando y factorizando:

$$y'=x^{4}(x+3)^{4}\left(\sqrt{x}+5\right)^{3}\left(12x^{\frac{3}{2}}+50x+21\sqrt{x}+75\right)$$

Con MATLAB

```
>> syms x
>> A=(x^(1/2)+5)^(4)

A =

(x^(1/2) + 5)^4

>> B=(x^2+3*x)^(5)

B =

(x^2 + 3*x)^5

>> diff((A*B),x)

ans =

(2*(x^2 + 3*x)^5*(x^(1/2) + 5)^3)/x^(1/2) + 5*(x^2 + 3*x)^4*(2*x + 3)*(x^(1/2) + 5)^4

>>pretty(ans)

      2       5   1/2     3
  2 (x  + 3 x)  (x    + 5)        2       4             1/2     4
  ------------------------- + 5 (x  + 3 x)  (2 x + 3) (x    + 5)
              1/2
             x

>> simple(ans)

simplify:

x^4*(x^(1/2) + 5)^3*(x + 3)^4*(50*x + 21*x^(1/2) + 12*x^(3/2) + 75)

radsimp:

(2*(x^2 + 3*x)^5*(x^(1/2) + 5)^3)/x^(1/2) + 5*(x^2 + 3*x)^4*(2*x + 3)*(x^(1/2) + 5)^4

simplify(100):

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

combine(sincos):

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

combine(sinhcosh):

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

combine(ln)

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

factor:

(x^(1/2))^8*(50*x+21*x^(1/2)+12*x^(3/2)+75)*(x^(1/2)+5)^3*(x+3)^4

expand:

759375*x^4 + 1737450*x^5 + 1608201*x^6 + 777240*x^7 + 208305*x^8 + 29650*x^9+1815*x^10
+ 668250*x^(9/2) + 12*x^11 + 1347840*x^(11/2) + 1073250*x^(13/2) + 428400*x^(15/2)
+ 88350*x^(17/2) + 8400*x^(19/2) + 230*x^(21/2)
```

```
combine:

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+(x^2+3*x)^4*(10*x+15)*(x^(1/2)+5)^4

rewrite(exp):

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

rewrite(sincos):

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

rewrite(sinhcosh):

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

rewrite(tan):

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

collect(x):

x^9*(10*(x^(1/2)+5)^4+(30*(x^(1/2)+5)^3)/x^(1/2))+x^8*(135*(x^(1/2)+5)^4+(180*
(x^(1/2)+5)^3)/x^(1/2))+x^7*(720*(x^(1/2)+5)^4+(540*(x^(1/2)+5)^3)/x^(1/2))+x^6
*(1890*(x^(1/2)+5)^4+(810*(x^(1/2)+5)^3)/x^(1/2))+x^5*(2430*(x^(1/2)+5)^4+(486*
(x^(1/2)+5)^3)/x^(1/2))+1215*x^4*(x^(1/2)+5)^4+2*x^(19/2)*(x^(1/2)+5)^3

mwcos2sin:

(2*(x^2+3*x)^5*(x^(1/2)+5)^3)/x^(1/2)+5*(x^2+3*x)^4*(2*x+3)*(x^(1/2)+5)^4

ans=

x^4*(x^(1/2)+5)^3*(x+3)^4*(50*x+21*x^(1/2)+12*x^(3/2)+75)

>> pretty(ans)

   4   1/2      3        4             1/2        3/2
  x  (x    + 5)  (x + 3)  (50 x + 21 x    + 12 x    + 75)
```

Ejemplo 4

$$y=\frac{x^2+x+x^{0,5}}{\sqrt[4]{x^2+3}}$$

$$y=\frac{x^2+x+x^{0,5}}{\left(x^2+3\right)^{\frac{1}{4}}}$$

$$y'=\frac{\left(2x+1+0,5x^{-0,5}\right)\left(x^2+3\right)^{\frac{1}{4}}-\left(\frac{1}{4}\right)\left(x^2+3\right)^{\frac{-3}{4}}(2x)\left(x^2+x+x^{0,5}\right)}{\left(x^2+3\right)^{\frac{2}{4}}}$$

$$y'=\frac{\left(2x+1+0,5x^{-0,5}\right)\left(x^2+3\right)^{\frac{1}{4}}-\left(\frac{x}{2}\right)\left(x^2+x+x^{0,5}\right)\left(x^2+3\right)^{\frac{-3}{4}}}{\left(x^2+3\right)^{\frac{1}{2}}}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{3x^{\frac{7}{2}} + x^{\frac{5}{2}} + 12x^{\frac{3}{2}} + 6\sqrt{x} + 3}{2\sqrt{x}\left(x^2+3\right)^{\frac{5}{4}}}$$

Con MATLAB

```
>> syms x
>> A=(x^2+x+(x^(0.5)))
A =
x + x^2 + x^(1/2)

>> B=(x^2+3)^(1/4)
B =
(x^2 + 3)^(1/4)

>> diff((A/B),x)

ans =
(2*x + 1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4) - (x*(x + x^2 + x^(1/2)))/(2*(x^2 + 3)^(5/4))

>> pretty(ans)

          1
  2 x + ------ + 1
          1/2                  2    1/2
       2 x           x (x + x   + x    )
  ---------------- - ----------------
      2     1/4              2     5/4
    (x   + 3)           2 (x   + 3)

>> collect(ans)

ans =
(-1/(2*(x^2 + 3)^(5/4)))*x^3 + (-1/(2*(x^2 + 3)^(5/4)))*x^2 + (2/(x^2 + 3)^(1/4)
- x^(1/2)/(2*(x^2 + 3)^(5/4)))*x + (1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4)

>> pretty(ans)

                                                          1
                                                       ------ + 1
                      /                     1/2      \     1/2
      3       2       |      2               x         |   2 x
  #1 x   + #1 x   +   | ----------- - -------------| x + ----------
                      |   2     1/4       2     5/4 |        2     1/4
                      \ (x   + 3)      2 (x   + 3)    /     (x   + 3)

  where
                     1
     #1 == - -------------
                   2     5/4
              2 (x   + 3)

>> simple(ans)
simplify:
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(2*x^(1/2)*(x^2 + 3)^(5/4))
```

```
radsimp:
(1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4) - x^2/(2*(x^2 + 3)^(5/4)) - x^3/(2*(x^2 + 2)^(5/4))
+ x^(2/(x^2 + 3)^(1/4) - x^(1/2)/(2*(x^2 + 3)^(5/4)))

simplify(100):
(12*x + x^2 + 3/x^(1/2) + 3*x^3 + 6)/(2*(x^2 + 3)^(5/4))

combine(sincos):
(1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4) - x^2/(2*(x^2 + 3)^(5/4)) - x^3/(2*(x^2 + 3)^(5/4))
+ x*(2/(x^2 + 3)^(1/4) - x^(1/2)/(2*(x^2 + 3)^(5/4)))

combine(sinhcosh):
(1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4) - x^2/(2*(x^2 + 3)^(5/4)) - x^3/(2*(x^2 + 3)^(5/4))
+ x*(2/(x^2 + 3)^(1/4) - x^(1/2)/(2*(x^2 + 3)^(5/4)))

combine(ln)
(-1/(2*(x^2 + 3)^(5/4)))*x^3 + (-1/(2*(x^2 + 3)^(5/4)))*x^2 + (2/(x^2 + 3)^(1/4) -
x^(1/2)/(2*(x^2 + 3)^(5/4)))*x + (1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4)

factor:
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(2*x^(1/2)*(x^2 + 3)^(5/4))

expand:
(2*x)/(x^2 + 3)^(1/4) + 1/(x^2 + 3)^(1/4) + 1/(2*x^(1/2)*(x^2 + 3)^(1/4)) - x^2/
(2*(x^2 + 3)^(5/4)) - x^3/(2*(x^2 + 3)^(5/4)) - x^(3/2)/(2*(x^2 + 3)^(5/4))

combine:
(1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4) - x^2/(2*(x^2 + 3)^(5/4)) - x^3/(2*(x^2 + 3)^(5/4))
+ x*(2/(x^2 + 3)^(1/4) - x^(1/2)/(2*(x^2 + 3)^(5/4)))

rewrite(exp):
(1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4) - x^2/(2*(x^2 + 3)^(5/4)) - x^3/(2*(x^2 + 3)^(5/4))
+ x*(2/(x^2 + 3)^(1/4) - x^(1/2)/(2*(x^2 + 3)^(5/4)))

rewrite(sincos):
(1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4) - x^2/(2*(x^2 + 3)^(5/4)) - x^3/(2*(x^2 + 3)^(5/4))
+ x*(2/(x^2 + 3)^(1/4) - x^(1/2)/(2*(x^2 + 3)^(5/4)))

rewrite(sinhcosh):
(-1/(2*(x^2 + 3)^(5/4)))*x^3 + (-1/(2*(x^2 + 3)^(5/4)))*x^2 + (2/(x^2 + 3)^(1/4)
- x^(1/2)/(2*(x^2 + 3)^(5/4)))*x + (1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4)

rewrite(tan):
(-1/(2*(x^2 + 3)^(5/4)))*x^3 + (-1/(2*(x^2 + 3)^(5/4)))*x^2 + (2/(x^2 + 3)^(1/4)
- x^(1/2)/(2*(x^2 + 3)^(5/4)))*x + (1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4)

mwcos2sin:
(1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4) - x^2/(2*(x^2 + 3)^(5/4)) - x^3/(2*(x^2 +
3)^(5/4)) + x*(2/(x^2 + 3)^(1/4) - x^(1/2)/(2*(x^2 + 3)^(5/4)))

collect(x):
(-1/(2*(x^2 + 3)^(5/4)))*x^3 + (-1/(2*(x^2 + 3)^(5/4)))*x^2 + (2/(x^2 + 3)^(1/4)
- x^(1/2)/(2*(x^2 + 3)^(5/4)))*x + (1/(2*x^(1/2)) + 1)/(x^2 + 3)^(1/4)

ans =
(12*x + x^2 + 3/x^(1/2) + 3*x^3 + 6)/(2*(x^2 + 3)^(5/4))
>> pretty(ans)

          2    3        3
  12 x + x  + ---- + 3 x  + 6
               1/2
              x
 ---------------------------
             2    5/4
         2 (x  + 3)
```

Manualmente:

```
>> E=(3*(x^(7/2)))
E =
3*x^(7/2)

>> F=x^(5/2)
F =
x^(5/2)

>> G=12*x^(3/2)
G =
12*x^(3/2)

>> H=(6*(x^(1/2)))
H =
6*x^(1/2)

>> I=3
I =
     3

>> J=x^(1/2)
J =
x^(1/2)
>> K=(x^2+3)^(5/4)

K =
(x^2 + 3)^(5/4)

>> collect (((E+F+G+H+I)/(J*K)),x)
ans =
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

>> pretty(ans)

     1/2       3/2     5/2       7/2
  6 x    + 12 x    + x    + 3 x    + 3
  ------------------------------------
             1/2   2     5/4
            x    (x  + 3)

>> simple(ans)
simplify:
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

radsimp:
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

simplify(100):
(12*x + x^2 + 3/x^(1/2) + 3*x^3 + 6)/(x^2 + 3)^(5/4)

combine(sincos):
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

combine(sinhcosh):
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

combine(ln):
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))
```

```
factor:
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

expand:
(12*x)/(x^2 + 3)^(5/4) + 6/(x^2 + 3)^(5/4) + x^2/(x^2 + 3)^(5/4) + 3/(x^(1/2)*(x^2
+ 3)^(5/4)) + (3*x^3)/(x^2 + 3)^(5/4)

combine:
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

rewrite(exp):
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

rewrite(sincos):
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

rewrite(sinhcosh):
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

rewrite(tan):
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

mwcos2sin:
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

collect(x):
(6*x^(1/2) + 12*x^(3/2) + x^(5/2) + 3*x^(7/2) + 3)/(x^(1/2)*(x^2 + 3)^(5/4))

ans =
(12*x + x^2 + 3/x^(1/2) + 3*x^3 + 6)/(x^2 + 3)^(5/4)

>> pretty(ans)

          2    3        3
  12 x + x  + ---- + 3 x  + 6
               1/2
              x
  ---------------------------
            2     5/4
          (x  + 3)
```

EJEMPLO 5

$$y = \left(x^2+5\right)^3\left(x^3+2x+3\right)^4$$

$$y' = 3\left(x^2+5\right)^2(2x)\left(x^3+2x+3\right)^4 + \left(x^2+5\right)^3(4)\left(x^3+2x+3\right)^3\left(3x^2+2\right)$$

Factorizando y simplificando:

$$y' = 2(x^2+5)^2\left(x^3+2x+3\right)^3\left(9x^4+40x^2+9x+20\right)$$

Con MATLAB

```
>> syms x
>> diff(((x^2+5)^(3))*((x^3+2*x+3)^(4)),x)
ans =
6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4 + 4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

>> pretty(ans)

         2     2   3           4       2     3     2        3           3
  6 x (x  + 5)  (x  + 2 x + 3)  + 4 (x  + 5)  (3 x  + 2) (x  + 2 x + 3)

>> simple(ans)
simplify:
2*(x^2 + 5)^2*(x^3 + 2*x + 3)^3*(9*x^4 + 40*x^2 + 9*x + 20)

radsimp:
6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4 + 4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

simplify(100):
2*(x^2 + 5)^2*(x^3 + 2*x + 3)^3*(9*x^4 + 40*x^2 + 9*x + 20)

combine(sincos):
6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4 + 4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

combine(sinhcosh):
6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4 + 4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

combine(ln):
6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4 + 4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

factor:
2*(9*x^4 + 40*x^2 + 9*x + 20)*(x^2 + 5)^2*(x + 1)^3*(x^2 - x + 3)^3

expand:

18*x^17 + 368*x^15 + 180*x^14 + 3066*x^13 + 3276*x^12 + 14052*x^11 + 23364*x^10 +
43220*x^9 + 83376*x^8 + 105168*x^7 + 161532*x^6 + 188826*x^5 + 182700*x^4 +
185660*x^3 + 125100*x^2 + 66150*x + 27000

combine:

6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4+(x^2 + 5)^3*(12*x^2 + 8)*(x^3 + 2*x + 3)^3

rewrite(exp):

6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4+4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

rewrite(sincos):

6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4+4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

rewrite(sinhcosh):

6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4+4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

rewrite(tan):

6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4+4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3
```

```
mwcos2sin:

6*x*(x^2 + 5)^2*(x^3 + 2*x + 3)^4+4*(x^2 + 5)^3*(3*x^2 + 2)*(x^3 + 2*x + 3)^3

collect(x):

18*x^17 + 368*x^15 + 180*x^14 + 3066*x^13 + 3276*x^12 + 14052*x^11 + 23364*x^10+
43220*x^9 + 83376*x^8 + 105168*x^7 + 161532*x^6 + 188826*x^5 + 182700*x^4 +
185660*x^3 + 125100*x^2 + 66150*x + 27000

ans =

2*(x^2 + 5)^29(x^3 + 2*x + 3)^3*(9*x^4 + 40*x^2 + 9*x + 20)

>> pretty(ans)

         2      2    3            3       4        2
   2 (x   + 5)  (x   + 2 x + 3)  (9 x   + 40 x   + 9 x + 20)
```

EJEMPLO 6

$$y=\frac{\sqrt{x^2+1}}{x^2-1}$$

$$y=\frac{\left(x^2+1\right)^{\frac{1}{2}}}{x^2-1}$$

$$y'=\frac{\frac{1}{2}\left(x^2+1\right)^{\frac{-1}{2}}(2x)\left(x^2-1\right)-\left(x^2+1\right)^{\frac{1}{2}}(2x)}{\left(x^2-1\right)^2}$$

Operando, eliminando exponentes negativos y simplificando:

$$y'=-\frac{x\left(x^2+3\right)}{\left(x^2-1\right)^2\sqrt{\left(x^2+1\right)}}$$

Con MATLAB

```
>> syms x

>> A=(x^2+1)^(1/2)

A =
(x^2 + 1)^(1/2)

>> B=(x^2)-1

B =
x^2 - 1
```

```
>> diff((A/B),x)

ans =
x/((x^2 - 1)*(x^2 + 1)^(1/2)) - (2*x*(x^2 + 1)^(1/2))/(x^2 - 1)^2

>> pretty(ans)

                                2     1/2
           x              2 x (x  + 1)
  -------------------- - ---------------
    2        2     1/2        2     2
  (x  - 1) (x  + 1)         (x  - 1)

>> collect(ans)

ans =
(- x^3 - 3*x)/((x^2 + 1)^(1/2)*x^4 + (-2*(x^2 + 1)^(1/2))*x^2 + (x^2 + 1)^(1/2))

>> pretty(ans)

                          3
                       - x  - 3 x
  ---------------------------------------------------
    2     1/2  4           2     1/2   2      2     1/2
  (x  + 1)    x  + (- 2 (x  + 1)   ) x  + (x  + 1)

>> simple(ans)

simplify:
-(x*(x^2 + 3))/((x^2 - 1)^2*(x^2 + 1)^(1/2))

radsimp:
-((x^3 + 3*x)*((x^2 + 1)^(1/2) + 2*x^2*(x^2 + 1)^(1/2) - x^4*(x^2 + 1)^(1/2)))/
(x^2 - (2*x^2*(x^2 + 1)^(1/2) - x^4*(x^2 + 1)^(1/2))^2 + 1)

simplify(100):
-(x*(x^2 + 3))/((x^2 - 1)^2*(x^2 + 1)^(1/2))

combine(sincos):
-(x^3 + 3*x)/((x^2 + 1)^(1/2) - 2*x^2*(x^2 + 1)^(1/2) + x^4*(x^2 + 1)^(1/2))

combine(sinhcosh):
-(x^3 + 3*x)/((x^2 + 1)^(1/2) - 2*x^2*(x^2 + 1)^(1/2) + x^4*(x^2 + 1)^(1/2))

combine(ln):
(- x^3 - 3*x)/((x^2 + 1)^(1/2)*x^4 + (-2*(x^2 + 1)^(1/2))*x^2 + (x^2 + 1)^(1/2))

factor:
-(x*(x^2 + 3))/((x^2 + 1)^(1/2)*(x - 1)^2*(x + 1)^2)

expand:
- x^3/((x^2 + 1)^(1/2) - 2*x^2*(x^2 + 1)^(1/2) + x^4*(x^2 + 1)^(1/2)) - (3*x)/((x^2
+ 1)^(1/2) - 2*x^2*(x^2 + 1)^(1/2) + x^4*(x^2 + 1)^(1/2))

combine:
-(x^3 + 3*x)/((x^2 + 1)^(1/2) - 2*x^2*(x^2 + 1)^(1/2) + x^4*(x^2 + 1)^(1/2))

rewrite(exp):
-(x^3 + 3*x)/((x^2 + 1)^(1/2) - 2*x^2*(x^2 + 1)^(1/2) + x^4*(x^2 + 1)^(1/2))
```

```
rewrite(sincos):
-(x^3 + 3*x)/((x^2 + 1)^(1/2) - 2*x^2*(x^2 + 1)^(1/2) + x^4*(x^2 + 1)^(1/2))

rewrite(sinhcosh):
(- x^3 - 3*x)/((x^2 + 1)^(1/2)*x^4 + (-2*(x^2 + 1)^(1/2))*x^2 + (x^2 + 1)^(1/2))

rewrite(tan):
(- x^3 - 3*x)/((x^2 + 1)^(1/2)*x^4 + (-2*(x^2 + 1)^(1/2))*x^2 + (x^2 + 1)^(1/2))

mwcos2sin:
-(x^3 + 3*x)/((x^2 + 1)^(1/2) - 2*x^2*(x^2 + 1)^(1/2) + x^4*(x^2 + 1)^(1/2))

collect(x):
(-(x^3 + 3*x)/((x^2 + 1)^(1/2)*x^4 + (-2*(x^2 + 1)^(1/2))*x^2 + (x^2 + 1)^(1/2))

ans =
-(x*(x^2 + 3))/((x^2 - 1)^2*(x^2 + 1)^(1/2))

>> pretty(ans)

              2
          x (x  + 3)
  - ---------------------
        2      2   2     1/2
      (x  - 1)  (x  + 1)
```

EJEMPLO 7

$$y = \frac{x\sqrt{x^2+1}}{x^2+1}$$

$$y = \frac{x}{\sqrt{x^2+1}} = \frac{x}{\left(x^2+1\right)^{\frac{1}{2}}}$$

$$y' = \frac{(1)\left(x^2+1\right)^{\frac{1}{2}} - x\left(\frac{1}{2}\right)\left(x^2+1\right)^{\frac{-1}{2}}(2x)}{\left(x^2+1\right)}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{1}{\left(x^2+1\right)^{\frac{3}{2}}}$$

Con MATLAB

```
>> syms x
>> A=(x*((x^2+1)^(1/2)))
A =
x*(x^2 + 1)^(1/2)
>> B=(x^2)+1
B =
x^2 + 1
```

```
>> diff((A/B),x)
ans =
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

>> pretty(ans)

                     2
     1              x
 ----------- - -----------
   2     1/2     2     3/2
 (x  + 1)      (x  + 1)

>> collect(ans)
ans =
(-1/(x^2 + 1)^(3/2))*x^2 + 1/(x^2 + 1)^(1/2)

>> pretty(ans)

 /        1      \  2         1
 | - ----------- | x  + ----------
 |     2     3/2 |        2     1/2
 \   (x  + 1)    /      (x  + 1)

>> simple(ans)

simplify:
1/(x^2 + 1)^(3/2)

radsimp:
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

simplify(100):
1/(x^2 + 1)^(3/2)

combine(sincos):
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

combine(sinhcosh):
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

combine(ln):
(-1/(x^2 + 1)^(3/2))*x^2 + 1/(x^2 + 1)^(1/2)

factor:
1/(x^2 + 1)^(3/2)

expand:
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

combine:
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

rewrite(exp):
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

rewrite(sincos):
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

rewrite(sinhcosh):
(-1/(x^2 + 1)^(3/2))*x^2 + 1/(x^2 + 1)^(1/2)+
```

```
rewrite(tan):
(-1/(x^2 + 1)^(3/2))*x^2 + 1/(x^2 + 1)^(1/2)+

mwcos2sin:
1/(x^2 + 1)^(1/2) - x^2/(x^2 + 1)^(3/2)

collect(x):
(-1/(x^2 + 1)^(3/2))*x^2 + 1/(x^2 + 1)^(1/2)

ans =
1/(x^2 + 1)^(3/2)

>> pretty(ans)

       1
  -----------
    2     3/2
  (x  + 1)
```

Ejemplo 8

$$y=\frac{x^2+5x+6}{\left(x^2+\sqrt{x}\right)^3}$$

$$y'=\frac{(2x+5)\left(x^2+\sqrt{x}\right)^3-\left(x^2+5x+6\right)(3)\left(x^2+\sqrt{x}\right)^2\left(2x+\frac{1}{2\sqrt{x}}\right)}{\left(x^2+\sqrt{x}\right)^6}$$

Operando, eliminando exponentes negativos y simplificando:

$$y'=-\frac{8x^{\frac{7}{2}}+50x^{\frac{5}{2}}-x^2+72x^{\frac{3}{2}}+5x+18}{2x^{\frac{5}{2}}\left(x^{\frac{3}{2}}+1\right)^4}$$

Con MATLAB

```
>> A=(x^2+5*x+6)

A =
x^2 + 5*x + 6

>> B=((x^2+(x^(1/2)))^3)

B =
(x^2 + x^(1/2))^3

>> diff((A/B),x)

ans =
(2*x + 5)/(x^2 + x^(1/2))^3 - (3*(2*x + 1/(2*x^(1/2)))*(x^2 + 5*x + 6))/(x^2 +
x^(1/2))^4
```

```
>> pretty(ans)

                    /         1   \    2
                  3 | 2 x + ------ | (x   + 5 x + 6)
                    |          1/2 |
    2 x + 5         \       2 x    /
  ------------ - ----------------------------------
    2    1/2 3               2    1/2 4
  (x   + x    )            (x   + x    )

>> collect(ans)
ans =
(x^2 - 5*x - 72*x^(3/2) - 50*x^(5/2) - 8*x^(7/2) - 18)/(8*x^7 + 8*x^4 + 2*x^(5/2)
+ 12*x^(11/2) + 2*x^(17/2))

>> pretty(ans)

   2             3/2       5/2      7/2
  x  - 5 x - 72 x    - 50 x    - 8 x    - 18
  -------------------------------------------
     7      4      5/2        11/2       17/2
  8 x  + 8 x  + 2 x    + 12 x     + 2 x

>> simple(ans)

simplify:
-(5*x - x^2 + 72*x^(3/2) + 50*x^(5/2) + 8*x^(7/2) + 18)/(2*x^(5/2)*(x^(3/2) + 1)^4)

radsimp:
-(5*x - x^2 + 72*x^(3/2) + 50*x^(5/2) + 8*x^(7/2) + 18)/(8*x^4 + 2*x^(5/2) + 8*x^7
+ 12*x^(11/2) + 2*x^(17/2))

simplify(100):
-((5*x - x^2 + 50*x^(5/2) + 8*x^(7/2) + 18)/(2*x^(5/2)) + 36/x)/(x^(3/2) + 1)^4

combine(sincos):

-(5*x-x^2+72*x^(3/2)+50*x^(5/2)+8*^(7/2)+18)/(8*x^4+2*x^(5/2)+8*x^7+12*x^(11/2)+
2*x^(17/2))

combine(sinhcosh):

-(5*x-x^2+72*x^(3/2)+50*x^(5/2)+8*x^(7/2)+18)/(8*x^4+2*x^(5/2)+8*x^7+12*x^(11/2)
+2*x^(17/2))

combine(ln):

(x^2-5*x-72*x^(3/2)-50*x^(5/2)-8*x^(7/2)-18)/(8*x^7+8*x^4+2*x^(5/2)+12*x^(11/2)
+2*x^(17/2))

factor:
(x^2-5*x-72*x^(3/2)-50*x^(5/2)-8*x^(7/2)-18)/(2*x^(5/2)*(x^(1/2)+1)^4*(x-
x^(1/2)+1)^4)

expand:
x^2/(8*x^4+2*x^(5/2)+8*x^7+12*x^(11/2)+2*x^(17/2))-(72*x^(3/2))/(8*x^4+2*x^(5/2
)+8*x^7+12*x^(11/2)+2*x^(17/2))-(50*x^(5/2))/(8*x^4+2*x^(5/2)+8*x^7+12*x^(11/2)
+2*x^(17/2))-(8*x^(7/2))/(8*x^4+2*x^(5/2)+8*x^7+12*x^(11/2)+2*x^(17/2))-(5*x)/
(8*x^4+2*x^(5/2)+8*x^7+12*x^(11/2)+2*x^(17/2))-18/(8*x^4+2*x^(5/2)+8*x^7+12*x^(1
1/2)+2*x^(17/2))
```

```
combine:
-(5*x-x^2+72*x^(3/2)+50*x^(5/2)+8*x^(7/2)+18)/(8*x^4+2*x^(5/2)+8*x^7+12*x^(11/2)
+2*x^(17/2))

rewrite(exp):
-(5*x-x^2+72*x^(3/2)+50*x^(5/2)+8*x^(7/2)+18)/(8*x^4+2*x^(5/2)+8*x^7+12*x^(11/2)
+2*x^(17/2))

rewrite(sincos):
-(5*x - x^2 + 72*x^(3/2) + 50*x^(5/2) + 8*x^(7/2) + 18)/(8*x^4 + 2*x^(5/2) + 8*x^7
+ 12*x^(11/2) + 2*x^(17/2))

rewrite(sinhcosh):
(x^2 - 5*x - 72*x^(3/2) - 50*x^(5/2) - 8*x^(7/2) - 18)/(8*x^7 + 8*x^4 + 2*x^(5/2)
+ 12*x^(11/2) + 2*x^(17/2))

rewrite(tan):
(x^2 - 5*x - 72*x^(3/2) - 50*x^(5/2) - 8*x^(7/2) - 18)/(8*x^7 + 8*x^4 + 2*x^(5/2)
+ 12*x^(11/2) + 2*x^(17/2))

mwcos2sin:
-(5*x - x^2 + 72*x^(3/2) + 50*x^(5/2) + 8*x^(7/2) + 18)/(8*x^4 + 2*x^(5/2) + 8*x^7
+ 12*x^(11/2) + 2*x^(17/2))

collect(x):
(x^2 - 5*x - 72*x^(3/2) - 50*x^(5/2) - 8*x^(7/2) - 18)/(8*x^7 + 8*x^4 + 2*x^(5/2)
+ 12*x^(11/2) + 2*x^(17/2))

ans =
-((5*x - x^2 + 50*x^(5/2) + 8*x^(7/2) + 18)/(2*x^(5/2)) + 36/x)/(x^(3/2) + 1)^4

>> pretty(ans)

           2       5/2      7/2
    5 x - x  + 50 x    + 8 x    + 18   36
    ------------------------------ + --
                  5/2                x
               2 x
  - ------------------------------------
                 3/2     4
               (x    + 1)
```

Ejemplo 9

$$y = \frac{1}{\sqrt[3]{x^3 + 3x + 2}}$$

$$y = \frac{1}{\left(x^3 + 3x + 2\right)^{\frac{1}{3}}} = \left(x^3 + 3x + 2\right)^{\frac{-1}{3}}$$

$$y' = -\frac{1}{3}\left(x^3 + 3x + 2\right)^{\frac{-4}{3}}\left(3x^2 + 3\right)$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = -\frac{x^2+1}{\sqrt[3]{\left(x^3+3x+2\right)^4}}$$

Con MATLAB

```
>> syms x
>> diff((1/((x^(3))+3*x+2)^(1/3)),x)

ans =
-(3*x^2 + 3)/(3*(x^3 + 3*x + 2)^(4/3))

>> pretty(ans)

             2
         3 x  + 3
  - -------------------
        3           4/3
    3 (x  + 3 x + 2)

>> collect(ans)

ans =
(-1/(x^3 + 3*x + 2)^(4/3))*x^2 - 1/(x^3 + 3*x + 2)^(4/3)

>> pretty(ans)

  /           1         \  2           1
  | - ----------------- | x  - -----------------
  |     3           4/3 |        3           4/3
  \   (x  + 3 x + 2)    /      (x  + 3 x + 2)

>> simple(ans)

simplify:
-(x^2 + 1)/(x^3 + 3*x + 2)^(4/3)

radsimp:
- 1/(x^3 + 3*x + 2)^(4/3) - x^2/(x^3 + 3*x + 2)^(4/3)

simplify(100):
-(x^2 + 1)/(x^3 + 3*x + 2)^(4/3)

combine(sincos):
- 1/(x^3 + 3*x + 2)^(4/3) - x^2/(x^3 + 3*x + 2)^(4/3)

combine(sinhcosh):
- 1/(x^3 + 3*x + 2)^(4/3) - x^2/(x^3 + 3*x + 2)^(4/3)

combine(ln):
(-1/(x^3 + 3*x + 2)^(4/3))*x^2 - 1/(x^3 + 3*x + 2)^(4/3)

factor:
-(x^2 + 1)/(x^3 + 3*x + 2)^(4/3)

expand:
- x^2/(x^3 + 3*x + 2)^(4/3) - 1/(x^3 + 3*x + 2)^(4/3)
```

```
combine:
- 1/(x^3 + 3*x + 2)^(4/3) - x^2/(x^3 + 3*x + 2)^(4/3)

rewrite(exp):
- 1/(x^3 + 3*x + 2)^(4/3) - x^2/(x^3 + 3*x + 2)^(4/3)

rewrite(sincos):
- 1/(x^3 + 3*x + 2)^(4/3) - x^2/(x^3 + 3*x + 2)^(4/3)

rewrite(sinhcosh):
(-1/(x^3 + 3*x + 2)^(4/3))*x^2 - 1/(x^3 + 3*x + 2)^(4/3)

rewrite(tan):
(-1/(x^3 + 3*x + 2)^(4/3))*x^2 - 1/(x^3 + 3*x + 2)^(4/3)

mwcos2sin:
- 1/(x^3 + 3*x + 2)^(4/3) - x^2/(x^3 + 3*x + 2)^(4/3)

collect(x):
(-1/(x^3 + 3*x + 2)^(4/3))*x^2 - 1/(x^3 + 3*x + 2)^(4/3)

ans =
-(x^2 + 1)/(x^3 + 3*x + 2)^(4/3)

>> pretty(ans)

          2
         x  + 1
  - -----------------
      3           4/3
    (x  + 3 x + 2)
```

Ejemplo 10

$$y = \frac{x^2+3}{\sqrt{x+1}}$$

$$y = \frac{x^2+3}{(x+1)^{\frac{1}{2}}}$$

$$y' = \frac{(2x)(x+1)^{\frac{1}{2}} - (x^2+3)\left(\frac{1}{2}\right)(x+1)^{\frac{-1}{2}}}{x+1}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{3x^2+4x-3}{2(x+1)^{\frac{3}{2}}}$$

Con MATLAB

```
>> syms x
>> A=(x^2+3)
A =
x^2 + 3

>> B=(x+1)^(1/2)
B =
(x + 1)^(1/2)

>> diff((A/B),x)
ans =
(2*x)/(x + 1)^(1/2) - (x^2 + 3)/(2*(x + 1)^(3/2))

>> pretty(ans)

                   2
     2 x          x  + 3
  ---------- - ------------
        1/2             3/2
  (x + 1)      2 (x + 1)

>> collect(ans)

ans =
(-1/(2*(x + 1)^(3/2)))*x^2 + (2/(x + 1)^(1/2))*x - 3/(2*(x + 1)^(3/2))

>> pretty(ans)

  /        1       \  2       2               3
  | - ------------ | x  + ---------- x - ------------
  |             3/2 |             1/2              3/2
  \   2 (x + 1)    /      (x + 1)        2 (x + 1)

>> simple(ans)

simplify:
(3*x^2 + 4*x - 3)/(2*(x + 1)^(3/2))

radsimp:
(2*x)/(x + 1)^(1/2) - 3/(2*(x + 1)^(3/2)) - x^2/(2*(x + 1)^(3/2))

simplify(100):
(3*x^2 + 4*x - 3)/(2*(x + 1)^(3/2))

combine(sincos):
(2*x)/(x + 1)^(1/2) - 3/(2*(x + 1)^(3/2)) - x^2/(2*(x + 1)^(3/2))

combine(sinhcosh):
(2*x)/(x + 1)^(1/2) - 3/(2*(x + 1)^(3/2)) - x^2/(2*(x + 1)^(3/2))

combine(ln):
(-1/(2*(x + 1)^(3/2)))*x^2 + (2/(x + 1)^(1/2))*x - 3/(2*(x + 1)^(3/2))

factor:
(3*x^2 + 4*x - 3)/(2*(x + 1)^(3/2))

expand:
(2*x)/(x + 1)^(1/2) - x^2/(2*(x + 1)^(3/2)) - 3/(2*(x + 1)^(3/2))
```

```
combine:
(2*x)/(x + 1)^(1/2) - 3/(2*(x + 1)^(3/2)) - x^2/(2*(x + 1)^(3/2))

rewrite(exp):
(2*x)/(x + 1)^(1/2) - 3/(2*(x + 1)^(3/2)) - x^2/(2*(x + 1)^(3/2))

rewrite(sincos):
(2*x)/(x + 1)^(1/2) - 3/(2*(x + 1)^(3/2)) - x^2/(2*(x + 1)^(3/2))

rewrite(sinhcosh):
(-1/(2*(x + 1)^(3/2)))*x^2 + (2/(x + 1)^(1/2))*x - 3/(2*(x + 1)^(3/2))

rewrite(tan):
(-1/(2*(x + 1)^(3/2)))*x^2 + (2/(x + 1)^(1/2))*x - 3/(2*(x + 1)^(3/2))

mwcos2sin:
(2*x)/(x + 1)^(1/2) - 3/(2*(x + 1)^(3/2)) - x^2/(2*(x + 1)^(3/2))

collect(x):
(-1/(2*(x + 1)^(3/2)))*x^2 + (2/(x + 1)^(1/2))*x - 3/(2*(x + 1)^(3/2))

ans =
(3*x^2 + 4*x - 3)/(2*(x + 1)^(3/2))

>> pretty(ans)

     2
  3 x  + 4 x - 3
  --------------
             3/2
   2 (x + 1)
```

Ejemplo 11

$$y = \sqrt{x^2+2} \cdot \sqrt[4]{x+1}$$

$$y = \left(x^2+2\right)^{\frac{1}{2}} (x+1)^{\frac{1}{4}}$$

$$y' = \frac{1}{2}\left(x^2+2\right)^{\frac{-1}{2}}(2x)(x+1)^{\frac{1}{4}} + \left(x^2+2\right)^{\frac{1}{2}}\left(\frac{1}{4}\right)(x+1)^{\frac{-3}{4}}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{5x^2+4x+2}{4\sqrt[4]{(x+1)^3}\sqrt{x^2+2}}$$

Con MATLAB

```
>> syms x
>> A=(x^2+2)^(1/2)

A =
(x^2 + 2)^(1/2)

>> B=(x+1)^(1/4)

B =
(x + 1)^(1/4)

>> pretty(ans)

    2     1/2              1/4
  (x  + 2)       x (x + 1)
  ------------ + ------------
           3/4     2     1/2
  4 (x + 1)      (x  + 2)

>> collect(ans)

ans =
((x + 1)^(1/4)/(x^2 + 2)^(1/2))*x + (x^2 + 2)^(1/2)/(4*(x + 1)^(3/4))

>> pretty(ans)

         1/4        2     1/2
  (x + 1)         (x  + 2)
  ----------- x + ------------
    2     1/2              3/4
  (x  + 2)        4 (x + 1)

>> simple(ans)

simplify:
(5*x^2 + 4*x + 2)/(4*(x^2 + 2)^(1/2)*(x + 1)^(3/4))

radsimp:
(x^2 + 2)^(1/2)/(4*(x + 1)^(3/4)) + (x*(x + 1)^(1/4))/(x^2 + 2)^(1/2)

simplify(100):
((5*x^2)/4 + x + 1/2)/((x^2 + 2)^(1/2)*(x + 1)^(3/4))

combine(sincos):
(x^2 + 2)^(1/2)/(4*(x + 1)^(3/4)) + (x*(x + 1)^(1/4))/(x^2 + 2)^(1/2)

combine(sinhcosh):
(x^2 + 2)^(1/2)/(4*(x + 1)^(3/4)) + (x*(x + 1)^(1/4))/(x^2 + 2)^(1/2)

combine(ln):
((x + 1)^(1/4)/(x^2 + 2)^(1/2))*x + (x^2 + 2)^(1/2)/(4*(x + 1)^(3/4))

factor:
(5*x^2 + 4*x + 2)/(4*(x^2 + 2)^(1/2)*(x + 1)^(3/4))

expand:
(x*(x + 1)^(1/4))/(x^2 + 2)^(1/2) + (x^2 + 2)^(1/2)/(4*(x + 1)^(3/4))
```

```
combine:
(x^2 + 2)^(1/2)/(4*(x + 1)^(3/4)) + (x*(x + 1)^(1/4))/(x^2 + 2)^(1/2)

rewrite(exp):
(x^2 + 2)^(1/2)/(4*(x + 1)^(3/4)) + (x*(x + 1)^(1/4))/(x^2 + 2)^(1/2)

rewrite(sincos):
(x^2 + 2)^(1/2)/(4*(x + 1)^(3/4)) + (x*(x + 1)^(1/4))/(x^2 + 2)^(1/2)

rewrite(sinhcosh):
((x + 1)^(1/4)/(x^2 + 2)^(1/2))*x + (x^2 + 2)^(1/2)/(4*(x + 1)^(3/4))

rewrite(tan):
((x + 1)^(1/4)/(x^2 + 2)^(1/2))*x + (x^2 + 2)^(1/2)/(4*(x + 1)^(3/4))

mwcos2sin:
(x^2 + 2)^(1/2)/(4*(x + 1)^(3/4)) + (x*(x + 1)^(1/4))/(x^2 + 2)^(1/2)

collect(x):
((x + 1)^(1/4)/(x^2 + 2)^(1/2))*x + (x^2 + 2)^(1/2)/(4*(x + 1)^(3/4))

ans =
(5*x^2 + 4*x + 2)/(4*(x^2 + 2)^(1/2)*(x + 1)^(3/4))

>> pretty(ans)

           2
        5 x  + 4 x + 2
  ------------------------
       2     1/2        3/4
   4 (x  + 2)    (x + 1)
```

EJEMPLO 12

$$y=\frac{x^2+3x+6}{\left(x^4+5\right)^{\frac{1}{3}}}$$

$$y'=\frac{(2x+3)\left(x^4+5\right)^{\frac{1}{3}}-\left(x^2+3x+6\right)\left(\frac{1}{3}\right)\left(x^4+5\right)^{\frac{-2}{3}}\left(4x^3\right)}{\left(x^4+5\right)^{\frac{2}{3}}}$$

Operando, eliminando exponentes negativos y simplificando:

$$y'=\frac{2\,x^5-\ 3\,x^4-\ 24\,x^3\ +30\,x+45}{3\left(x^4+5\right)^{\frac{4}{3}}}$$

Con MATLAB

```
>> syms x
>> A=(x^2+3*x+6)

A =
x^2 + 3*x + 6

>> B=(x^4+5)^(1/3)

B =
(x^4 + 5)^(1/3)

>> diff((A/B),x)

ans =
(2*x + 3)/(x^4 + 5)^(1/3) - (4*x^3*(x^2 + 3*x + 6))/(3*(x^4 + 5)^(4/3))

>> pretty(ans)

                   3   2
    2 x + 3     4 x  (x  + 3 x + 6)
  ----------- - ------------------
    4     1/3           4     4/3
  (x  + 5)          3 (x  + 5)

>> collect(ans)

ans =
(-4/(3*(x^4 + 5)^(4/3)))*x^5 + (-4/(x^4 + 5)^(4/3))*x^4 + (-8/(x^4 + 5)^(4/3))*x^3
+ (2/(x^4 + 5)^(1/3))*x + 3/(x^4 + 5)^(1/3)

>> pretty(ans)

  /    4    \  5   /   4  \  4   /   8  \  3        2                3
  | - ---- | x  + | - -- | x  + | - -- | x  + ----------- x + -----------
  \   3 #1 /      \   #1 /      \   #1 /        4     1/3        4     1/3
                                              (x  + 5)        (x  + 5)

  where

           4     4/3
     #1 == (x  + 5)

>> simple(ans)

simplify:
(2*x^5 - 3*x^4 - 24*x^3 + 30*x + 45)/(3*(x^4 + 5)^(4/3))

radsimp:
(2*x)/(x^4 + 5)^(1/3) + 3/(x^4 + 5)^(1/3) - (8*x^3)/(x^4 + 5)^(4/3) - (4*x^4)/(x^4
+ 5)^(4/3) - (4*x^5)/(3*(x^4 + 5)^(4/3))

simplify(100):
((2*x^5)/3 - x^4 - 8*x^3 + 10*x + 15)/(x^4 + 5)^(4/3)

combine(sincos):
(2*x)/(x^4 + 5)^(1/3) + 3/(x^4 + 5)^(1/3) - (8*x^3)/(x^4 + 5)^(4/3) - (4*x^4)/(x^4
+ 5)^(4/3) - (4*x^5)/(3*(x^4 + 5)^(4/3))
```

```
combine(sinhcosh):
(2*x)/(x^4 + 5)^(1/3) + 3/(x^4 + 5)^(1/3) - (8*x^3)/(x^4 + 5)^(4/3) - (4*x^4)/(x^4
+ 5)^(4/3) - (4*x^5)/(3*(x^4 + 5)^(4/3))

combine(ln):
(-4/(3*(x^4 + 5)^(4/3)))*x^5 + (-4/(x^4 + 5)^(4/3))*x^4 + (-8/(x^4 + 5)^(4/3))*x^3
+ (2/(x^4 + 5)^(1/3))*x + 3/(x^4 + 5)^(1/3)

factor:
(2*x^5 - 3*x^4 - 25*x^3 + 30*x + 45)/(3*(x^4 + 5)^(4/3))

expand:
(2*x)/(x^4 + 5)^(1/3) - (4*x^4)/(x^4 + 5)^(4/3) - (8*x^3)/(x^4 + 5)^(4/3) -
(4*x^5)/(3*(x^4 + 5)^(4/3)) + 3/(x^4 + 5)^(1/3)

combine:
(2*x)/(x^4 + 5)^(1/3) + 3/(x^4 + 5)^(1/3) - (8*x^3)/(x^4 + 5)^(4/3) - (4*x^4)/(x^4
+ 5)^(4/3) - (4*x^5)/(3*(x^4 + 5)^(4/3))

rewrite(exp):
(2*x)/(x^4 + 5)^(1/3) + 3/(x^4 + 5)^(1/3) - (8*x^3)/(x^4 + 5)^(4/3) - (4*x^4)/(x^4
+ 5)^(4/3) - (4*x^5)/(3*(x^4 + 5)^(4/3))

rewrite(sincos):
(2*x)/(x^4 + 5)^(1/3) + 3/(x^4 + 5)^(1/3) - (8*x^3)/(x^4 + 5)^(4/3) - (4*x^4)/(x^4
+ 5)^(4/3) - (4*x^5)/(3*(x^4 + 5)^(4/3))

rewrite(sinhcosh):
(-4/(3*(x^4 + 5)^(4/3)))*x^5 + (-4/(x^4 + 5)^(4/3))*x^4 + (-8/(x^4 + 5)^(4/3))*x^3
+ (2/(x^4 + 5)^(1/3))*x + 3/(x^4 + 5)^(1/3)

rewrite(tan):
(-4/(3*(x^4 + 5)^(4/3)))*x^5 + (-4/(x^4 + 5)^(4/3))*x^4 + (-8/(x^4 + 5)^(4/3))*x^3
+ (2/(x^4 + 5)^(1/3))*x + 3/(x^4 + 5)^(1/3)

mwcos2sin:
(2*x)/(x^4 + 5)^(1/3) + 3/(x^4 + 5)^(1/3) - (8*x^3)/(x^4 + 5)^(4/3) - (4*x^4)/(x^4
+ 5)^(4/3) - (4*x^5)/(3*(x^4 + 5)^(4/3))

collect(x):
(-4/(3*(x^4 + 5)^(4/3)))*x^5 + (-4/(x^4 + 5)^(4/3))*x^4 + (-8/(x^4 + 5)^(4/3))*x^3
+ (2/(x^4 + 5)^(1/3))*x + 3/(x^4 + 5)^(1/3)

ans =
((2*x^5)/3 - x^4 - 8*x^3 + 10*x + 15)/(x^4 + 5)^(4/3)

>> pretty(ans)

     5
  2 x     4      3
  ---- - x  - 8 x  + 10 x + 15
   3
  ----------------------------
            4    4/3
          (x  + 5)
```

EJEMPLO 13

$$y = \frac{(x+4)^2}{\sqrt{x^2+5x-6}}$$

$$y = \frac{(x+4)^2}{(x^2+5x-6)^{\frac{1}{2}}}$$

$$y' = \frac{2(x+4)(x^2+5x-6)^{\frac{1}{2}} - (x+4)^2\left(\frac{1}{2}\right)(x^2+5x-6)^{\frac{-1}{2}}(2x+5)}{(x^2+5x-6)}$$

Operando, eliminando exponentes negativos y simplificando:

$$y' = \frac{(x+4)(2x^2+7x-44)}{2\sqrt{(x^2+5x-6)^3}}$$

Con MATLAB

```
>> syms x
>> A=(x+4)^2

A =
(x + 4)^2

>> B=(x^2+5*x-6)^(1/2)

B =
(x^2 + 5*x - 6)^(1/2)

>> diff((A/B),x)
ans =
(2*x + 8)/(x^2 + 5*x - 6)^(1/2) - ((2*x + 5)*(x + 4)^2)/(2*(x^2 + 5*x - 6)^(3/2))

>> pretty(ans)

                                      2
       2 x + 8        (2 x + 5) (x + 4)
  ----------------- - -------------------
    2           1/2       2           3/2
  (x  + 5 x - 6)      2 (x  + 5 x - 6)

>> simple(ans)

simplify:
-(- 2*x^3 - 15*x^2 + 16*x + 176)/(2*(x^2 + 5*x - 6)^(3/2))

radsimp:
(2*x + 8)/(x^2 + 5*x - 6)^(1/2) - ((2*x + 5)*(x + 4)^2)/(2*(x^2 + 5*x - 6)^(3/2))

simplify(100):
((x + 4)*(2*x^2 + 7*x - 44))/(2*(x^2 + 5*x - 6)^(3/2))
```

```
combine(sincos):
(2*x + 8)/(x^2 + 5*x - 6)^(1/2) - ((2*x + 5)*(x + 4)^2)/(2*(x^2 + 5*x - 6)^(3/2))

combine(sinhcosh):
(2*x + 8)/(x^2 + 5*x - 6)^(1/2) - ((2*x + 5)*(x + 4)^2)/(2*(x^2 + 5*x - 6)^(3/2))

combine(ln):
(2*x + 8)/(x^2 + 5*x - 6)^(1/2) - ((2*x + 5)*(x + 4)^2)/(2*(x^2 + 5*x - 6)^(3/2))

factor:
((x + 4)*(2*x^2 + 7*x - 44))/(2*(x^2 + 5*x - 6)^(3/2))

factor:
((x+4)*(2*x^2+7*x-44))/(2*(x^2+5*x-6)^(3/2))

expand:
(2*x)/(x^2+5*x-6)^(1/2)-(36*x)/(x^2+5*x-6)^(3/2)+8/(x^2+5*x-6)^(1/2)-40/
(x^2+5*x-6)^(3/2)-(21*x^2)/(2*(x^2+5*x-6)^(3/2))-x^3/(x^2+5*x-6)^(3/2)

combine:
(2*x+8)/(x^2+5*x-6)^(1/2)-((x+4)^2*(x+5/2))/(x^2+5*x-6)^(3/2)

rewrite(exp):
(2*x+8)/(x^2+5*x-6)^(1/2)-((2*x+5)*(x+4)^2)/(2*(x^2+5*x-6)^(3/2))

rewrite(sincos):
(2*x+8)/(x^2+5*x-6)^(1/2)-((2*x+5)*(x+4)^2)/(2*(x^2+5*x-6)^(3/2))

rewrite(sinhcosh):
(2*x+8)/(x^2+5*x-6)^(1/2)-((2*x+5)*(x+4)^2)/(2*(x^2+5*x-6)^(3/2))

rewrite(tan):
(2*x+8)/(x^2+5*x-6)^(1/2)-((2*x+5)*(x+4)^2)/(2*(x^2+5*x-6)^(3/2))

mwcos2sin:
(2*x+8)/(x^2+5*x-6)^(1/2)-((2*x+5)*(x+4)^2)/(2*(x^2+5*x-6)^(3/2))

collect
(-1/(x^2+5*x-6)^(3/2))*x^3+(-21/(2*(x^2+5*x-6)^(3/2)))*x^2+(2/(x^2+5*x-6)^(1/2)-
36/(x^2+5*x-6)^(3/2))*x+8/(x^2+5*x-6)^(1/2)-40/(x^2+5*x-6)^(3/2)

ans =
((x+4)*(2*x^2+7*x-44))/(2*(x^2+5*x-6)^(3/2))

>> pretty(ans)

                2
  (x + 4) (2 x   + 7 x - 44)
  -------------------------
          2            3/2
     2 (x  + 5 x - 6)
```

Ejemplo 14

$$y=\frac{\left(x^2+3x+5\right)^5\left(x+3x^{-1}\right)}{\sqrt[3]{x^{-3}+3x+2}}$$

$$y=\frac{\left(x^2+3x+5\right)^5\left(x+3x^{-1}\right)}{\left(x^{-3}+3x+2\right)^{\frac{1}{3}}}$$

Para facilitar el proceso de derivación vamos a redefinir "y":

$$y=\frac{A(x)\cdot B(x)}{C(x)}$$

Donde: $A(x)=\left(x^2+3x+5\right)^5$, $B(x)=\left(x+3x^{-1}\right)$ y $C(x)=\sqrt[3]{x^{-3}+3x+2}$

Derivando la nueva expresión de "y":

$$y'=\left\{\frac{\left[A'(x)\cdot B(x)+A(x)\cdot B'(x)\right]\cdot C(x)-\left[A(x)\cdot B(x)\right]\cdot C'(x)}{\left[C(x)\right]^2}\right\}$$

Donde:

$$A'(x)=5\left(x^2+3x+5\right)^4(2x+3)$$

$$B'(x)=\left(1-3x^{-2}\right)$$

$$C'(x)=\left(\frac{1}{3}\right)\left(x^{-3}+3x+2\right)^{\frac{-2}{3}}\left(-3x^{-4}+3\right)$$

Reemplazando $A(x)$, $B(x)$, $C(x)$, $A'(x)$, $B'(x)$ y $C'(x)$ en " y'", operando y simplificando, tenemos:

$$y'=\frac{\left(x^2+3x+5\right)^4\left(32x^7+73x^6+124x^5+163x^4+24x^3-9x^2+40x+45\right)}{\left(3x^4+2x^3+1\right)^{\frac{4}{3}}}$$

Con MATLAB

```
>> syms x
>> A=(x^2+3*x+5)^(5)

A =

(x^2 + 3*x + 5)^5

>> B=(x+((3*x)^(-1)))

B =

X + 1/(3*x)

>> C=((x^(-3))+3*x+2)^(1/3)
```

```
C =

(3*x + 1/x^3 + 2)^(1/3)

>> diff(((A*B)/C),x

ans=

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

>> pretty(ans)

                                          /   1      \   2           5
  /      1  \             2           4   | ---- - 1 | (x  + 3 x + 5)
5 | x + --- | (2 x + 3) (x  + 3 x + 5)    |    2     |
  \     3 x /                             \ 3 x      /
--------------------------------------- - ---------------------------- +
          /        1     \1/3                 /        1     \1/3
          | 3 x + -- + 2 |                    | 3 x + -- + 2 |
          |        3     |                    |        3     |
          \       x      /                    \       x      /

   /      1  \ /  3     \   2           5
   | x + --- | | -- - 3 | (x  + 3 x + 5)
   \     3 x / |  4     |
               \ x      /
   --------------------------------------
              /        1     \4/3
            3 | 3 x + -- + 2 |
              |        3     |
              \       x      /

>> simple(ans)

simplify:

((x^2  +  3*x  +  5)^4*(32*x^7  +  73*x^6  +  (164*x^5)/3  +  27*x^4  +  (40*x^3)/3  +
(53*x^2)/3 + (40*x)/3 + 5))/(x^4*((3*x^4 + 2*x^3 + 1)/x^3)^(4/3))

radsimp:

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

simplify(100):

((32*x^8+73*x^7+(164*x^6)/3+27*x^5+(40*x^4)/3+(53*x^3)/3+(40*x^2)/3+5*x)*(x^
8+12*x^7+74*x^6+288*x^5+771*x^4+1440*x^3+1850*x^2+1500*x+625))/(x^5*(3*x+1/
x^3+2)^(4/3))

combine(sincos):

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))
```

```
combine(sinhcosh):

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

combine(In):

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

factor:

((96*x^7+219*x^6+164*x^5+81*x^4+40*x^3+53*x^2+40*x+15)*(x^2+3*x+5)^4)/
(3*x^4*((2*x^3+3*x^4+1)/x^3)^(4/3))

expand:

(28250*x)/(3*x+1/x^3+2)^(1/3)+(8330*x)/(3*(3*x+1/x^3+2)^(4/3))+23750/
(3*(3*x+1/x^3+2)^(1/3))+12856/(3*x+1/x^3+2)^(4/3)-3125/(3*x^2*(3*x+1/
x^3+2)^(1/3))+(53150*x^2)/(3*x+1/x^3+2)^(1/3)+(63924*x^3)/(3*x+1/
x^3+2)^(1/3)+16675/(x*(3*x+1/x^3+2)^(4/3))+(160400*x^4)/(3*(3*x+1/
x^3+2)^(1/3))+14125/(x^2*(3*x+1/x^3+2)^(4/3))-(8742*x^2)/(3*x+1/
x^3+2)^(4/3)+(32298*x^5)/(3*x+1/x^3+2)^(1/3)+23750/(3*x^3*(3*x+1/
x^3+2)^(4/3))-(47020*x^3)/(3*(3*x+1/x^3+2)^(4/3))+(42910*x^6)/(3*(3*x+1/
x^3+2)^(1/3))+3125/(x^4*(3*x+1/x^3+2)^(4/3))-(15406*x^4)/(3*x+1/
x^3+2)^(4/3)+(4600*x^7)/(3*x+1/x^3+2)^(1/3)+3125/(3*x^5*(3*x+1/x^3+2)^(4/3))-
(10578*x^5)/(3*x+1/x^3+2)^(4/3)+(1038*x^8)/(3*x+1/x^3+2)^(1/3)-(5368*x^6)/
(3*x+1/x^3+2)^(4/3)+(150*x^9)/(3*x+1/x^3+2)^(1/3)-(6127*x^7)/(3*(3*x+1/
x^3+2)^(4/3))+(11*x^10)/(3*x+1/x^3+2)^(1/3)-(575*x^8)/(3*x+1/x^3+2)^(4/3)-
(346*x^9)/(3*(3*x+1/x^3+2)^(4/3))-(15*x^10)/(3*x+1/x^3+2)^(4/3)-x^11/(3*x+1/
x^3+2)^(4/3)

combine:

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

rewrite(exp):

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

rewrite(sincos):

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

rewrite(sinhcosh):

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))
```

```
rewrite(tan):

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

mwcos2sin:

(5*(x+1/(3*x))*(2*x+3)*(x^2+3*x+5)^4)/(3*x+1/x^3+2)^(1/3)-((1/(3*x^2)-
1)*(x^2+3*x+5)^5)/(3*x+1/x^3+2)^(1/3)+((x+1/(3*x))*(3/x^4-3)*(x^2+3*x+5)^5)/
(3*(3*x+1/x^3+2)^(4/3))

ans=

((96*x^7+219*x^6+164*x^5+81*x^4+40*x^3+53*x^2+40*x+15)*(x^2+3*x+5)^4)/
(3*x^4*((2*x^3+3*x^4+1)/x^3)^(4/3))

>> pretty(ans)

      7         6         5        4        3        2                 2            4
((96 x  + 219 x  + 164 x  + 81 x  + 40 x  + 53 x  + 40 x + 15) (x  + 3 x + 5)

       /      / /    4      3     \1/3 \4 \
       |    4 | | 3 x  + 2 x  + 1 |    |  |
   ) / | 3 x  | | ---------------- |    |  |
       |      | |        3        |    |  |
       \      \ \       x         /    /  /
```

EJEMPLO 15

$$y=\frac{\sqrt{x^3+7x^2+4}}{x^2+x^{-2}+x}$$

$$y=\frac{\left(x^3+7x^2+4\right)^{\frac{1}{2}}}{x^2+x^{-2}+x}$$

$$y'=\frac{\frac{1}{2}\left(x^3+7x^2+4\right)^{\frac{-1}{2}}\left(3x^2+14x\right)\left(x^2+x^{-2}+x\right)-\left(x^3+7x^2+4\right)^{\frac{1}{2}}\left(2x-2x^{-3}+1\right)}{\left(x^2+x^{-2}+x\right)^2}$$

Operando, eliminando exponentes negativos y simplificando:

$$y'=-\frac{x\left(x^7+13x^6+16x^4+x^3-42x^2-16\right)}{2\sqrt{x^3+7x^2+4}\left(x^4+x^3+1\right)^2}$$

Con MATLAB

```
>> syms x
>> A=(x^3+(7*x^2)+4)^(1/3)

A =

(x^3 + 7*x^2 + 4)^(1/2)

>> B=(x^2+(x^(-2))+x)

B =

X + 1/x^2 + x^2

>> diff((A/B),x)

ans =

(3*x^2 + 14*x)/(2*(x^3 + 7*x^2 + 4)^(1/2)*(x + 1/x^2 + x^2)) - ((2*x - 2/x^3 +
1)*(x^3 + 7*x^2 + 4)^(1/2))/(x + 1/x^2 + x^2)^2

>> pretty(ans)

                                          /         2     \   3      2     1/2
                                          | 2 x - -- + 1 | (x  + 7 x  + 4)
                  2                       |         3     |
              3 x  + 14 x                 \        x      /
  -------------------------------------- ----------------------------------
     3      2     1/2 /      1    2 \             /      1    2 \2
  2 (x  + 7 x  + 4)   | x + -- + x  |             | x + -- + x  |
                      |      2      |             |      2      |
                      \     x       /             \     x       /

>> collect(ans)

ans=

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^2+
4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2+4)^
(1/2)+2*(x^3+7*x^2+4)^/1/2))

>> pretty(ans)

    8       7       5    4       3
- (x  + 13 x  + 16 x  + x  - 42 x  - 16 x) /

       3   3      2     1/2      4   3      2     1/2
   (4 x  (x  + 7 x  + 4)    + 4 x  (x  + 7 x  + 4)    +

      6   3      2     1/2      7   3      2     1/2
   2 x  (x  + 7 x  + 4)    + 4 x  (x  + 7 x  + 4)    +

      8   3      2     1/2        3      2     1/2
   2 x  (x  + 7 x  + 4)    + 2 (x  + 7 x  + 4)    )

>> simple(ans)

Simplify:

-(x*(x^7 + 13*x^6 + 16*x^4 + x^3 - 42*x^2 - 16))/(2*(x^4 + x^3 + 1)^2*(x^3 + 7*x^2
+ 4)^(1/2))
```

```
radsimp:

((x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)*(4*x^4*(x^3+7*x^2+4)^(1/2)-4*x^3*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2)))/(16*x^6*(x^3+7*x^2+4)-(4*x^4*(x^3+7*x^2+4)^(1
/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2+4)^(1/
2)+2*(x^3+7*x^2+4)^(1/2))^2)

simplify(100):

-(x*(x^7+13*x^6+16*x^4+x^3-42*x^2-16))/(2*(x^4+x^3+1)^2*(x^3+7*x^2+4)^(1/2))

combine(sincos):

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

combine(sinhcosh):

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

combine(In):

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

factor:

-(x*(x^7+13*x^6+16*x^4+x^3-42*x^2-16))/(2*(x^3+7*x^2+4)^(1/2)*(x^4+x^3+1)^2)

expand:

(42*x^3)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^2+4)^(1/2)+2*x^6*(x^3+7
*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2+4)^(1/2)+2*(x^3+7
*x^2+4)^(1/2))-x^4/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^2+4)^(1/2)+2*x^
6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2+4)^(1/2)+2*
(x^3+7*x^2+4)^(1/2))-(16*x^5)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^2+4)
^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2+
4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))-(13*x^7)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x
^3+7*x^2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*
(x^3+7*x^2+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))-x^8/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*
x^4*(x^3+7*x^2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+
2*x^8*(x^3+7*x^2+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))+(16*x)/(4*x^3*(x^3+7*x^2+4)
^(1/2)+4*x^4*(x^3+7*x^2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+
4)^(1/2)+2*x^8*(x^3+7*x^2+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

combine:

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^2
+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2+4
)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

rewrite(exp):

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))
```

```
rewrite(sincos):

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

rewrite(sinhcosh):

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^2
+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^((1/2)+2*x^8*(x^3+7*x^2+
4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

rewrite(tan):

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

collect(x)

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

mwcos2sin:

-(x^8+13*x^7+16*x^5+x^4-42*x^3-16*x)/(4*x^3*(x^3+7*x^2+4)^(1/2)+4*x^4*(x^3+7*x^
2+4)^(1/2)+2*x^6*(x^3+7*x^2+4)^(1/2)+4*x^7*(x^3+7*x^2+4)^(1/2)+2*x^8*(x^3+7*x^2
+4)^(1/2)+2*(x^3+7*x^2+4)^(1/2))

>> pretty(ans)

        7       6       4    3       2
    x (x  + 13 x  + 16 x  + x  - 42 x  - 16)
  - --------------------------------------
           4    3     2   3       2     1/2
       2 (x  + x  + 1)  (x  + 7 x  + 4)
```

1.3. DERIVADAS DE FUNCIONES EXPONENCIALES

Calcular la derivada de los siguientes ejercicios:

EJEMPLO 1

$$y = \frac{e^{x^2}}{x^2 + 1}$$

Aplicamos la fórmula de derivación del cociente entre dos funciones:

$$y' = \frac{2xe^{x^2}(x^2+1) - e^{x^2}(2x)}{(x^2+1)^2}$$

Simplificando:

$$y' = \frac{2xe^{x^2}(x^2)}{(x^2+1)^2} = \frac{2x^3e^{x^2}}{(x^2+1)^2}$$

Con MATLAB

```
>> syms x
>> A=exp(x^2)

A =
exp(x^2)

>> B=(x^2)+1

B =
x^2 + 1

>> diff((A/B),x)

ans =
(2*x*exp(x^2))/(x^2 + 1) - (2*x*exp(x^2))/(X^2 + 1)^2

>> pretty(ans)

           2              2
  2 x exp(x )   2 x exp(x )
  ----------- - -----------
     2              2     2
    x  + 1       (x  + 1)

>> collect(ans)

ans =
((2*exp(x^2))*x^3)/(x^4 + 2*x^2 + 1)

>> pretty(ans)

          2    3
  (2 exp(x )) x
  --------------
   4      2
  x  + 2 x + 1
```

```
>> simple(ans)

Simplify:
(2*x^3*exp(x^2))/(x^2 + 1)^2

radsimp:
(2*x^3*exp(x^2))/(x^4 + 2*x^2 +1)

simplify(100)
(2*x^3*exp(x^2))/(x^2 + 1)^2

combine(sincos):
(2*x^3*exp(x^2))/(x^4 + 2*x^2 +1)

combine(sinhcosh):
(2*x^3*exp(x^2))/(x^4 + 2*x^2 +1)

combine(ln):
((2*exp(x^2))*x^3)/(x^4 + 2*x^2 +1)

factor:
(2*exp(x^2)*x^3)/(x^2 + 1)^2

expand:
(2*x^3*exp(x^2))/(x^4 + 2*x^2 +1)

combine:
(2*x^3*exp(x^2))/(x^4 + 2*x^2 +1)

rewrite:
(2*x^3*exp(x^2))/(x^4 + 2*x^2 +1)

rewrite(sincos):
(x^3*(2*cos(x^2*i) - sin(x^2*i)*2*i))/(x^4 + 2*x^2 +1)

rewrite(sinhcosh):
(x^3*(2*cosh(x^2) + 2*sinh(x^2)))/(x^4 + 2*x^2 +1)

rewrite(tan)
-(2*x^3*(tan((x^2*i)/2) + i))/((tan((x^2*i)/2) - i)*(x^4 + 2*x^2 + 1))

mwcos2sin:
-(x^3*(sin(x^2*i)*2*i + 4*sin((x^2*i)/2)^2 - 2))/(x^4 + 2*x^2 + 1)

collect(x):
((2*exp(x^2))*x^3)/(x^4 + 2*x^2 + 1)

ans =
(2*x^3*exp(x^2))/(x^2 + 1)^2

>> pretty(ans)

     3      2
  2 x  exp(x )
  ------------
     2     2
   (x  + 1)
```

EJEMPLO 2

$$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Aplicamos la fórmula de derivación del cociente entre dos funciones:

$$y' = \frac{\left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right)\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}$$

$$y' = \frac{\left(e^x + e^{-x}\right)^2 - \left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$

Operando y simplificando obtenemos:

$$y' = \frac{4e^{2x}}{\left(e^{2x} + 1\right)^2}$$

Con MATLAB

```
>> syms x
>> A=exp(x)-exp(-x)

A =
exp(x) - exp(-x)

>> B=exp(x)+(exp(-x))

B =
exp(-x) + exp(x)

>> diff((A/B),x)

ans =
1 - (exp(-x) - exp(x))^2/(exp(-x) + exp(x))^2

>> pretty(ans)

                       2
      (exp(-x) - exp(x))
  1 - -------------------
                       2
      (exp(-x) + exp(x))

>> simple(ans)

simplify:
(4*exp(2*x))/(exp(2*x) + 1)^2

radsimp:
1 - (exp(-x) - exp(x))^2/(exp(-x) + exp(x))^2

simplify(100):
1/cosh(x)^2

combine(sincos):
1 - (exp(-x) - exp(x))^2/(exp(-x) + exp(x))^2
```

```
combine(sinhcosh):
1 - (exp(-x) - exp(x))^2/(exp(-x) + exp(x))^2

combine(ln):
1 - (exp(-x) - exp(x))^2/(exp(-x) + exp(x))^2

factor:
(4*exp(2*x))/(exp(2*x) + 1)^2

expand:
2/(exp(-2*x) + exp(2*x) + 2) - exp(-2*x)/(exp(-2*x) + exp(2*x) + 2) - exp(2*x)/
(exp(-2*x) + exp(2*x) + 2) + 1

combine:
1 - (exp(-x) - exp(x))^2/(exp(-x) + exp(x))^2

rewrite(exp):
1 - (exp(-x) - exp(x))^2/(exp(-x) + exp(x))^2

rewrite(sincos):
sin(x*i)^2/cos(x*i)^2 + 1

rewrite(sinhcosh):
1 - sinh(x)^2/cosh(x)^2

rewrite(tan):
1 - ((tan((x*i)/2) + i)/(tan((x*i)/2) - i) - (tan((x*i)/2) - i)/(tan((x*1)/2) +
i))^2/((tan(x*(i/2)) - i)/(tan(x*(i/2)) - i) + (tan(x*(i/2)) - i)/(tan(x*(1/2))
+ i))^2

mwcos2sin:
1 - sin(x*i)^2/(sin(x*i)^2 - 1)

collect(x)
1 - (exp(-x) - exp(x))^2/(exp(-x) + exp(x))^2

ans =
1/cosh(x)^2
```

Nota

Es importante recordar que:

$$\cosh = \frac{e^{x} + e^{-x}}{2}$$

La cual reemplazando en la solución final de MATLAB:

$$\frac{1}{\cosh(x)\,^\wedge 2}$$

Se obtiene

$$\frac{4e^{2x}}{\left(e^{2x} + 1\right)^{2}}$$

La cual es idéntica a la solución realizada manualmente.

EJEMPLO 3

$$y=\frac{Ae^{x^2+1}+B}{x^2+9x+\frac{5}{x^2}}$$

Aplicamos la fórmula de derivación del cociente entre dos funciones:

$$y'=\frac{(2x)\left(Ae^{x^2+1}\right)\left(x^2+9x+\frac{5}{x^2}\right)-\left(Ae^{x^2+1}+B\right)\left(2x+9-\frac{10}{x^3}\right)}{\left(x^2+9x+\frac{5}{x^2}\right)^2}$$

Factorizando y efectuando:

$$y'=\frac{x\left[Ae^{x^2+1}\left(2x^6+18x^5-2x^4-9x^3+10x^2+10\right)-B\left(2x^4+9x^3-10\right)\right]}{\left(x^4+9x^3+5\right)^2}$$

Con MATLAB

```
>> syms x A B
>> C=(A*(exp(x^2+1)))+B

C =
B + A*exp(x^2 + 1)

>> D=(x^2+9*x+(5/(x^2)))

D =
9*x + 5/x^2 + x^2

>> diff((C/D),x)

ans =
(2*A*x*exp(x^2 + 1))/(9*x + 5/x^2 + x^2) – ((B + A*exp(x^2 + 1))*(2*x – 10/x^3 +
9))/(9*x + 5/x^2 + x^2)^2

>> pretty(ans)

                                2        /       10     \
                   (B + A exp(x   + 1)) | 2 x - -- + 9 |
             2                           |        3     |
  2 A x exp(x   + 1)                     \       x      /
  ----------------- - -------------------------------------
          5     2               /      5     2 \2
    9 x + -- + x                | 9 x + -- + x  |
           2                    |        2      |
          X                     \       x       /

>> collect(ans)

ans=

((2*A*exp(x^2+1))*x^7+(18*A*exp(x^2+1))*x^6+(-2*B-2*A*exp(x^2+1))*x^5+(-9*B-
9*A*exp(x^2+1))*x^4+(10*A*exp(x^2+1))*x^3+(10*B+10*A*exp(x^2+1))*x)/(x^8+18*x^7+
81*x^6+10*x^4+90*x^3+25)
```

```
>> pretty(ans)

            7             6                      5                      4
  ((2 A #1) x  + (18 A #1) x  + (- 2 B – 2 A #1) x  + (- 9 B – 9 A #1) x  +

             3                            8       7       6       4       3
     (10 A #1) x  + (10 B + 10 A #1) x) / (x  + 18 x  + 81 x  + 10 x  + 90 x  +

     25)

  Where

                 2
     #1 == exp(x  + 1)

>> simple(ans)

simplify:

(x*(10*B+10*A*exp(x^2+1))-x^5*(2*B+2*A*exp(x^2+1))-x^4*(9*B+9*A*exp(x^2+1))+10*A
*x^3*exp(x^2+1)+18*A*x^6*exp(x^2+1)+2*A*x^7*exp(x^2+1))/(x^4+9*x^3+5)^2

radsimp:

(x*(10*B+10*A*exp(x^2+1))-x^5*(2*B+2*A*exp(x^2+1))-x^4*(9*B+9*A*exp(x^2+1))+10*A
*x^3*exp(x^2+1)+18*A*x^6*exp(x^2+1)+2*A*x^7*exp(x^2+1))/(x^8+18*x^7+81*x^6+10*x^
4+90*x^3+25)

simplify(100):

(9*B-2*B*x+9*A*exp(x^2+1)+2*A*x^3*exp(x^2+1)-2*A*x*exp(x^2+1))/(x^4+9*x^3+5)-
((B+A*exp(x^2+1))*(81*x^3-20*x+45))/(x^4+9*x^3+5)^2

combine(sincos):

(x*(10*B+10*A*exp(x^2+1))-x^5*(2*B+2*A*exp(x^2+1))-x^4*(9*B+9*A*exp(x^2+1))+10*A
*x^3*exp(x^2+1)+18*A*x^6*exp(x^2+1)+2*A*x^7*exp(x^2+1))/(x^8+18*x^7+81*x^6+10*x^
4+90*x^3+25)

combine(sinhcosh):

(x*(10*B+10*A*exp(x^2+1))-x^5*(2*B+2*A*exp(x^2+1))-x^4*(9*B+9*A*exp(x^2+1))+10*A
*x^3*exp(x^2+1)+18*A*x^6*exp(x^2+1)+2*A*x^7*exp(x^2+1))/(x^8+18*x^7+81*x^6+10*x^
4+90*x^3+25)

conbine(ln):

((2*A*exp(x^2+1))*x^7+(18*A*exp(x^2+1))*x^6+(-2*B-2*A*exp(x^2+1))*x^5+(-9*B-
9*A*exp(x^2+1))*x^4+(10*A*exp(x^2+1))*x^3+(10*B+10*A*exp(x^2+1))*x)/(x^8+18*x^7+
81*x^6+10*x^4+90*x^3+25)

factor:

(x*(10*B+10*A*exp(x^2+1)-9*B*x^3-2*B*x^4+10*A*x^2*exp(x^2+1)-9*A*x^3*exp(x^2+1)-
2*A*x^4*exp(x^2+1)+18*A*x^5*exp(x^2+1)+2*A*x^6*exp(x^2+1)))/(x^4+9*x^3+5)^2
```

```
expand:

(10*B*x)/(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)-(9*B*x^4)/(x^8+18*x^7+81*x^6+10*x
^4+90*x^3+25)-(2*B*x^5)/(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)+(10*A*x*esp(1)*esp
(x^2))/(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)+(10*A*x^3*exp(1)*exp(x^2))/(x^8+18*
x^7+81*x^6+10*x^4+90*x^3+25)-(9*A*x^4*exp(1)*esp(x^2))/(x^8+18*x^7+81*x^6+10*x
^4+90*x^3+25)-(2*A*x^5*exp(1)*esp(x^2))/(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)+(
18*A*x^6*exp(1)*esp(x^2))/(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)+(2*A*x^7*exp(1)
*esp(x^2))/(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)(18*A*x^6*exp(1)*exp(x^2))/(x^8+1
8*x^7+81*x^6+10*x^4+90*x^3+25)+(2*A*x^7*exp(1)*exp(x^2))/(x^8+18*x^7+81*x^6+10*x
^4+90*x^3+25)

combine:

(x*(10*B+10*A*exp(x^2+1))-x^5*(2*B+2*A*exp(x^2+1))-x^4*(9*B+9*A*exp(x^2+1))+10*A
*x^3*exp(x^2+1)+18*A*x^6*exp(x^2+1)+2*A*x^7*exp(x^2+1))/(x^8+18*x^7+81*x^6+10*x^
4+90*x^3+25)

rewrite(exp):

(x*(10*B+10*A*exp(x^2+1))-x^5*(2*B+2*A*exp(x^2+1))-x^4*(9*B+9*A*exp(x^2+1))+10*A
*x^3*exp(x^2+1)+18*A*x^6*exp(x^2+1)+2*A*x^7*exp(x^2+1))/(x^8+18*x^7+81*x^6+10*x^
4+90*x^3+25)

rewrite(sincos):

-(-x*(10*B-10*A*(sin(x^2*i+i)*i-cos(x^2*i+i)))+x^5*(2*B-2*A*(sin(x^2*i+i)*i-
cos(x^2*i+i)))+x^4*(9*B-9*A*(sin(x^2*i+i)*i-cos(x^2*i+i)))+10*A*x^3*(sin(x-
^2*i+i)*i-cos(x^2*i+i))+18*A*x^6*(sin(x^2*i+i)*i-cos(x^2*i+i))+2*A*x^7*(sin(x^2*
i+i)*i-cos(x^2*i+i)))/(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)

rewrite(sinhcosh):

(x*(10*B+10*A*(cosh(x^2+1)+sinh(x^2+1)))-x^5*(2*B+2*A*(cosh(x^2+1)+sinh
(x^2+1)))-x^4*(9*B+9*A*(cosh(x^2+1)+sinh(x^2+1)))+10*A*x^3*(cosh(x^2+1)+sinh(x^-
2+1))+18*A*x^6*(cosh(x^2+1)+sinh(x^2+1))+2*A*x^7*(cosh(x^2+1)+sinh(x^2+1)))/(x^8
+18*x^7+81*x^6+10*x^4+90*x^3+25)

rewrite(tan)

-(x^5*(2*B-(2*A*(tan(x^2*(i/2)+i/2)+i))/(tan(x^2*(i/2)+i/2)-i))+x^4*(9*B-
(9*A*(tan(x^2*(i/2)+i/2)+i))/(tan(x^2*(i/2)+i/2)-i))-x*(10*B-
(10*A*(tan(x^2*(i/2)+i/2)+i))/(tan(x^2*(i/2)+i/2)-i))+(10*A*x^3*(tan(x^2
*(i/2)+i/2)+i))/(tan(x^2*(i/2)+i/2)-i)+(18*A*x^6*(tan(x^2*(i/2)+i/2)+i))/
(tan(x^2*(i/2)+i/2)-i)+(2*A*x^7*(tan(x^2*(i/2)+i/2)+i))/(tan(x^2*(i/2)+i/2)-i))/
(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)

mwcos2sin:

-(-x*(10*B-10*A*(sin(x^2*i+i)*i+2*sin(x^2*(i/2)+(i/2)^2-1))+x^5*(2*B-2*A*(si
n(x^2*i+i)*i+2*sin(x^2*(i/2)+(i/2)^2-1))+x^4*(9*B-9*A*(sin(x^2*i+i)*i+2*si-
n(x^2*(i/2)+(i/2)^2-1))+10*A*x^3*(sin(x^2*i+i)*i+2*sin(x^2*(i/2)+(i/2)^2-
1)+18*A*x^6*(sin(x^2*i+i)*i+2*sin(x^2*(i/2)+(i/2)^2-1)+2*A*x^7*(sin(x^2*i+i)*-
i+2*sin(x^2*(i/2)+(i/2)^2-1))/(x^8+18*x^7+81*x^6+10*x^4+90*x^3+25)

collect(x):

((2*A*exp(x^2+1))*x^7+(18*A*exp(x^2+1))*x^6+(-2*B-2*A*exp(x^2+1))*x^5+(-9*B-
9*A*exp(x^2+1))*x^4+(10*A*exp(x^2+1))*x^3+(10*B+10*A*exp(x^2+1))*x)/(x^8+18*x^7+
81*x^6+10*x^4+90*x^3+25)
```

```
ans=

(9*B-2*B*x+9*A*exp(x^2+1)+2*A*x^3*exp(x^2+1)-2*A*x*exp(x^2+1))/(x^4+9*x^3+5)-
((B+A*exp(x^2+1))*(81*x^3-20*x+45))/(x^4+9*x^3+5)^2

>> pretty(ans)

                               3                                  3
  9 B - 2 B x + 9 A #1 + 2 A x  #1 - 2 A x #1   (B + A #1) (81 x  - 20 x + 45)
  ----------------------------------------- - ------------------------------
                  4      3                                4      3     2
                 X  + 9 x  + 5                          (x  + 9 x  + 5)

  where

                 2
     #1 == exp(x  + 1)
```

Ejemplo 4

$$y = \frac{e^x + e^{x^2} + e^{2x}}{e^x}$$

$$y = e^{-x}(e^x + e^{x^2} + e^{2x})$$

Aplicamos la fórmula de derivación del producto de dos funciones:

$$y' = -e^{-x}\left(e^x + e^{x^2} + e^{2x}\right) + e^{-x}\left(e^x + 2xe^{x^2} + 2e^{2x}\right)$$

Factorizando y efectuando:

$$y' = e^{-x}\left[e^{x^2}(2x-1) + e^{2x}\right]$$

$$y' = e^{x^2-x}(2x-1) + e^x$$

Con MATLAB

```
>> syms x
>> A=exp(x)+exp(x^2)+exp(2*x)

A =
exp(2*x) + exp(x^2) + exp(x)

>> B=exp(x)

B =
exp(x)

>> diff((A/B),x)

ans =

exp(-x)*(2*exp(2*x) + exp(x) + 2*x*exp(x^2)) - exp(-x)*(exp(2*x) + exp(x^2) +
exp(x))
```

```
>> pretty(ans)
                                         2
  exp(-x) (2 exp(2 x) + exp(x) + 2 x exp(x  )) -

                             2
     exp(-x) (exp(2 x) + exp(x  ) + exp(x))

>> collect(ans)

ans =
(2*exp(-x)*exp(x^2))*x  +  exp(-x)*(2*exp(2*x)  +  exp(x))  -  exp(-x)*(exp(2*x)  +
exp(x^2) + exp(x))

>> pretty(ans)

                 2
  (2 exp(-x) exp(x  )) x + exp(-x) (2 exp(2 x) + exp(x)) -

                             2
     exp(-x) (exp(2 x) + exp(x  ) + exp(x))

>> simple(ans)

simplify:
exp(x) - exp(x*(x - 1)) + 2*x*exp(x*(x - 1))

radsimp:
exp(-x)*(2*exp(2*x) + exp(x)) - exp(-x)*(exp(2*x) + exp(x^2) + exp(x)) + 2*x*exp(-
x)*exp(x^2)

simplify(100):
exp(x) - exp(x^2 - x) + 2*x*exp(x^2 - x)

combine(sincos):
exp(-x)*(2*exp(2*x) + exp(x)) - exp(-x)*(exp(2*x) + exp(x^2) + exp(x)) + 2*x*exp(-
x)*exp(x^2)

combine(sinhcosh):
exp(-x)*(2*exp(2*x) + exp(x)) - exp(-x)*(exp(2*x) + exp(x^2) + exp(x)) + 2*x*exp(-
x)*exp(x^2)

combine(ln):
(2*exp(-x)*exp(x^2))*x  +  exp(-x)*(2*exp(2*x)  +  exp(x))  -  exp(-x)*(exp(2*x)  +
exp(x^2) + exp(x))

factor:
(exp(2*x) - exp(x^2) + 2*x*exp(x^2))/exp(x)

expand:
exp(x) - exp(-x)*exp(x^2) + 2*x*exp(-x)*exp(x^2)

combine:
exp(-x)*(2*exp(2*x) + exp(x)) - exp(-x)*(exp(2*x) + exp(x^2) + exp(x)) + 2*x*exp(-
x)*exp(x^2)

rewrite(exp):
exp(-x)*(2*exp(2*x) + exp(x)) - exp(-x)*(exp(2*x) + exp(x^2) + exp(x)) + 2*x*exp(-
x)*exp(x^2)
```

```
rewrite(sincos):
(cos(x*i)+sin(x*i)*i)*(cos(x*i)+2*cos(x*2*i)-sin(x*i)*i-sin(x*2*i)*2*i)-
(cos(x*i)+sin(x*i)*i)*(cos(x*i)+cos(x*2*i)-sin(x*i)*i-sin(x*2*i)*i+cos(x^2*i)-
sin(x^2*i)*i)+2*x*(cos(x^2*i)-sin(x^2*i)*i)*(cos(x*i)+sin(x*i)*i)

rewrite(sinhcosh):
(cosh(x)-sinh(x))*(2*cosh(2*x)+2*sinh(2*x)+cosh(x)+sinh(x))-(cosh(x)-sinh(x
))*(cosh(2*x)+cosh(x^2)+sinh(2*x)+sinh(x^2)+cosh(x)+sinh(x))+2*x*(cosh(x)-
sinh(x))*(cosh(x^2)+sinh(x^2))

rewrite(tan):
-((tan((x*i)/2)-i)*((tan(x*i)+i)/(tan(x*i)-i)+(tan((x*i)/2)+i)/(tan((x*i)/2)-
i)+(tan((x^2*i)/2)+i)/(tan((x^2*i)/2)-i)))/(tan((x*i)/2)+i)+(((2*(tan(x*
i)+i))/(tan(x*i)-i)+(tan((x*i)/2)+i)/(tan((x*i)/2)-i))*(tan((x*i)/2)-i))/
(tan((x*i)/2)+i)+(2*x*(tan((x^2*i)/2)+i)*(tan((x*i)/2)-i))/((tan((x^2*i)/2)-
i)*(tan((x*i)/2)+i))

mwcos2sin:
-(sin(x*i)*i-2*sin((x*i)/2)^2+1)*(sin(x*i)*i+sin(x*2*i)*2*i+4*sin(x*i)^2+2*sin((
x*i)/2)^2-3)+(sin(x*i)*i-2*sin((x*i)/2)^2+1)*(sin(x*i)*i+sin(x*2*i)*i+sin(x^2*i)
*i+2*sin((x^2*i)/2)^2+2*sin(x*i)^2+2*sin((x*i)/2)^2-3)-2*x*(sin(x*i)*i-2*sin((x*
i)/2)^2+1)*(sin(x^2*i)*i+2*sin((x^2*i)/2)^2-1)

collect(x):
(2*exp(-x)*exp(x^2))*x+exp(-x)*(2*exp(2*x)+exp(x))-exp(-x)
*(exp(2*x)+exp(x^2)+exp(x))

ans=
exp(x)-exp(x^2-x)+2*x*exp(x^2-x)

>> pretty(ans)

                 2                  2
  exp(x) - exp(x  - x) + 2 x exp(x  -x)
```

Ejemplo 5

$$f(\theta) = e^{-\theta/2}\cos(2\pi\,\theta)$$

$$f'(\theta) = (-1/2)\ e^{-\theta/2}\cos(2\pi\theta) - 2\pi e^{-\theta/2}\operatorname{sen}(2\pi\theta)$$

Con MATLAB

```
>> syms x
>> diff(exp(-x/2)*(cos(2*pi*x)),x)

ans =
- (exp(-x/2)*cos(2*pi*x))/2 - 2*pi*exp(-x/2)*sin(2*pi*x)

>> pretty(ans)

        /   x \
    exp| - - | cos(2 pi x)
        \   2 /                        /   x \
  - --------------------- - 2 pi exp| - - | sin(2 pi x)
              2                        \   2 /
```

```
>> collect(ans)
ans =
- (exp(-x/2)*cos(2*pi*x))/2 - 2*pi*exp(-x/2)*sin(2*pi*x)

>> pretty(ans)

       /   x \
    exp| - - | cos(2 pi x)
       \   2 /                     /   x \
  - --------------------- - 2 pi exp| - - | sin(2 pi x)
              2                    \   2 /

>> simple(ans)

simplify:
-(exp(-x/2)*(cos(2*pi*x) + 4*pi*sin(2*pi*x)))/2

radsimp:
- (exp(-x/2)*cos(2*pi*x))/2 - 2*pi*exp(-x/2)*sin(2*pi*x)

simplify(100):
-(exp(-x/2)*(cos(2*pi*x) + 4*pi*sin(2*pi*x)))/2

combine(sincos):
- (exp(-x/2)*cos(2*pi*x))/2 - 2*pi*exp(-x/2)*sin(2*pi*x)

combinw(sinhcosh):
- (exp(-x/2)*cos(2*pi*x))/2 - 2*pi*exp(-x/2)*sin(2*pi*x)

combine(ln):
- (exp(-x/2)*cos(2*pi*x))/2 - 2*pi*exp(-x/2)*sin(2*pi*x)

factor:
-(cos(2*pi*x) + 4*pi*sin(2*pi*x))/(2*exp(x/2))

expand:
exp(-x/2)/2 - exp(-x/2)*cos(pi*x)^2 - 4*pi*exp(-x/2)*cos(pi*x)*sin(pi*x)

combine
- (exp(-x/2)*cos(2*pi*x))/2 - 2*pi*exp(-x/2)*sin(2*pi*x)

rewrite(exp):
-  (exp(-x/2)*(exp(-pi*x*2*i)/2  +  exp(pi*x*2*i)/2))/2  -  2*pi*exp(-x/2)*((exp(-
pi*x*2*i)*i)/2 - (exp(pi*x*2*i)*i)/2)

rewrite(sincos):
- (cos(2*pi*x)*(cos((x*i)/2) + sin((x*i)/2)*i))/2 - 2*pi*sin(2*pi*x)*(cos((x*i)/2)
+ sin((x*i)/2)*i)

rewrite(sinhcosh):
-  (cosh(pi*x*2*i)*(cosh(x/2)  -  sinh(x/2)))/2  +  pi*sinh(pi*x*2*i)*(cosh(x/2)  -
sinh(x/2))*2*i

rewrite(tan):
- ((tan(pi*x)^2 - 1)*(tan((x*i)/4) - i))/(2*(tan(pi*x)^2 + 1)*(tan((x*i)/4) + i))
+ (4*pi*tan(pi*x)*(tan((x*i)/4) - i))/((tan(pi*x)^2 + 1)*(tan((x*i)/4) + i))

mwcos2sin:
((2*sin(pi*x)^2 - 1)*(sin((x*i)/2)*i - 2*sin((x*i)/4)^2 + 1))/2 - 2*pi*sin(2*pi*
x)*(sin((x*i)/2)*i - 2*sin((x*i)/4)^2 + 1)
```

```
collect(x):
- (exp(-x/2)*cos(2*pi*x))/2 - 2*pi*exp(-x/2)*sin(2*pi*x)

ans =
-(exp(-x/2)*(cos(2*pi*x) + 4*pi*sin(2*pi*x)))/2

>> pretty(ans)

       /   x \
    exp| - - | (cos(2 pi x) + 4 pi sin(2 pi x))
       \   2 /
  - -------------------------------------------
                         2
```

EJEMPLO 6

$$f(\theta)=\frac{\operatorname{sen}\theta^2-\cos 2\theta}{\sqrt{\theta^2+e^{\theta^2}}}=\frac{\operatorname{sen}\theta^2-\cos 2\theta}{\left(\theta^2+e^{\theta^2}\right)^{\frac{1}{2}}}$$

$$f'(\theta)=\frac{\left(2\theta\cos\theta^2+2\operatorname{sen}2\theta\right)\sqrt{\theta^2+e^{\theta^2}}-\left(\operatorname{sen}\theta^2-\cos 2\theta\right)\left(\dfrac{2\theta+2\theta\, e^{\theta^2}}{2\sqrt{\theta^2+e^{\theta^2}}}\right)}{\left(\sqrt{\theta^2+e^{\theta^2}}\right)^2}$$

$$f'(\theta)=\frac{\dfrac{\left(2\theta\cos\theta^2+2\operatorname{sen}2\theta\right)\left(\theta^2+e^{\theta^2}\right)-\left(\operatorname{sen}\theta^2-\cos 2\theta\right)\left(\theta+\theta\, e^{\theta^2}\right)}{\sqrt{\theta^2+e^{\theta^2}}}}{\left(\sqrt{\theta^2+e^{\theta^2}}\right)^2}$$

$$f'(\theta)=\frac{(2)\left(\theta\cos\theta^2+\operatorname{sen}2\theta\right)\left(\theta^2+e^{\theta^2}\right)-(\theta)\ \left(\operatorname{sen}\theta^2-\cos 2\theta\right)\left(1+e^{\theta^2}\right)}{\left(\theta^2+e^{\theta^2}\right)^{\frac{3}{2}}}$$

Con MATLAB

```
>> syms x
>> A=(sin(x^2))-(cos(2*x))

A =
sin(x^2) - cos(2*x)

>> B=((x^2)+(exp(x^2)))^(1/2)

B =
(exp(x^2) + x^2)^(1/2)

>> diff((A/B),x)

ans =
(2*sin(2*x) + 2*x*cos(x^2))/(exp(x^2) + x^2)^(1/2) + ((cos(2*x) - sin(x^2))*(2*x
+ 2*x*exp(x^2)))/(2*(exp(x^2) + x^2)^(3/2))
```

```
>> pretty(ans)

                          2                      2                    2
  2 sin(2 x) + 2 x cos(x )   (cos(2 x) - sin(x )) (2 x + 2 x exp(x ))
  ------------------------ + ----------------------------------------
             2    2 1/2                          2    2 3/2
     (exp(x ) + x )                      2 (exp(x ) + x )

>> simple(ans)

simplify:
(2*sin(2*x)   +   2*x*cos(x^2))/(exp(x^2)   +   x^2)^(1/2)   +   (x*(cos(2*s)   -
sin(x^2))*(exp(x^2) + 1))/(exp(x^2) + x^2)^(3/2)

radsimp:
(2*sin(2*x) + 2*x*cos(x^2))/(exp(x^2) + x^2)^(1/2) + ((cos(2*s) - sin(x^2))*(2*x
+ 2*x*exp(x^2)))/(2*(exp(x^2) + x^2)^(3/2))

simplify(100):
(2*sin(2*x) + 2*x*cos(x^2))/(exp(x^2) + x^2)^(1/2) + ((cos(2*s) - sin(x^2))*(2*x
+ 2*x*exp(x^2)))/(2*(exp(x^2) + x^2)^(3/2))

combine(sincos):
(2*sin(2*x) + 2*x*cos(x^2))/(exp(x^2) + x^2)^(1/2) + ((cos(2*s) - sin(x^2))*(2*x
+ 2*x*exp(x^2)))/(2*(exp(x^2) + x^2)^(3/2))

combine(sinhcosh):
(2*sin(2*x) + 2*x*cos(x^2))/(exp(x^2) + x^2)^(1/2) + ((cos(2*s) - sin(x^2))*(2*x
+ 2*x*exp(x^2)))/(2*(exp(x^2) + x^2)^(3/2))

combine(ln):
(2*sin(2*x) + 2*x*cos(x^2))/(exp(x^2) + x^2)^(1/2) + ((cos(2*s) - sin(x^2))*(2*x
+ 2*x*exp(x^2)))/(2*(exp(x^2) + x^2)^(3/2))

factor:

(x*cos(2*x)-x*sin(x^2)+2*x^3*cos(x^3)+2*x^2*sin(2*x)+2*sin(2*x)*exp(x^2)+x*cos(2
*x)*exp(x^2)+2*x*cos(x^2)*exp(x^2)-x*sin(x^2)*exp(x^2))/(exp(x^2)+x^2)^(3/2)

expand:

(2*x*cos(x^2))/(exp(x^2)+x^2)^(1/2)-x/(exp(x^2)+x^2)^(3/2)+(2*x*cos(x)^2)/
(exp(x^2)+x^2)^(3/2)-(x*exp(x^2))/(exp(x^2)+x^2)^(3/2)-(x*sin(x^2))/(exp(x^2
)+x^2)^(3/2)+(4*cos(x)*sin(x))/(exp(x^2)+x^2)^(1/2)+(2*x*exp(x^2)*cos(x)^2)/
(exp(x^2)+x^2)^(3/2)-(x*sin(x^2)*exp(x^2))/(exp(x^2)+x^2)^(3/2)

combine:

(2*sin(2*x)+2*x*cos(x^2))/(exp(x^2)+x^2)^(1/2)+((cos(2*x)-
sin(x^2))*(2*x+2*x*exp(x^2)))/(2*(exp(x^2)+x^2)^(3/2))

rewrite(exp):

(exp(-x*2*i)*i-exp(x*2*i)*i+2*x*(exp(-x^2*i)/2+exp(x^2*i)/2))/(exp(x^
2)+x^2)^(1/2)+((2*x+2*x*exp(x^2))*(exp(-x*2*i)/2+exp(x*2*i)/2-(exp(-
x^2*i)*i)/2+(exp(x^2*i)*i)/2))/(2*(exp(x^2)+x^2)^(3/2))

rewrite(sincos):

(2*sin(2*x)+2*x*cos(x^2))/(cos(x^2*i)+sin(x^2*i)*(-i)+x^2)^(1/2)+((2*x+2*x*(
cos(x^2*i)-sin(x^2*i)*i))*(cos(2*x)-sin(x^2)))/(2*(cos(x^2*i)+sin(x^2*i)*(-
i)+x^2)^(3/2))
```

```
rewrite(sinhcosh):

-(sinh(x*2*i)*2*i-2*x*cosh(x^2*i))/(cosh(x^2)+sinh(x^2)+x^2)^(1/2)+((cosh(x*
2*i)+sinh(x^2*i)*i)*(2*x+2*x*(cosh(x^2)+sinh(x^2))))/(2*(cosh(x^2)+sinh(x^2)
+x^2)^(3/2))

rewrite(tan):

((4*tan(x))/(tan(x)^2+1)-(2*x*(tan(x^2/2)^2-1))/(tan(x^2/2)^2+1))/
(-(tan(x^2*(i/2))+i)/(tan(x^2*(i/2))-i)+x^2)^(1/2)-(((tan(x)^2-1)/
(tan(x)^2+1)+(2*tan(x^2/2))/(tan(x^2/2)^2+1))*(2*x-(2*x*(tan((x^2*i)/2)+i))/
(tan((x^2*i)/2)-i)))/(2*(-(tan(x^2*(i/2))+i)/(tan(x^2*(i/2))-i)+x^2)^(3/2))

mwcos2sin:

(2*sin(2*x)-2*x*(2*sin(x^2/2)^2-1))/(sin(x^2*i)*(-i)-
2*sin(x^2*(i/2))^2+x^2+1)^(1/2)-((2*x-2*x*(sin(x^2*i)*i+2*sin((x^2*i)/2)^2-
1))*(sin(x^2)+2*sin(x)^2-1))/(2*(sin(x^2*i)*(-i)-2*sin(x^2*(i/2))^2
+x^2+1)^(3/2))

collect(x):

((2*cos(x^2))/(exp(x^2)+x^2)^(1/2)+((cos(2*x)-sin(x^2))*(2*exp(x^2)+2))/(2*(exp(
x^2)+x^2)^(3/2)))*x+(2*sin(2*x))/(exp(x^2)+x^2)^(1/2)

ans =

(2*sin(2*x)+2*x*cos(x^2))/(exp(x^2)+x^2)^(1/2)+(x*(cos(2*x)-
sin(x^2))*(exp(x^2)+1))/(exp(x^2)+x^2)^(3/2)

>> pretty(ans)

                       2                        2         2
  2 sin(2 x) + 2 x cos(x )   x (cos(2 x) - sin(x )) (exp(x ) + 1)
  ------------------------ + -------------------------------------
            2     2 1/2                      2     2 3/2
     (exp(x ) + x )                    (exp(x ) + x )
```

Ejemplo 7

$$f(\theta)=\frac{\cos\theta \operatorname{sen}\left(e^{2\theta-1}\right)}{\operatorname{ctg}\theta}=\operatorname{sen}\theta \operatorname{sen}\left(e^{2\theta-1}\right)$$

$$f'(\theta)=\cos\theta \operatorname{sen}\left(e^{2\theta-1}\right)+2e^{2\theta-1}\operatorname{sen}\theta\cos\left(e^{2\theta-1}\right)$$

Con MATLAB

```
>> syms x

>> A=cos(x)*(sin(exp(2*x-1)))

A =

sin(exp(2*x-1))*cos(x)
```

```
>> B=cot(x)

B =

cot(x)

>> diff((A/B),x)

ans =

(2*exp(2*x-1)*cos(x)*cos(exp(2*x-1)))/cot(x)-(sin(exp(2*x-1))*sin(x))/
cot(x)+(sin(exp(2*x-1))*cos(x)*(cot(x)^2+1))/cot(x)^2

>> pretty(ans)

2 exp(2 x - 1) cos(x) cos(exp(2 x - 1))   sin(exp(2 x - 1)) sin(x)
--------------------------------------- - ------------------------ +
                 cot(x)                            cot(x)

                                    2
   sin(exp(2 x - 1)) cos(x) (cot(x)  + 1)
  ---------------------------------------
                          2
                   cot(x)

>> simple(ans)

>> simple(ans)

simplify:

(sin(exp(2*x-1))*cos(x)-sin(exp(2*x-1))*cot(x)*sin(x)+sin(exp(2*x-
1))*cos(x)*cot(x)^2+2*exp(2*x-1)*cos(x)*cot(x)*cos(exp(2*x-1)))/cot(x)^2)

radsimp:

(2*exp(2*x-1)*cos(x)*cos(exp(2*x-1)))/cot(x)-(sin(exp(2*x-1))*sin(x))/
cot(x)+(sin(exp(2*x-1))*cos(x)*(cot(x)^2+1))/cot(x)^2

simplify(100):

sin(exp(2*x-1))*cos(x)+2*exp(2*x-1)*sin(x)*cos(exp(2*x-1))

combine(sincos):

cos(x+exp(2*x-1))/(2*cot(x))-cos(exp(2*x-1)-x)/(2*cot(x))+(exp(2*x-
1)*cos(x+exp(2*x-1)))/cot(x)+(sin(x+exp(2*x-1))*(cot(x)^2+1))/
(2*cot(x)^2)+(cos(x-exp(2*x-1))*exp(2*x-1))/cot(x)-(sin(x-exp(2*x-
1))*(cot(x)^2+1))/(2*cot(x)^2)

combine(sinhcosh):

(2*exp(2*x-1)*cos(x)*cos(exp(2*x-1)))/cot(x)-(sin(exp(2*x-1))*sin(x))/
cot(x)+(sin(exp(2*x-1))*cos(x)*(cot(x)^2+1))/cot(x)^2

combine(ln):

(2*exp(2*x-1)*cos(x)*cos(exp(2*x-1)))/cot(x)-(sin(exp(2*x-1))*sin(x))/
cot(x)+(sin(exp(2*x-1))*cos(x)*(cot(x)^2+1))/cot(x)^2
```

```
factor:

(sin(exp(2*x-1))*cos(x)-sin(exp(2*x-1))*cot(x)*sin(x)+sin(exp(2*x-
1))*cos(x)*cot(x)^2+2*exp(2*x-1)*cos(x)*cot(x)*cos(exp(2*x-1)))/cot(x)^2

expand:

sin(exp(2*x)/exp(1))*cos(x)+(sin(exp(2*x)/exp(1))*cos(x))/cot(x)^2-
(sin(exp(2*x)/exp(1))*sin(x))/cot(x)+(2*cos(exp(2*x)/exp(1))*exp(2*x)*cos(x))/
(exp(1)*cot(x))

combine:

(2*exp(2*x-1)*cos(x)*cos(exp(2*x-1)))/cot(x)-(sin(exp(2*x-1))*sin(x))/
cot(x)+(sin(exp(2*x-1))*cos(x)*(cot(x)^2+1))/cot(x)^2

rewrite(exp):

(2*exp(2*x-1)*(exp(2*i*x)-1)*(exp(i*x)/2+1/(2*exp(i*x)))*(exp(i*exp
(2*x-1))/2+1/(2*exp(i*exp(2*x-1)))))/(i+i*exp(2*i*x))-((exp(2*i*x)-
1)^2*(exp(i*x)/2+1/(2*exp(i*x)))*((i*exp(i*exp(2*x-1)))/2-i/(2*exp(i*exp(2*x-
1))))*((i+i*exp(2*i*x))^2/(exp(2*i*x)-1)^2+1))/(i+i*exp(2*i*x))^2-
((exp(2*i*x)-1)*((i*exp(i*x))/2-i/(2*exp(i*x)))*((i*exp(i*exp(2*x-1)))/2-i/
(2*exp(i*exp(2*x-1)))))/(i+i*exp(2*i*x))

rewrite(sincos):

2*cos(cos(i-2*i*x)+i*sin(i-2*i*x))*sin(x)*(cos(i-2*i*x)+i*sin(i-2*i*x))-
(sin(cos(i-2*i*x)+i*sin(i-2*i*x))*sin(x)^2)/cos(x)+(sin(cos(i-2*i*x)+i*sin(i-
2*i*x))*sin(x)^2*(cos(x)^2/sin(x)^2+1))/cos(x)

rewrite(sinhcosh):

(i*sinh(-i*cosh(2*x-1)-i*sinh(2*x-1))*sinh(-i*x)^2)/cosh(-i*x)+2*i*cosh(-i*cosh(2*x-
1)-i*sinh(2*x-1))*sinh(-i*x)*(cosh(2*x-1)+sinh(2*x-1))+(i*sinh(-i*cosh(2*x-1)-
i*(sinh(2*x-1))*sinh(-i*x)^2*(cosh(-i*x)^2/sinh(-i*x)^2-1))/cosh(-i*x)

rewrite(tan):

(4*tan(x/2)*tan(-(i+tan(i*x-i/2))/(2*(i-tan(i*x-1/2*i))))*tan(x))/
((tan(x/2)^2+1)*(tan(-(i+tan(i*x-i/2))/(2*(i-tan(i*x-1/2*i))))^2+1))+(2*tan(-
(i+tan(i*x-i/2))/(2*(i-tan(i*x-1/2*i))))*tan(x)^2*(tan(x/2)^2-1)*(1/
tan(x)^2+1))/((tan(x/2)^2+1)*(tan(-(i+tan(i*x-i/2))/(2*(i-tan(i*x-1/2*i))))^2+
1))+(2*tan(x)*(tan(x/2)^2-1)*(tan(-(i+tan(i*x-i/2))/(2*(i-tan(i*x-1/2*i))))^2-
1)*(i+tan(i*x-i/2)))/((i-tan(i*x-1/2*i))*(tan(x/2)^2+1)*(tan(-(i+tan(i*x-i/2))/
(2*(i-tan(i*x-1/2*i))))^2+1))

collect(x):
(2*exp(2*x-1)*cos(x)*cos(exp(2*x-1)))/cot(x)-(sin(exp(2*x-1))*sin(x))/
cot(x)+(sin(exp(2*x-1))*cos(x)*(cot(x)^2+1))/cot(x)^2

mwcos2sin:

2*cos(cos(i-2*i*x)+i*sin(i-2*i*x))*sin(x)*(cos(i-2*i*x)+i*sin(i-2*i*x))-
(sin(cos(i-2*i*x)+i*sin(i-2*i*x))*sin(x)^2)/cos(x)+(sin(cos(i-2*i*x)+i*sin(i-
2*i*x))*sin(x)^2*(cos(x)^2/sin(x)^2+1))/cos(x)

ans =

sin(exp(2*x-1))*cos(x)+2*exp(2*x-1)*sin(x)*cos(exp(2*x-1))
```

EJEMPLO 8

$$f(\theta)=\frac{\csc\theta\ \text{sen}\left(e^{\theta^2-3\theta-1}\right)}{\sec\theta}=\text{ctg}\,\theta\ \text{sen}\left(e^{\theta^2-3\theta-1}\right)$$

$$f'(\theta)=\left[-\csc^2\theta\ \text{sen}\left(e^{\theta^2-3\theta-1}\right)+(2\theta-3)e^{\theta^2-3\theta-1}\ \text{ctg}\,\theta\ \cos\left(e^{\theta^2-3\theta-1}\right)\right]$$

Con MATLAB

```
>> syms x
>> A=cot(x)*(sin(exp(x^2-3*x-1)))

A =

cot(x)*sin(exp(x^2 - 3*x - 1))

>> diff((A),x)

ans =

exp(x^2 - 3*x - 1)*cos(exp(x^2 - 3*x - 1))*cot(x)*(2*x - 3) - sin(exp(x^2 - 3*x
- 1))*(cot(x)^2 + 1)

>> pretty(ans)

      2                      2
exp(x  - 3 x - 1) cos(exp(x  - 3 x - 1)) cot(x) (2 x - 3) -

             2                   2
   sin(exp(x  - 3 x - 1)) (cot(x)  + 1)
```

1.4. DERIVADAS DE FUNCIONES LOGARÍTMICAS I

Calcular la derivada de:

EJEMPLO 1

$$y=\ln\left(x+\sqrt{x^2-1}\right)$$

$$y'=\frac{1+\dfrac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}=\frac{1}{\sqrt{x^2-1}}$$

Con MATLAB

```
>> syms x
>> y=log(x+((x^2-1)^(1/2)))

y =

log(x + (x^2 - 1)^(1/2))
```

```
>> diff(y)

ans =

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

>> simple(ans)

simplify:

1/(x^2 - 1)^(1/2)

radsimp:

(x - (x^2 - 1)^(1/2))*(x/(x^2 - 1)^(1/2) + 1)

simplify(100):

1/(x^2 - 1)^(1/2)

combine(sincos):

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

combine(sinhcosh):

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

combine(ln):

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

factor:

1/(x^2 - 1)^(1/2)

expand:

1/(x + (x^2 - 1)^(1/2)) + x/((x^2 - 1)^(1/2)*(x + (x^2 - 1)^(1/2)))

combine:

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

rewrite(exp):

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

rewrite(sincos):

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

rewrite(sinhcosh):

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

rewrite(tan):

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

collect(x):

1/(x^2 - 1)^(1/2)
```

```
mwcos2sin:

(x/(x^2 - 1)^(1/2) + 1)/(x + (x^2 - 1)^(1/2))

ans=

1/(x^2 - 1)^(1/2)

>> pretty(ans)

       1
  -----------
    2     1/2
  (x  - 1)
```

Ejemplo 2

$$y = \ln\sqrt{\frac{x^2+2}{x-1}}$$

$$y = \frac{1}{2}\ln\left(\frac{x^2+2}{x-1}\right) = \frac{1}{2}\left[\ln\left(x^2+2\right) - \ln\left(x-1\right)\right]$$

$$y' = \frac{1}{2}\left[\frac{2x}{x^2+2} - \frac{1}{x-1}\right]$$

Con MATLAB

```
>> syms x
>> y=log(((x^2+2)/(x-1))^(1/2))

y =

log(((x^2 + 2)/(x - 1))^(1/2))

>> diff(y)

ans =

(((2*x)/(x - 1) - (x^2 + 2)/(x - 1)^2)*(x - 1))/(2*(x^2 + 2))

>> pretty(ans)

  /          2       \
  |  2 x    x  + 2   |
  | ----- - -------- | (x - 1)
  | x - 1          2 |
  \         (x - 1)  /
  -----------------------------
                2
           2 (x  + 2)
```

```
>> simple(ans)

simplify:

-(2*x - x^2 + 2)/(2*x^3 - 2*x^2 + 4*x - 4)

radsimp:

(((2*x)/(x - 1) - (x^2 + 2)/(x - 1)^2)*(x - 1))/(2*(x^2 + 2))

simplify(100):

x/(x^2 + 2) - 1/(2*(x - 1))

combine(sincos):

(((2*x)/(x-1)-(x^2+2)/(x-1)^2)*(x-1))/(2*(x^2 + 2))

combine(sinhcosh):

(((2*x)/(x-1)-(x^2+2)/(x-1)^2)*(x-1))/(2*(x^2 + 2))

combine(ln):

(((2*x)/(x-1)-(x^2+2)/(x-1)^2)*(x-1))/(2*(x^2 + 2))

factor:

(x^2-2*x-2)/(2*(x-1)*(x^2+2))

expand:

x^2/(2*(x^4-2*x^3+3*x^2-4*x+2))-x^3/(2*(x^4-2*x^3+3*x^2-4*x+2))-x/(x^3-x^2+2*x-
2)-x/(x^4-2*x^3+3*x^2-4*x+2)+x^2/(x^3-x^2+2*x-2)+1/(x^4-2*x^3+3*x^2-4*x+2)

combine:

(((2*x)/(x-1)-(x^2+2)/(x-1)^2)*(x-1))/(2*(x^2 + 2))

rewrite(exp):

(((2*x)/(x-1)-(x^2+2)/(x-1)^2)*(x-1))/(2*(x^2 + 2))

rewrite(sincos):

(((2*x)/(x-1)-(x^2+2)/(x-1)^2)*(x-1))/(2*(x^2 + 2))

rewrite(sinhcosh):

(((2*x)/(x - 1) - (x^2 + 2)/(x - 1)^2)*(x - 1))/(2*(x^2 + 2))

rewrite(tan):

(((2*x)/(x - 1) - (x^2 + 2)/(x - 1)^2)*(x - 1))/(2*(x^2 + 2))

collect(x):

-(2*x - x^2 + 2)/(2*x^3 - 2*x^2 + 4*x - 4)

mwcos2sin:

(((2*x)/(x - 1) - (x^2 + 2)/(x - 1)^2)*(x - 1))/(2*x^2 + 4)
```

```
ans =

x/(x^2 + 2) - 1/(2*(x - 1))

>> pretty(ans)

     x         1
  ------ - ----------
   2       2 (x - 1)
  x  + 2
```

EJEMPLO 3

$$y = x^2 \log\sqrt{\frac{x^3+1}{x^{0,5}+2x}}$$

$$y = x^2 \log\left(\frac{x^3+1}{x^{0,5}+2x}\right)^{\frac{1}{2}} = \frac{x^2}{2}\log\left(\frac{x^3+1}{x^{0,5}+2x}\right)$$

$$y = \frac{x^2}{2}\left[\log\left(x^3+1\right)-\log\left(x^{0,5}+2x\right)\right] = \frac{x^2}{2\ln 10}\left[\ln\left(x^3+1\right)-\ln\left(x^{0,5}+2x\right)\right]$$

$$y' = x\log\left(\frac{x^3+1}{x^{0,5}+2x}\right)+\frac{x^2}{2\ln 10}\left[\frac{3x^2}{\left(x^3+1\right)}-\frac{\left(0,5x^{-0,5}+2\right)}{\left(x^{0,5}+2x\right)}\right]$$

Eliminando exponentes negativos:

$$y' = x\log\left(\frac{x^3+1}{x^{0,5}+2x}\right)+\frac{x^2}{2\ln 10}\left[\frac{3x^2}{x^3+1}-\frac{1+4\sqrt{x}}{2x+4\sqrt{x^3}}\right]$$

Con MATLAB

```
>> syms x
>> A=x^3+1

A =

x^3 + 1

>> B=x^(0.5)+2*x

B =

2*x + x^(1/2)

>> y=x^2*log10((A/B)^(1/2))

y =

(x^2*log(((x^3 + 1)/(2*x + x^(1/2)))^(1/2)))/log(10)
```

```
>> diff(y)

ans =

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

>> pretty(ans)

      / /    3       \1/2 \
      | |   x  + 1   |    |
2 x ln| | ---------- |    |
      | |        1/2 |    |
      \ \ 2 x + x    /    /
--------------------------- +
           ln(10)

                   /               3      /    1      \ \
                   |             (x  + 1) | ------ + 2 | |
                   |      2               |    1/2     | |
    2         1/2  |   3 x                \ 2 x        / |
   x  (2 x + x   ) | ---------- - ---------------------- |
                   |       1/2                1/2 2      |
                   \ 2 x + x          (2 x + x   )       /
   -----------------------------------------------------
                                 3
                      2 ln(10) (x  + 1)

>> simple(ans)

simplify:

x*log(((x^3 + 1)/(2*x + x^(1/2)))^(1/log(10))) - (x/4 - (5*x^4)/4 + x^(3/2) -
2*x^(9/2))/(log(10)*(x^3 + 1)*(2*x^(1/2) + 1))

radsimp:

(2*x*log((-((x^3+1)*(2*x-x^(1/2)))/(x-4*x^2))^(1/2)))/log(10)-
(x^2*((3*x^2*(2*x-x^(1/2)))/(x-4*x^2)+((x^3+1)*(1/(2*x^(1/2))+2))/
(2*x+x^(1/2))^2)*(2*x+x^(1/2)))/(2*log(10)*(x^3+1))

simplify(100):

(x*log((x^3+1)/(2*x+x^(1/2))))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

combine(sincos):

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

combine(sinhcosh):

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

combine(ln):

x*log((((x^3+1)/(2*x+x^(1/2)))^(1/2))^(2/log(10)))+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))
```

```
factor:

((x^(1/2))^2*(8*log(((x^3+1)/(2*x+x^(1/2)))^(1/2))+16*x^(1/2)*log((
(x^3+1)/(2*x+x^(1/2)))^(1/2))+8*x^3*log(((x^3+1)/(2*x+x^(1/2)))^(1/2))+16*
x^(7/2)*log(((x^3+1)/(2*x+x^(1/2)))^(1/2))-4*x^(1/2)+5*x^3+8*x^(7/2)-1))/
(4*log(10)*(2*x^(1/2)+1)*(x+1)*(x^2-x+1))

expand:

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)-(x^2/(4*(x+4*x^2+4*x^(3/2)))-
(3*x^(9/2))/(2*(2*x+x^(1/2)))-(3*x^5)/(2*x+x^(1/2))+(2*x^3)/
(x+4*x^2+4*x^(3/2))+x^5/(4*(x+4*x^2+4*x^(3/2)))+(2*x^6)/
(x+4*x^2+4*x^(3/2))+(3*x^(5/2))/(2*(x+4*x^2+4*x^(3/2)))+(3*x^(11/2))/
(2*(x+4*x^2+4*x^(3/2))))/(log(10)*x^3+log(10))

combine:

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

rewrite(exp):

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

rewrite(sincos):

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

rewrite(sinhcosh):

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

rewrite(tan):

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

collect(x):

(16*x^(3/2)*log(((x^3+1)/(2*x+x^(1/2)))^(1/2))+16*x^(9/2)*log(((x^3+1)/(2*x+x^
(1/2)))^(1/2))+x*(8*log(((x^3+1)/(2*x+x^(1/2)))^(1/2))-1)+x^4*(8*log(((x^3+1)/
(2*x+x^(1/2)))^(1/2))+5)-4*x^(3/2)+8*x^(9/2))/(4*log(10)+8*x^(1/2)*log(10)+4*x^3
*log(10)+8*x^(7/2)*log(10))

mwcos2sin:

(2*x*log(((x^3+1)/(2*x+x^(1/2)))^(1/2)))/log(10)+(x^2*(2*x+x^(1/2))*((3*x^2)/
(2*x+x^(1/2))-((x^3+1)*(1/(2*x^(1/2))+2))/(2*x+x^(1/2))^2))/(2*log(10)*(x^3+1))

ans =

x*log(((x^3+1)/(2*x+x^(1/2)))^(1/log(10)))-(x/4-(5*x^4)/4+x^(3/2)-2*x^(9/2))/
(log(10)*(x^3+1)*(2*x^(1/2)+1))

>> pretty(ans)

       /                  1   \            4
       |               ------ |     x   5 x     3/2      9/2
       | /    3       \ln(10) |     - - ---- + x    - 2 x
       | |   x  + 1   |       |     4     4
   x ln| | ---------- |       | - ----------------------------
       | |        1/2 |       |            3          1/2
       \ \ 2 x + x    /       /   ln(10) (x  + 1) (2 x    + 1)
```

Ejemplo 4

$$y = \frac{\ln\left(x^2 + 6x - 3\right)}{\sqrt[3]{x^2 + 2x + 3}}$$

$$y = \left(x^2 + 2x + 3\right)^{\frac{-1}{3}} \cdot \ln\left(x^2 + 6x - 3\right)$$

$$y' = \frac{\left(-\frac{1}{3}\right)(2x+2)\cdot \ln\left(x^2 + 6x - 3\right)}{\sqrt[3]{\left(x^2 + 2x + 3\right)^4}} + \left(x^2 + 2x + 3\right)^{\frac{-1}{3}} \cdot \left(\frac{2x+6}{x^2 + 6x - 3}\right)$$

$$y' = -\frac{(2x+2)\cdot \ln\left(x^2 + 6x - 3\right)}{3\sqrt[3]{\left(x^2 + 2x + 3\right)^4}} + \frac{2x+6}{\left(x^2 + 6x - 3\right)\sqrt[3]{\left(x^2 + 2x + 3\right)}}$$

Con MATLAB

```
>> syms x
>> A=log(x^2+6*x-3)

A =

log(x^2 + 6*x - 3)

>> B=(x^2+2*x+3)^(1/3)

B =

(x^2 + 2*x + 3)^(1/3)

>> y=(A/B)

y =

log(x^2 + 6*x - 3)/(x^2 + 2*x + 3)^(1/3)

>> diff(y)

ans =

(2*x + 6)/((x^2 + 2*x + 3)^(1/3)*(x^2 + 6*x - 3)) - (log(x^2 + 6*x - 3)*(2*x +
2))/(3*(x^2 + 2*x + 3)^(4/3))

>> pretty(ans)

                                                  2
               2 x + 6                ln(x   + 6 x - 3) (2 x + 2)
  ------------------------------ - --------------------------
    2           1/3   2                      2           4/3
  (x   + 2 x + 3)    (x   + 6 x - 3)       3 (x   + 2 x + 3)
```

EJEMPLO 5

$$y=\frac{\log\left(\dfrac{\sqrt{x}+x}{x^2+3x+1}\right)}{\ln\left(\dfrac{x^2+1}{\sqrt{x}}\right)}$$

$$y=\frac{\log\left(\sqrt{x}+x\right)-\log\left(x^2+3x+1\right)}{\ln\left(x^2+1\right)-\dfrac{1}{2}\ln x}$$

$$y=\frac{\left(\dfrac{1}{\ln 10}\right)\left[\ln\left(\sqrt{x}+x\right)-\ln\left(x^2+3x+1\right)\right]}{\ln\left(x^2+1\right)-\dfrac{1}{2}\ln x}$$

$$y'=\frac{\left(\dfrac{1}{\ln 10}\right)\left[\dfrac{\dfrac{1}{2\sqrt{x}}+1}{\sqrt{x}+x}-\dfrac{2x+3}{x^2+3x+1}\right]\ln\left(\dfrac{x^2+1}{\sqrt{x}}\right)-\log\left(\dfrac{\sqrt{x}+x}{x^2+3x+1}\right)\left(\dfrac{2x}{x^2+1}-\dfrac{1}{2x}\right)}{\left[\ln\left(\dfrac{x^2+1}{\sqrt{x}}\right)\right]^2}$$

$$y'=\frac{\dfrac{\ln\left(\dfrac{x^2+1}{\sqrt{x}}\right)}{\ln 10}\left[\dfrac{2\sqrt{x}+1}{\sqrt{x^3}+2x}-\dfrac{2x+3}{x^2+3x+1}\right]-\log\left(\dfrac{\sqrt{x}+x}{x^2+3x+1}\right)\left(\dfrac{3x^2-1}{2x^3+2x}\right)}{\left[\ln\left(\dfrac{x^2+1}{\sqrt{x}}\right)\right]^2}$$

Con MATLAB

```
>> syms x
>> A=x^(1/2)+x

A =

x + x^(1/2)

>> B=x^2+3*x+1

B =

x^2 + 3*x + 1

>> C=x^2+1

C =

x^2 + 1
```

```
>> D=x^(1/2)

D =

x^(1/2)

>> y=log10(A/B)/(log(C/D))

y =

log((x + x^(1/2))/(x^2 + 3*x + 1))/(log(10)*log((x^2 + 1)/x^(1/2)))

>> diff(y)

ans=

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

>> pretty(ans)

/     1                                \
|  ------ + 1                          |
|     1/2                       1/2    |
|  2 x           (2 x + 3) (x + x   )  |   2
| ------------ - -------------------   | (X  + 3 X + 1)
|  2                  2           2    |
\ x  + 3 x + 1      (x  + 3 x + 1)     /
--------------------------------------------------------- +
                     /  2     \
                     | z  + 1 |      1/2
            ln(10) ln| ------ | (x + x   )
                     |   1/2  |
                     \  x     /

          /        1/2    \ /  2            \
    1/2   |    x + x      | | x  + 1     1/2 |
   x    ln| ------------  | | ------ - 2 x   |
          |  2            | |    3/2         |
          \ x  + 3 x + 1 / \ 2 x            /
   -------------------------------------------
                     /  2     \2
                     | x  + 1 |    2
            ln(10) ln| ------ |  (x  + 1)
                     |   1/2  |
                     \  x     /
>> simple(ans)

simplify:

-((3*x)/2+(3*x^2)/2-x^(1/2)+x^(5/2)-1/2)/(x^(1/2)*log(10)*log((x^2+1)/x^(1/2
))*(x+x^(1/2))*(x^2+3*x+1))-(log((x+x^(1/2))/(x^2+3*x+1))*((3*x^2)/2-1/2))/
(x*log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

radsimp:
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))-((x-x^(1/2))*((1/(2*x^(1/2))+1)/
(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/(x^2+3*x+1)^2)*(x^2+3*x+1))/
(log(10)*log((x^2+1)/x^(1/2))*(x-x^2))
```

```
simplify(100):

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

combine(sincos):

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

combine(sinhcosh):

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

combine(ln):
(x^(1/2)*log(((x+x^(1/2))/(x^2+3*x+1))^(1/log(10)))*((x^2+1)/(2*x^(3/2))-
2*x^(1/2)))/(log((x^2+1)/x^(1/2))^2*(x^2+1))+(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-
((2*x+3)*(x+x^(1/2)))/(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/
x^(1/2))*(x+x^(1/2)))

factor:
-(2*x^2*log((x^2+1)/x^(1/2))-log((x+x^(1/2))/(x^2+3*x+1))-log(1/
x^(1/2)*(x^2+1))-2*x^(1/2)*log((x^2+1)/x^(1/2))+3*x^3*log((x^2+1)/
x^(1/2))+3*x^4*log((x^2+1)/x^(1/2))+2*x^(9/2)*log((x^2+1)/
x^(1/2))+3*x^2*log((x+x^(1/2))/(x^2+3*x+1))-x^(1/2)*log((x+x^(1/2))/(x^2+3*x
+1))+9*x^3*log((x+x^(1/2))/(x^2+3*x+1))+3*x^4*log((x+x^(1/2))/(x^2+3*x+1))-
3*x^(3/2)*log((x+x^(1/2))/(x^2+3*x+1))+2*x^(5/2)*log((x+x^(1/2))/(x^2+3*
x+1))+9*x^(7/2)*log((x+x^(1/2))/(x^2+3*x+1))+3*x^(9/2)*log((x+x^(1/2))/
(x^2+3*x+1))+3*x*log((x^2+1)/x^(1/2))-3*x*log((x+x^(1/2))/(x^2+3*x+1)))/
(2*log((x^2+1)/x^(1/2))^2*log(10)*x*(x^(1/2)+1)*(x^2+3*x+1)*(x^2+1))

expand:

(log((x+x^(1/2))/(x^2+3*x+1))/(2*x)-(3*x*log((x+x^(1/2))/(x^2+3*x+1)))/2)/
(log(10)*x^2*log((x^2+1)/x^(1/2))^2+log(10)*log((x^2+1)/x^(1/2))^2)-
((11*x^2)/(x^4+6*x^3+11*x^2+6*x+1)+(3*x^(1/2))/(x^4+6*x^3+11*x^2+6*x+1
)+(9*x^3)/(x^4+6*x^3+11*x^2+6*x+1)+(2*x^4)/(x^4+6*x^3+11*x^2+6*x+1)+(11
*x^(3/2))/(x^4+6*x^3+11*x^2+6*x+1)+(9*x^(5/2))/(x^4+6*x^3+11*x^2+6*x+1)+(
2*x^(7/2))/(x^4+6*x^3+11*x^2+6*x+1)-(3*x)/(x^2+3*x+1)-1/(x^2+3*x+1)-x^2/
(x^2+3*x+1)-1/(2*x^(1/2)*(x^2+3*x+1))-(3*x^(1/2))/(2*(x^2+3*x+1))-x^(3/2)/
(2*(x^2+3*x+1))+(3*x)/(x^4+6*x^3+11*x^2+6*x+1))/(x*log(10)*log((x^2+1)/x^(1/2))+
x^(1/2)*log(10)*log((x^2+1)/x^(1/2)))

combine:

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2/2+1/2)/x^(3/2)-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))
```

```
rewrite(exp):

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

rewrite(sincos):

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

rewrite(sinhcosh):

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

rewrite(tan):

(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

collect(x):

-(2*x^(9/2)*log((x^2+1)/x^(1/2))-log((x+x^(1/2))/(x^2+3*x+1))-
2*x^(1/2)*log((x^2+1)/x^(1/2))-log(1/x^(1/2)*(x^2+1))+x^2*(2*lo
g((x^2+1)/x^(1/2))+2*log((x+x^(1/2))/(x^2+3*x+1)))+x^4*(3*log(
(x^2+1)/x^(1/2))+3*log((x+x^(1/2))/(x^2+3*x+1)))+x^3*(3*log((x^2+1)/
x^(1/2))+9*log((x+x^(1/2))/(x^2+3*x+1)))-x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))-
3*x^(3/2)*log((x+x^(1/2))/(x^2+3*x+1))+2*x^(5/2)*log((x+x^(1/2))/(x^2+3*
x+1))+9*x^(7/2)*log((x+x^(1/2))/(x^2+3*x+1))+3*x^(9/2)*log((x+x^(1/2))/
(x^2+3*x+1))+x*(3*log((x^2+1)/x^(1/2))-3*log((x+x^(1/2))/(x^2+3*x+1))))/
(2*x*log(10)*log((x^2+1)/x^(1/2))^2+6*x^2*log(10)*log((x^2+1)/x^(1/2))^2+4*x^3*l
og(10)*log((x^2+1)/x^(1/2))^2+6*x^4*log(10)*log((x^2+1)/x^(1/2))^2+2*x^(3/2)*log
(10)*log((x^2+1)/x^(1/2))^2+2*x^5*log(10)*log((x^2+1)/x^(1/2))^2+6*x^(5/2)*log(1
0)*log((x^2+1)/x^(1/2))^2+4*x^(7/2)*log(10)*log((x^2+1)/x^(1/2))^2+6*x^(9/2)*log
(10)*log((x^2+1)/x^(1/2))^2+2*x^(11/2)*log(10)*log((x^2+1)/x^(1/2))^2)

mwcos2sin:
(((1/(2*x^(1/2))+1)/(x^2+3*x+1)-((2*x+3)*(x+x^(1/2)))/
(x^2+3*x+1)^2)*(x^2+3*x+1))/(log(10)*log((x^2+1)/x^(1/2))*(x+x^(1/2)))+
(x^(1/2)*log((x+x^(1/2))/(x^2+3*x+1))*((x^2+1)/(2*x^(3/2))-2*x^(1/2)))/
(log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))

ans=

-((3*x)/2+(3*x^2)/2-x^(1/2)+x^(5/2)-1/2)/(x^(1/2)*log(10)*log((x^2+1)/x^(1/2
))*(x+x^(1/2))*(x^2+3*x+1))-(log((x+x^(1/2))/(x^2+3*x+1))*((3*x^2)/2-1/2))/
(x*log(10)*log((x^2+1)/x^(1/2))^2*(x^2+1))
```

```
>> pretty(ans)

                     2
           3 x   3 x      1/2    5/2
           --- + ---- - x     + x     - 1/2
            2      2
- -------------------------------------------------- -
                / 2     \
  1/2           | x  + 1 |       1/2    2
 x    ln(10) ln| ------ | (X + x    ) (x  + 3 x + 1)
                |  1/2   |
                \  x     /

     /        1/2   \ /    2       \
     |   x + x      | | 3 x        |
   ln| ------------ | | ---- - 1/2 |
     |  2           | \   2        /
     \ x  + 3 x + 1 /
   ---------------------------------
               / 2     \2
               | x  + 1 |    2
    x ln(10) ln| ------ |  (x  + 1)
               |  1/2   |
               \  x     /
```

EJEMPLO 6

$$y = \ln(\text{tg } x)^3$$

Por propiedades de los logaritmos: $y = 3\ln(\text{tgx})$

Derivando respecto de "x":

$$y' = 3\left(\frac{\sec^2 x}{\text{tgx}}\right)$$

Con MATLAB

```
>> syms x
>> y=log(tan(x))^(3)

y =

log(tan(x))^3

>> diff(y)

ans =

(3*log(tan(x))^2*(tan(x)^2 + 1))/tan(x)

>> pretty(ans)

             2         2
  3 ln(tan(x))  (tan(x)  + 1)
  ---------------------------
           tan(x)
```

Ejemplo 7

$$y = \ln\left(\frac{\operatorname{sen} x}{1-\cos^2 x}\right)$$

$$y = \ln\left(\frac{\operatorname{sen} x}{\operatorname{sen}^2 x}\right) = \ln\left(\frac{1}{\operatorname{sen} x}\right) = \ln(\operatorname{sen} x)^{-1} = -\ln(\operatorname{sen} x)$$

Derivando respecto de "x":

$$y' = -\left(\frac{\cos x}{\operatorname{sen} x}\right) = -\operatorname{ctg} x$$

Con MATLAB

```
>> syms x
>> A=sin(x)

A =

sin(x)

>> B=1-(cos(x)^2)

B =

1 - cos(x)^2

>> y=log(A/B)

y =

log(-xin(x)/(cos(x)^2 - 1))

>> diff(y)

ans =

((cos(x)/(cos(x)^2 - 1) + (2*cos(x)*sin(x)^2)/(cos(x)^2 - 1)^2)*(cos(x)^2 - 1))/sin(x)

>> pretty(ans)

  /                              2 \
  |    cos(x)      2 cos(x) sin(x)  |         2
  | ----------- + ---------------- | (cos(x)   - 1)
  |        2                2     2 |
  \ cos(x)   - 1     (cos(x)   - 1)   /
  ------------------------------------------------
                      sin(x)

>> simple(ans)

simplify:

-(cos(3*x)-cos(x))/(sin(3*x)-3*sin(x))

radsimp:

((cos(x)/(cos(x)^2-1)+(2*cos(x)*sin(x)^2)/(cos(x)^2-1)^2)*(cos(x)^2-1))/sin(x)
```

```
simplify(100):

-cot(x)

combine(sincos):

((cos(2*x)/2-1/2)*(cos(x)/(cos(2*x)/2-1/2)+cos(x)/(2*(cos(2*x)/2-1/2)^2)-
cos(3*x)/(2*(cos(2*x)/2-1/2)^2)))/sin(x)

combine(sinhcosh):

((cos(x)/(cos(x)^2-1)+(2*cos(x)*sin(x)^2)/(cos(x)^2-1)^2)*(cos(x)^2-1))/sin(x)

combine(ln):

((cos(x)/(cos(x)^2-1)+(2*cos(x)*sin(x)^2)/(cos(x)^2-1)^2)*(cos(x)^2-1))/sin(x)

factor:

(cos(x)*(cos(x)^2+2*sin(x)^2-1))/((cos(x)-1)*(cos(x)+1)*sin(x))

expand:

cos(x)/(sin(x)-cos(x)^2*sin(x))-cos(x)^3/(sin(x)-cos(x)^2*sin(x))+(2*cos(x)^3*sin
(x))/(cos(x)^4-2*cos(x)^2+1)-(2*cos(x)*sin(x))/(cos(x)^4-2*cos(x)^2+1)

combine(ln):

((cos(x)/(cos(x)^2-1)+(2*cos(x)*sin(x)^2)/(cos(x)^2-1)^2)*(cos(x)^2-1))/sin(x)

rewrite(exp):

-(((exp(i*x)/2+1/(2*exp(i*x)))^2-1)*((exp(i*x)/2+1/(2*exp(i*x)))/((exp(i*x)/2+1/
(2*exp(i*x)))^2-1)+(2*((i*exp(i*x))/2-i/(2*exp(i*x)))^2*(exp(i*x)/2+1/)2*exp(i
*x))))/((exp(i*x)/2+1/(2*exp(i*x)))^2-1)^2))/((i*exp(i*x))/2-i/(2*exp(i*x)))

rewrite(sincos):

((cos(x)/(cos(x)^2-1)+(2*cos(x)*sin(x)^2)/(cos(x)^2-1)^2)*(cos(x)^2-1))/sin(x)

rewrite(sinhcosh):

-(i*(cosh(-i*x)^2-1)*(cosh(-i*x)/(cosh(-i*x)^2-1)-(2*cosh(-i*x)*sinh(-i*x)^2)/
(cosh(-i*x)^2-1)^2))/sinh(-i*x)

rewrite(tan):

-((tan(x/2)^2+1)*((tan(x/2)^2-1)^2/(tan(x/2)^2+1)^2-1)*((tan(x/2)^2-1)/((tan(x
/2)^2+1)*((tan(x/2)^2-1)^2/(tan(x/2)^2+1)^2-1))+(8*tan(x/2)^2*(tan(x/2)^2-1))/
((tan(x/2)^2+1)^3*((tan(x/2)^2-1)^2/(tan(x/2)^2+1)^2-1)^2)))/(2*tan(x/2))

collect(x):

((cos(x)/(cos(x)^2 - 1) + (2*cos(x)*sin(x)^2)/(cos(x)^2 - 1)^2)*(cos(x)^2 - 1))/
sin(x)

mwcos2sin:

-cos(x)/sin(x)

ans =

-cot(x)
```

Ejemplo 8

$$y = \ln\left(e^{\cos x} - 3\right)$$

$$y' = \left[\frac{(-\operatorname{sen} x)\left(e^{\cos x}\right)}{e^{\cos x} - 3}\right]$$

Con MATLAB

```
>> syms x
>> y=log(exp(cos(x))-3)

y =

log(exp(cos(x)) - 3)

>> diff(y)

ans =

-(exp(cos(x))*sin(x))/(exp(cos(x)) - 3)

>> pretty(ans)

    exp(cos(x)) sin(x)
  - ------------------
      exp(cos(x)) - 3
```

Ejemplo 9

$$y = \ln\sqrt{\sec^2 x + \operatorname{tg} x}$$

$$y = \ln\left(\sec^2 x + \operatorname{tg} x\right)^{1/2} = \frac{1}{2}\ln\left(\sec^2 x + \operatorname{tg} x\right)$$

Derivando respecto a "x":

$$y' = \frac{1}{2}\left[\frac{2\sec x\left(\sec x.\operatorname{tg} x\right) + \sec^2 x}{\left(\sec^2 x + \operatorname{tg} x\right)}\right]$$

$$y' = \frac{1}{2}\left[\frac{2\sec^2 x.\left(\operatorname{tg} x\right) + \sec^2 x}{\left(\sec^2 x + \operatorname{tg} x\right)}\right]$$

$$y' = \frac{1}{2}\left[\frac{\sec^2 x.\left(2\operatorname{tg} x + 1\right)}{\left(\sec^2 x + \operatorname{tg} x\right)}\right]$$

Con MATLAB

```
>> syms x
>> A=(((sec(x))^2)+tan(x))^(1/2)

A =
(tan(x) + 1/cos(x)^2)^(1/2)

>> y=log(A)

y =
log((tan(x) + 1/cos(x)^2)^(1/2))

>> diff(y)

ans =
(tan(x)^2 + (2*sin(x))/cos(x)^3 + 1)/(2*(tan(x) + 1/cos(x)^2))

>> pretty(ans)

        2   2 sin(x)
  tan(x)  + -------- + 1
                  3
            cos(x)
  ----------------------
    /             1    \
  2 | tan(x) + ------- |
    |                2 |
    \          cos(x)  /
```

EJEMPLO 10

$$f(\theta) = \ln\left(\text{sen}^2 \theta\right)$$

Derivando respecto a "θ":

$$f'(\theta) = \frac{2\text{sen}\,\theta.\cos\theta}{\text{sen}^2\theta} = \frac{\text{sen}\,2\theta}{\text{sen}^2\theta}$$

Con MATLAB

```
>> syms x
>> y=log((sin(x))^2)

y =

log(sin(x)^2)

>> diff(y)

ans =

(2*cos(x))/sin(x)
```

```
>> simple(ans)
simplify:
(2*cos(x))/sin(x)
radsimp:
(2*cos(x))/sin(x)
simplify(100):
2*cot(x)
combine(sincos):
(2*cos(x))/sin(x)
combine(sinhcosh):
(2*cos(x))/sin(x)
combine(ln):
(2*cos(x))/sin(x)
factor:
(2*cos(x))/sin(x)
expand:
(2*cos(x))/sin(x)
combine:
(2*cos(x))/sin(x)
rewrite(exp):
-(2*(exp(i*x)/2 + 1/(2*exp(i*x))))/((i*exp(i*x))/2 - i/(2*exp(i*x)))
rewrite(sincos):
(2*cos(x))/sin(x)
rewrite(sinhcosh):
-(2*i*cosh(-i*x))/ainh(-i*x)
rewrite(tan):
-(tan(x/2)^2 - 1)/tan(x/2)
collect(x):
(2*cos(x))/sin(x)
mwcos2sin:
(2*cos(x))/sin(x)
ans =
2*cot(x)
>> pretty(ans)
  2 cot(z)
```

Manualmente:

```
>> A=sin(2*x)

A =

sin(2*x)

>> B=(sin(x))^2

B =

sin(x)^2

>> collect(A/B)

ans =

sin(2*x)/sin(x)^2

>> simple(ans)

simplify:

sin(2*x)/sin(x)^2

radsimp:

sin(2*x)/sin(x)^2

simplify(100):

2*cot(x)

combine(sincos):

sin(2*x)/sin(x)^2

combine(sinhcosh):

sin(2*x)/sin(x)^2

combine(ln):

sin(2*x)/sin(x)^2

factor:

sin(2*x)/sin(x)^2

expand:

(2*cos(x))/sin(x)

combine:

sin(2*x)/sin(x)^2

rewrite(exp):

(i/(2*exp(2*i*x)) - (i*exp(2*i*x))/2)/((i*exp(i*x))/2 - i/(2*exp(i*x)))^2
```

```
rewrite(sincos):

sin(2*x)/sin(x)^2

rewrite(sinhcosh):

-(i*sinh((-2)*i*x))/sinh(-i*x)^2

rewrite(tan):

(tan(x)*(tan(x/2)^2 + 1)^2)/(2*tan(x/2)^2*(tan(x)^2 + 1))

collect(x):

sin(2*x)/sin(x)^2

mwcos2sin:

sin(2*x)/sin(x)^2

ans =

2*cot(x)
```

Como se puede observar el resultado obtenido con MATLAB es el mismo que se obtiene mediante el desarrollo "manual" del ejercicio.

Ejemplo 11

$$f(\theta) = \ln\left(tg^2\theta\right)$$

Derivando respecto a "θ":

$$f'(\theta) = \frac{2\,tg\,\theta \cdot \sec^2\theta}{tg^2\theta} = \frac{2\sec^2\theta}{tg\,\theta}$$

Con MATLAB

```
>> syms x
>> y=log((tan(x))^2)

y =

log(tan(x)^2)

>> diff(y)

ans =

(2*(tan(x)^2 + 1))/tan(x)

>> pretty(ans)

                2
  2 (tan(x)  + 1)
  ---------------
       tan(x)
```

```
>> simple(ans)
simplify:
(2*tan(x)^2 + 2)/tan(x)
radsimp:
(2*(tan(x)^2 + 1))/tan(x)
simplify(100):
4/sin(2*x)
combine(sincos):
(2*(tan(x)^2 + 1))/tan(x)
combine(sinhcosh):
(2*(tan(x)^2 + 1))/tan(x)
combine(ln):
(2*(tan(x)^2 + 1))/tan(x)
factor:
(2*(tan(x)^2 + 1))/tan(x)
expand:
2*tan(x) + 2/tan(x)
combine:
(2*tan(x)^2 + 2)/tan(x)
rewrite(exp):
(2*(exp(2*i*x) + 1)*((i - i*exp(2*i*x))^2/(exp(2*i*x) + 1)^2 + 1))/(i - i*exp(2*i*x))
rewrite(sincos):
(2*cos(x)*(sin(x)^2/cos(x)^2 + 1))/sin(x)
rewrite(sinhcosh):
(2*i*cosh(-i*x)*(sinh(-i*x)^2/cosh(-i*x)^2 - 1))/sinh(-i*x)
rewrite(tan):
(2*(tan(x)^2 + 1))/tan(x)
collect(x):
(2*(tan(x)^2 + 1))/tan(x)
mwcos2sin:
(cos(x)*((2*sin(x)^2)/cos(x)^2 + 2))/sin(x)
```

```
ans =

4/sin(2*x)

>> pretty(ans)

      4
  --------
  sin(2 x)
```

Manualmente:

```
>> A=(2*((sec(x))^2))

A =

2/cos(x)^2

>> B=tan(x)

B =

tan(x)

>> collect(A/B)

ans =

2/(cos(x)^2*tan(x))

>> pretty(ans)

         2
  --------------
        2
  cos(x)  tan(x)

>> simple(ans)

simplify:

2/(cos(x)^2*tan(x))

radsimp:

2/(cos(x)^2*tan(x))

simplify(100):

4/sin(2*x)

combine(sincos):

2/(cos(x)^2*tan(x))

combine(sinhcosh):

2/(cos(x)^2*tan(x))
```

```
combine(ln):

2/(cos(x)^2*tan(x))

factor:

2/(cos(x)^2*tan(x))

expand:

2/(cos(x)^2*tan(x))

combine:

2/(cos(x)^2*tan(x))

rewrite(exp):

(2*(exp(2*i*x) + 1))/((i - i*exp(2*i*x))*(exp(i*x)/2 + 1/(2*exp(i*x)))^2)

rewrite(sincos):

2/(cos(x)*sin(x))

rewrite(sinhcosh):

-(2*i)/(cosh(-i*x)*sinh(-i*x))

rewrite(tan):

(2*(tan(x/2)^2 + 1)^2)/(tan(x)*(tan(x/2)^2 - 1)^2)

collect(x):

2/(cos(x)^2*tan(x))

mwcos2sin:

-(2*cos(x))/(sin(x)*(sin(x)^2 - 1))

ans =

4/sin(2*x)

>> pretty(ans)

      4
 --------
  sin(2 x)
```

Como se puede observar el resultado obtenido en MATLAB es el mismo que el que se obtiene mediante el desarrollo “manual”.

1.5. DERIVADAS DE FUNCIONES LOGARÍTMICAS II

EJEMPLO 1

Derivar:

$$y = x^{3x}$$

Solución:

Tomamos logaritmos en los dos miembros y aplicamos que el logaritmo de una potencia es $\ln x^n = n \ln x$:

$$y = x^{3x} \rightarrow \ln y = 3x \ln x$$

Derivamos como función implícita:

$$\frac{y'}{y} = 3 \ln x + 3x.\frac{1}{x} = 3 \ln x + 3$$

Despejamos y':

$$y' = x^{3x}\left(3 \ln x + 3\right)$$

Con MATLAB

```
>> syms x

>> y = x^(3*x);

>> diff(y,x)

ans =

3*x*x^(3*x - 1) + 3*x^(3*x)*log(x)

>> pretty(ans)

       3 x - 1      3 x
  3 x x        + 3 x    log(x)

>> factor(ans)

ans =

3*x^(3*x)*(log(x) + 1)

>> pretty(ans)

     3 x
  3 x    (log(x) + 1)
```

EJEMPLO 2

Derivar:

$$y = x^{x+1}$$

Solución:

Tomamos logaritmos en los dos miembros y aplicamos que el logaritmo de una potencia es $\ln x^n = n \ln x$:

$$y = x^{x+1} \rightarrow \ln y = (x+1)\ln x$$

Derivamos como función implícita:

$$\frac{y'}{y} = \ln x + (x+1).\frac{1}{x} = \ln x + 1 + \frac{1}{x}$$

Despejamos y':

$$y' = x^{x+1}\left(\ln x + 1 + \frac{1}{x}\right)$$

Con MATLAB

```
>> syms x

>> y = x^(x+1);

>> diff(y,x)

ans =

x^x*(x + 1) + x^(x + 1)*log(x)

>> pretty(ans)

   x            x + 1
  x  (x + 1) + x      log(x)

>> factor(ans)

ans =

x^x*(x + x*log(x) + 1)

>> pretty(ans)

   x
  x  (x + x log(x) + 1)
```

La cual es equivalente a la solución obtenida “manualmente”.

EJEMPLO 3

Derivar:

$$y = x^{e^x}$$

Solución:

Tomamos logaritmos en los dos miembros y aplicamos que el logaritmo de una potencia es $\ln x^n = n \ln x$:

$$y = x^{e^x} \rightarrow \ln y = e^x . \ln x$$

Derivamos como función implícita:

$$\frac{y'}{y} = e^x . \ln x + e^x . \frac{1}{x} = e^x \left(\ln x + \frac{1}{x} \right)$$

Despejamos y':

$$y' = x^{e^x} . e^x \left(\ln x + \frac{1}{x} \right)$$

Con MATLAB

```
>> syms x

>> y = x^(exp(x));

>> diff(y,x)

ans =

x^(exp(x) - 1)*exp(x) + x^exp(x)*exp(x)*log(x)

>> pretty(ans)

   exp(x) - 1           exp(x)
  x           exp(x) + x       exp(x) log(x)

>> factor(ans)

ans =

(x^exp(x)*exp(x)*(x*log(x) + 1))/x

>> pretty(ans)

   exp(x)
  x       exp(x) (x log(x) + 1)
  -----------------------------
                x
```

La cual es equivalente a la solución obtenida “manualmente”.

EJEMPLO 4

Derivar:

$$y = (\ln x)^{x+1}$$

Solución:

Tomamos logaritmos en los dos miembros y aplicamos que el logaritmo de una potencia es $\ln x^n = n \ln x$:

$$y = (\ln x)^{x+1} \rightarrow \ln y = (x+1).\ln(\ln x)$$

Derivamos como función implícita:

$$\frac{y'}{y} = \ln(\ln x) + (x+1).\frac{1}{\ln x}.\frac{1}{x} = \ln(\ln x) + \frac{x+1}{x \ln x}$$

Despejamos y':

$$y' = (\ln x)^{x+1}.\left[\ln(\ln x) + \frac{x+1}{x \ln x}\right]$$

Con MATLAB

```
>> syms x

>> y = (log(x))^(x +1 );

>> diff(y,x)

ans =

log(log(x))*log(x)^(x + 1) + (log(x)^x*(x + 1))/x

>> pretty(ans)

                                    x
                     x + 1   log(x)  (x + 1)
  log(log(x)) log(x)       + ---------------
                                     x
>> factor(ans)

ans =

(log(x)^x*(x + x*log(log(x))*log(x) + 1))/x

>> pretty(ans)

        x
  log(x)  (x + x log(log(x)) log(x) + 1)
  --------------------------------------
                     x
```

La cual es equivalente a la solución obtenida “manualmente”, para probarlo podemos tomar la solución resuelta manualmente y la podemos comparar usando MATLAB:

```
>> B = (log(x)^(x +1 ))*(log(log(x)) + (x +1)/(x*log(x)));

>> expand(B)

ans =

log(x)^x + log(x)^x/x + log(log(x))*log(x)*log(x)^x

>> factor(ans)

ans =

(log(x)^x*(x + x*log(log(x))*log(x) + 1))/x

>> pretty(ans)

       x
  log(x)  (x + x log(log(x)) log(x) + 1)
  --------------------------------------
                    x
```

Se observa que son iguales.

EJEMPLO 5

Derivar:

$$y=\left(\frac{\operatorname{sen}x}{x}\right)^{x}$$

Solución:

Tomamos logaritmos en los dos miembros y aplicamos que el logaritmo de una potencia es $\ln x^{n}=n\ln x$ y que el logaritmo de un cociente es

$$\ln\left(\frac{a}{b}\right)=\ln a-\ln b$$

$$y=\left(\frac{\operatorname{sen}x}{x}\right)^{x}\rightarrow \ln y=x\ln\left(\frac{\operatorname{sen}x}{x}\right)=x\left(\ln(\operatorname{sen}x)-\ln x\right)$$

Derivamos como función implícita:

$$\frac{y'}{y}=\ln(\operatorname{sen}x)-\ln x+x\left(\frac{\cos x}{\operatorname{sen}x}-\frac{1}{x}\right)=\ln\left(\frac{\operatorname{sen}x}{x}\right)+\frac{x\cos x}{\operatorname{sen}x}-1$$

Despejamos y':

$$y'=\left(\frac{\operatorname{sen}x}{x}\right)^{x}.\left[\ln\left(\frac{\operatorname{sen}x}{x}\right)+\frac{x\cos x}{\operatorname{sen}x}-1\right]$$

Con MATLAB

```
>> syms x
>> y = (sin(x)/x)^(x);
>> diff(y,x)
ans =
log(sin(x)/x)*(sin(x)/x)^x + x*(sin(x)/x)^(x - 1)*(cos(x)/x - sin(x)/x^2)
>> pretty(ans)

     / sin(x) \ / sin(x) \x     / sin(x) \x - 1 / cos(x)   sin(x) \
  log| ------ | | ------ |  + x | ------ |      | ------ - ------ |
     \   x    / \   x    /      \   x    /      |   x        2    |
                                                \           x     /
>> factor(ans)
ans =
((sin(x)/x)^x*(x*cos(x) - sin(x) + log(sin(x)/x)*sin(x)))/sin(x)
>> pretty(ans)

  / sin(x) \x /                        / sin(x) \        \
  | ------ |  | x cos(x) - sin(x) + log| ------ | sin(x) |
  \   x    /  \                        \   x    /        /
  --------------------------------------------------------
                           sin(x)
```

La cual es equivalente a la solución obtenida “manualmente”.

CAPÍTULO

3

APLICACIONES DE LOS LÍMITES

3.1. CONTINUIDAD Y DERIVABILIDAD

EJEMPLO 1

Comprobar si la siguiente función es continua y derivable:

$$f(x)=\begin{cases}3x-1 & \text{si } x<1\\ x^2+x & \text{si } x\geq 1\end{cases}$$

Resolución:

Si $x \neq 1$, la función es continua y derivable, pues está formada por dos polinomios.

Continuidad en x = 1:

$$\left.\begin{array}{l}\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1}(3x-1)=2\\ \lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1}(x^2+x)=2\\ f(1)=2\end{array}\right\}\text{ f(x) es continua en x = 1}$$

Derivabilidad en x = 1:

$$\left.\begin{array}{l}\lim\limits_{x\to 1^-} f'(x)=\lim\limits_{x\to 1} 3=3=f'(1^-)\\ \lim\limits_{x\to 1^+} f'(x)=\lim\limits_{x\to 1}(2x+1)=3=f'(1^+)\end{array}\right\}\text{ Las derivadas laterales existen y coinciden}$$

Luego, f(x) es derivable en x = 1. Además, f’(1) = 3.

Por lo tanto f(x) es continua y derivable en todo **R**.

Con MATLAB

A) CONTINUIDAD

```
>> syms x
>> y = 3*x - 1;
>> z = x^2 + x;
>> limit(y,x,1,'left')
ans =
2
>> limit(z,x,1,'right')
ans =
2
```

Se observa que los límites laterales existen y son iguales.

B) DERIVABILIDAD

```
>> diff(y,x)
ans
3
>> diff(z,x)
ans =
2*x + 1
>> limit(3,x,1)
ans =
3
>> limit(2*x + 1,x,1)
ans =
3
```

Se observa que las derivadas laterales existen y coinciden.

Ejemplo 2

Comprobar si la siguiente función es continua y derivable:

$$f(x)=\begin{cases}\ln(x-1) & \text{si } x<2\\ 3x-6 & \text{si } x\geq 2\end{cases}$$

Resolución:

Si $x \neq 2$, la función es continua y derivable.

Continuidad en x = 2:

$$\left.\begin{aligned}&\lim_{x\to 2^-} f(x)=\lim_{x\to 2}\ln(x-1)=\ln 1=0\\ &\lim_{x\to 2^+} f(x)=\lim_{x\to 2}(3x-6)=0\\ &f(2)=0\end{aligned}\right\} \text{f(x) es continua en x = 2}$$

Derivabilidad en x = 2:

$$\left.\begin{aligned}&\lim_{x\to 2^-} f'(x)=\lim_{x\to 2}\frac{1}{x-1}=1=f'(2^-)\\ &\lim_{x\to 2^+} f'(x)=\lim_{x\to 2}3=3=f'(2^+)\end{aligned}\right\} \text{Las derivadas laterales existen pero no coinciden}$$

f(x) es derivable en x = 2.

Con MATLAB

A) CONTINUIDAD

```
>> syms x
>> y = log(x - 1);
>> z = 3*x -6;
>> limit(y,x,2,'left')
ans =
0
>> limit(z,x,2,'right')
ans =
0
```

Se observa que los límites laterales existen y son iguales.

B) DERIVABILIDAD

```
>> diff(y,x)
ans =
1/(x - 1)
>> diff(z,x)
ans =
3
>> limit(1/(x - 1),x,2,'left')
ans =
1
>> limit(3,x,2,'right')
ans =
3
```

Se observa que las derivadas laterales si existen pero son diferentes.

EJEMPLO 3

Analizar la continuidad y derivabilidad así como construir los gráficos de la función

$$f(x)=\begin{cases} x^2-9 & ; \ x<-3 \\ x+3 & ; \ -3\le x\le 0 \\ 3-x & ; \ 0<x<3 \\ x^2-9 & ; \ x\ge 3 \end{cases}$$

I. Análisis de continuidad:

a) Analizamos la continuidad de cada función que conforma f(x) en sus respectivos intervalos de estudio:

x^2-9 : función polinomial $\Rightarrow$ continua en su intervalo de estudio.

$x+3$: función polinomial $\Rightarrow$ continua en su intervalo de estudio.

$3-x$: función polinomial $\Rightarrow$ continua en su intervalo de estudio.

x^2-9 : función polinomial $\Rightarrow$ continua en su intervalo de estudio.

b) Puntos de interés:

- $x = -3$

$$f(-3) = -3 + 3 = 0$$

$$\lim_{x \to (-3)^-} f(x) = \lim_{x \to (-3)^-} \left(x^2 - 9\right) = \left\{\left[(-3)^-\right]^2 - 9\right\} = 9^+ - 9 = 0^+$$

$$\lim_{x \to (-3)^+} f(x) = \lim_{x \to (-3)^+} (x + 3) = -3^+ + 3 = 0^+$$

$$\lim_{x \to (-3)^+} f(x) = \lim_{x \to (-3)^-} f(x) = 0 \quad \Rightarrow \quad \lim_{x \to -3} f(x) = 0 = f(-3)$$

La función es continua en $x = -3$.

Con MATLAB

```
>> syms x
>> y = x^2 -9;
>> z = x +3;
>> limit(y,x,-3,'left')
ans =
0
>> limit(y,x,-3,'right')
ans =
0
>> limit(z,x,-3,'right')
ans =
0
```

- $x = 0$

$$f(0) = 0 + 3 = 3$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x + 3) = 0^- + 3 = 3^-$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3 - x) = 3 - 0^+ = 3^-$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 3 \quad \Rightarrow \quad \lim_{x \to 0} f(x) = 3 = f(0)$$

La función es continua en $x = 0$.

Con MATLAB

```
>> syms x
>> z = x +3;
>> w = 3 -x;
>> limit(z,x,0,'left')
ans =
3
>> limit(w,x,o,'right')
ans =
3
```

- $x = 0$

$$f(3) = 3^2 - 9 = 9 - 9 = 0$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (3 - x) = 3 - 3^- = 0^+$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x^2 - 9) = \left[(3^+)^2 - 9\right] = 9^+ - 9 = 0^+$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x) = 0 \quad \Rightarrow \quad \lim_{x\to 3} f(x) = 0 = f(3)$$

La función es continua en $x = 3$.

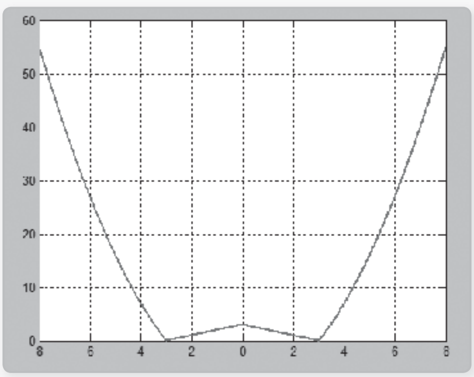
Con MATLAB

```
>> syms x
>> w = 3 -x;
>> v = x^2 -9;
>> limit(w,x,3,'left')
ans =
0
>> limit(v,x,3,'right')
ans =
0
```

II. Gráfica de $f(x)$**:**

Con MATLAB:

```
>> x1 = -8:0.01:-3;
>> y1 = x1.^2 - 9;
>> x2 = -3:0.01:0;
>> y2 = x2 + 3;
>> x3 = 0:0.01:3;
>> y3 = 3 - x3;
>> x4 = 3:0.01:8;
>> y4 = x4.^2 - 9;
>> x = [x1,x2,x3,x4];
>> y = [y1,y2,y3,y4];
>> plot(x,y)
>> grid on
```



III. Análisis de Derivabilidad:

Calculamos la derivada de cada función que conforma “f(x)” sin considerar los extremos de los intervalos:

$$f'(x)=\begin{cases}2x & ;\ x<-3\\ 1 & ;\ -3<x<0\\ -1 & ;\ 0<x<3\\ 2x & ;\ x>3\end{cases}$$

Cálculo de las derivadas de f(x) con MATLAB

```
>> syms x
>> y = x^2 - 9;
>> diff(y,x)
ans =
2*x
>> z = x + 3;
>> diff(z,x)
ans =
1
>> w = 3 - x;
>> diff(w,x)
ans =
-1
>> v = x^2 - 9;
>> diff(v,x)
ans =
2*x
```

Las derivadas están definidas en sus respectivos intervalos ya que todas son funciones polinómicas ahora debemos verificar si la derivada existe en los extremos de cada intervalo, para esto calcularemos los límites laterales en torno a cada extremo de cada intervalo de f '(x), si dichos límites existen la función será derivable en dichos puntos.

- $x=-3$

$$\lim_{x\to(-3)^-} f'(x)=\lim_{x\to(-3)^-}(2x)=2(-3)^-=(-6)^-$$

$$\lim_{x\to(-3)^+} f'(x)=\lim_{x\to(-3)^+}(1)=1$$

Como $\lim_{x\to(-3)^+} f'(x)\neq\lim_{x\to(-3)^-} f'(x)\Rightarrow$ la función f(x) no es derivable en $x=-3$

Con MATLAB

```
>> syms x
>> limit(2*x,x,-3,'left')
ans =
-6
>> limit(1,x,-3,'right')
ans =
1
```

Se observa que los limites laterales si existen pero son diferentes, por lo tanto no es derivable en $x = -3$

- $x = 0$

$$\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-} (1) = 1$$

$$\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} (-1) = -1$$

Como $\lim_{x\to 0^+} f'(x) \neq \lim_{x\to 0^-} f'(x) = 3 \Rightarrow$ la función f(x) no es derivable en $x = 0$.

Con MATLAB

```
>> syms x
>> limit(1,x,0,'left')
ans =
1
>> limit(-1,x,0,'right')
ans =
-1
```

Se observa que los limites laterales si existen pero son diferentes, por lo tanto no es derivable en x = 0

- $x = 3$

$$\lim_{x\to 3^-} f'(x) = \lim_{x\to 3^-} (-1) = -1$$

$$\lim_{x\to 3^+} f'(x) = \lim_{x\to 3^+} (2x) = 2(3^+) = 6^+$$

Como $\lim_{x\to 3^+} f'(x) \neq \lim_{x\to 3^-} f'(x) \Rightarrow$ la función f(x) no es derivable en $x = 3$.

Con MATLAB

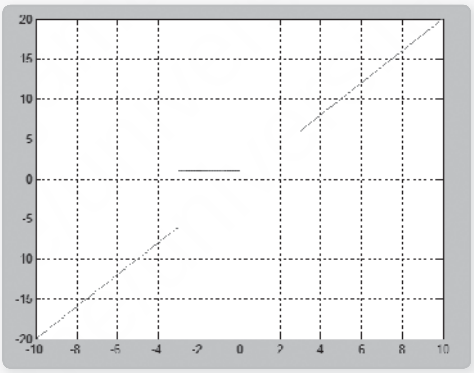
```
>> syms x
>> limit(-1,xx3,'left')
ans =
-1
>> limit(2*x,x,3,'right')
ans =
6
```

Se observa que los limites laterales si existen pero son diferentes, por lo tanto no es derivable en x = 3

IV. Gráfica de $f\,'(x)$:

Con MATLAB:

```
>> x1 = -10:0.01:-3;
>> y1 = 2.*x1;
>> plot(x1,y1)
>> hold on
>> x2 = -3:0.01:0;
>> y2 = 1;
>> plot(x2,y2)
>> x3 = 0:0.01:3;
>> y3 = -1;
>> plot(x3,y3)
>> x4 = 3:0.01:10;
>> y4 = 2.*x4;
>> plot(x4,y4)
>> grid on
```



EJEMPLO 4

$$f(x)=\begin{cases} -x & ;\ x<0 \\ \sqrt{6x-x^2} & ;\ 0\le x<3 \\ 3 & ;\ 3\le x\le 5 \\ \dfrac{3}{4}x^2-\dfrac{15}{2}x+\dfrac{87}{4} & ;\ x>5 \end{cases}$$

I. Análisis de continuidad:

a) Analizamos la continuidad de cada función que conforma $f(x)$ en sus respectivos intervalos de estudio:

$-x$: función polinomial $\Rightarrow$ continua en su intervalo de estudio.

$\sqrt{6x-x^2}$: función irracional $\Rightarrow$ continua en su intervalo de estudio.

3 : función polinomial $\Rightarrow$ continua en su intervalo de estudio.

$\frac{3}{4}x^2 - \frac{15}{2}x + \frac{87}{4}$: función polinomial $\Rightarrow$ continua en su intervalo de estudio.

b) Puntos de interés:

- $x = 0$

$$f(0) = \sqrt{6(0) - (0)^2} = 0$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x) = \left[-(0)^-\right] = 0^+$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(\sqrt{6\,x - x^2}\right) = \lim_{x \to 0^+} \left(\sqrt{x\,(6-x)}\right) = \sqrt{(0^+)(6-0^+)} = 0^+$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 0 \quad \Rightarrow \quad \lim_{x \to 0} f(x) = 0 = f(0)$$

La función es continua en $x = 0$.

Con MATLAB

```
>> syms x
>> y = -x;
>> z = (6*x-(x^2))^(1/2);
>> limit(y,x,0,'left')
ans =
0
>> limit(y,x,0,'right')
ans =
0
>> limit(z,x,0,'right')
ans =
0
```

- $x = 3$

$$f(3) = 3$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \left(\sqrt{6x - x^2}\right) = \lim_{x \to 3^-} \left(\sqrt{x(6-x)}\right) = \sqrt{(3^-)(6-3^-)}$$

$$= \sqrt{(3^-)(3^+)} = 3^-$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (3) = 3$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x) = 3 = f(3)$$

La función f(x) es continua en $x = 3$.

Con MATLAB

```
>> syms x
>> z = (6*x-(x^2))^(1/2);
>> w = 3;
>> limit(z,x,3,'left')
ans =
3
>> limit(w,x,3,'right')
ans =
3
```

- $x = 5$

$$f(5) = 3$$

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} (3) = 3$$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} \left(\frac{3}{4}x^2 - \frac{15}{2}x + \frac{87}{4} \right) = \lim_{x \to 5^+} \left(\frac{3x(x-10)+87}{4} \right)$$

$$= \frac{3(5^+)(5^+ - 10) + 87}{4} = \frac{15^+(-5^+) + 87}{4} = \frac{15^+(-5^+) + 87}{4} = \frac{-75^+ + 87}{4}$$

$$= \frac{12^+}{4} = 3^+$$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^-} f(x) = 3 \quad \Rightarrow \quad \lim_{x \to 5} f(x) = 3 = f(5)$$

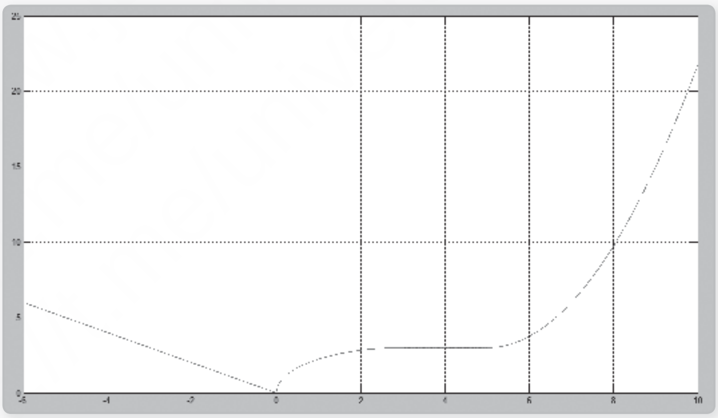
La función es continua en $x = 5$.

Con MATLAB

```
>> syms x
>> w = 3;
>> v = ((3/4)*(x^2))-((15/2)*x)+(87/84);
>> syms x
>> w = 3;
>> v = ((3/4)*(x^2))-((15/2)*x)+(87/4);
>> limit(w,x,5,'left')
ans =
3
>> limit(v,x,5,'right')
ans =
3
```

II. Gráfica de $f(x)$:

```
>> x1 = -6:0.01:0;
>> y1 = -x1;
>> plot(x1,y1)
>> grid on
>> hold on
>> x2 = 0:0.01:3;
>> y2 = ((6.*x2)-(x2.^2)).^(1/2);
>> plot(x2,y2)
>> grid on
>> hold on
>> x3 = 3:0.01:5;
>> y3 = 1.*3;
>> plot(x3,y3)
>> grid on
>> hold on
>> x4 = 5:0.01:10;
>> y4 = (3./4).*x4.^2-(15./2).*x4+(87./4);
>> plot(x4,y4)
>> grid on
>> hold off
```



III. Análisis de Derivabilidad:

Calculamos la derivada de cada función que conforma "f(x)" sin considerar los extremos de los intervalos:

$$f'(x) = \begin{cases} -1 & ; \ x < 0 \\ \dfrac{3-x}{\sqrt{6x - x^2}} & ; \ 0 < x \leq 3 \\ 0 & ; \ 3 \leq x \leq 5 \\ \dfrac{3}{2}x - \dfrac{15}{2} \ ; \ x \geq 5 & \end{cases}$$

Cálculo de las derivadas de f(x) con MATLAB

```
>> syms x
>> y = -x;
>> diff(y,x)
ans =
-1
>> z = (6*x-(x^2))^(1/2);
>> diff(z,x)
ans =
-(2*x - 6)/(2*(6*x - x^2)^(1/2))
>> w = 3;
>> diff(w,x)
ans =
0
>> v = ((3/4)*(x^2))-((15/2)*x)+(87/84);
>> diff(v,x)
ans =
(3*x)/2 - 15/2
```

Observamos que $\frac{3-x}{\sqrt{6x-x^2}}$ no está definida cuando $6x-x^2=0$, es decir, cuando $x=0$ ó cuando $x=6$, pero ambos valores no pertenecen al intervalo $(0,3]$. Por lo tanto $\frac{3-x}{\sqrt{6x-x^2}}$ estará definida en su intervalo de estudio.

Por otro lado el resto de funciones que conforman f'(x) están definidas en sus respectivos intervalos de estudio ya que son funciones polinomiales, ahora debemos verificar si la derivada existe en los extremos de cada intervalo, para esto calcularemos los límites laterales en torno a cada extremo de cada intervalo de f'(x), si dichos límites existen la función será derivable en dichos puntos.

- $x=0$

$$\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-}(-1) = -1$$

$$\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+}\left(\frac{3-x}{\sqrt{6x-x^2}}\right) = \lim_{x\to 0^+}\left(\frac{3-x}{\sqrt{x(6-x)}}\right) = \frac{3-0^+}{\sqrt{(0^+)(6-0^+)}}$$

$$= \frac{3^-}{\sqrt{(0^+)(6^-)}} = \frac{3^-}{\sqrt{0^+}} = \frac{3^-}{0^+} = +\infty$$

Como $\lim_{x\to 0^+} f'(x) \neq \lim_{x\to 0^-} f'(x) \Rightarrow$ la función f(x) no es derivable en $x=0$.

Con MATLAB

```
>> syms x
>> limit(-1,x,0,'left')
ans =
-1
>> limit((-(2*x-6)/(2*(6*x-x^2)^(1/2))),x,0,'right')
ans =
Inf
```

Se observa que los limites laterales si existen pero son diferentes, por lo tanto no es derivable en $x = 0$.

- $x = 3$

$$\lim_{x\to 3^-} f'(x) = \lim_{x\to 3^-}\left(\frac{3-x}{\sqrt{6x-x^2}}\right) = \lim_{x\to 3^-}\left(\frac{3-x}{\sqrt{x(6-x)}}\right) = \frac{3-3^-}{\sqrt{(3^-)(6-3^-)}}$$

$$= \frac{0^+}{\sqrt{(3^-)(3^+)}} = \frac{0^+}{\sqrt{9^-}} = \frac{0^+}{3^-} = 0^+$$

$$\lim_{x\to 3^+} f'(x) = \lim_{x\to 3^+}(0) = 0$$

Como $\lim_{x\to 3^+} f'(x) = \lim_{x\to 3^-} f'(x) \Rightarrow$ la función $f(x)$ es derivable en $x = 3$. Es decir en $x = 3$ se puede trazar una única recta tangente a $f(x)$. Es por eso que en la expresión de $f'(x)$ los signos de desigualdad no son estrictos en el entorno a 3.

Con MATLAB

```
>> syms x
>> limit((-(2*x-6)/(2*(6*x-x^2)^(1/2))),x,3,'left')
ans =
0
>> limit(0,x,3,'right')
ans =
0
```

Se observa que los limites laterales si existen y además son iguales, por lo tanto sí es derivable en $x = 3$.

- $x = 5$

$$\lim_{x\to 5^-} f'(x) = \lim_{x\to 5^-}(0) = 0$$

$$\lim_{x\to 5^+} f'(x) = \lim_{x\to 5^+}\left(\frac{3}{2}x - \frac{15}{2}\right) = \frac{3}{2}(5^+) - \frac{15}{2} = \frac{15^+}{2} - \frac{15}{2} = 0^+$$

Como $\lim_{x\to 5^+} f'(x) = \lim_{x\to 5^-} f'(x) \Rightarrow$ la función $f(x)$ es derivable en $x = 5$. Es decir en $x = 5$ se puede trazar una única recta tangente a $f(x)$. Es por eso que en la expresión de $f'(x)$ los signos de desigualdad no son estrictos en el entorno a 5.

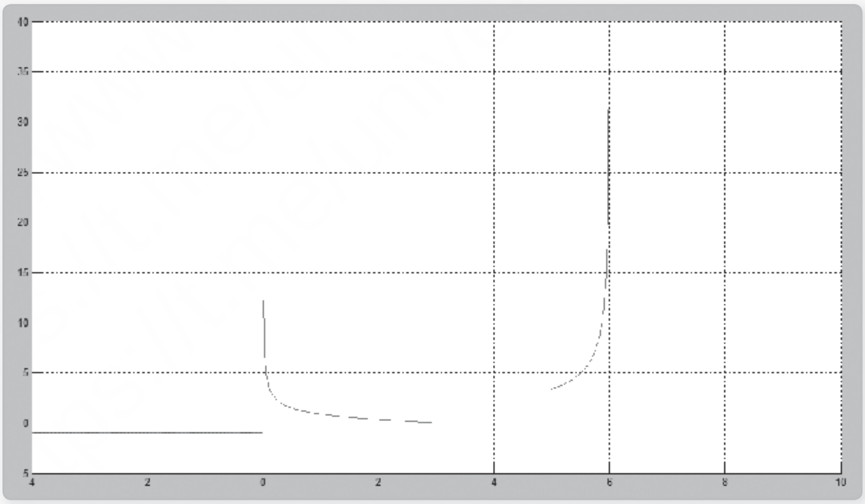
Con MATLAB

```
>> syms x
>> limit(0,x,5,'left')
ans =
0
>> limit((((3*x)/2)-(15/2)),x,5,'right')
ans =
0
```

Se observa que los limites laterales si existen y además son iguales, por lo tanto sí es derivable en $x = 5$.

Con MATLAB

```
>> x1=-4:0.01:0;
>> y1=-1;
>> plot(x1,y1)
>> grid on
>> hold on
>> x2=0:0.01:3;
>> y2 = ((-(2.*x2-6)./(2.*(6.*x2-x2.^2).^(1/2))));
>> plot(x2,y2)
>> grid on
>> hold on
>> x3=3:0.01:5;
>> y3=0;
>> plot(x3,y3)
>> grid on
>> hold on
>> x4=5:0.01:10;
>> y4=(3.*x4)./(2.*(6.*x4-x4.^2).^(1/2));
>> plot(x4,y4)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> hold off
```



EJEMPLO 5

$$f(x)=\begin{cases}\dfrac{2}{x+9} & ; \quad x< -2\\ \dfrac{2}{x^2+3} & ; \quad -2\le x\le 0\\ \dfrac{2}{3}+\dfrac{1}{9}x & ; \quad 0<x<2\\ \dfrac{x^2}{2}+\dfrac{7}{2}x-\dfrac{1}{4} & ; \quad 2\le x\le 6\\ \dfrac{1}{x-8} & ; \quad x>6\end{cases}$$

I. Análisis de continuidad:

a) Analizamos la continuidad de cada función que conforma $f(x)$ en sus respectivos intervalos de estudio:

$\dfrac{2}{x+9}$: función racional $\Rightarrow$ discontinua en $x=-9$.

$\dfrac{2}{x^2+3}$: función racional $\Rightarrow$ no es discontinua ya que $x^2+3>0$.

$\dfrac{2}{3}+\dfrac{1}{9}x$: función polinomial $\Rightarrow$ continua en su intervalo de estudio.

$\dfrac{x^2}{2}+\dfrac{7}{2}x-\dfrac{1}{4}$: función polinomial $\Rightarrow$ continua en su intervalo de estudio.

$\dfrac{1}{x-8}$: función irracional $\Rightarrow$ discontinua en $x=8$.

b) Puntos de interés:

- $x=-2$

$$f(-2)=\frac{2}{(-2)^2+3}=\frac{2}{4+3}=\frac{2}{7}$$

$$\lim_{x\to(-2)^-} f(x)=\lim_{x\to(-2)^-}\left(\frac{2}{x+9}\right)=\frac{2}{(-2)^-+9}=\frac{2}{7^+}\left(\frac{2}{7}\right)^-$$

$$\lim_{x\to(-2)^+} f(x)=\lim_{x\to(-2)^+}\left(\frac{2}{x^2+3}\right)=\frac{2}{\left[(-2)^+\right]^2+3}=\frac{2}{4^++3}=\frac{2}{7^+}=\left(\frac{2}{7}\right)^-$$

$$\lim_{x\to(-2)^+} f(x)=\lim_{x\to(-2)^-} f(x)=\frac{2}{7}\quad\Rightarrow\quad \lim_{x\to-2} f(x)=\frac{2}{7}=f(-2)$$

La función es continua en $x=-2$.

Con MATLAB

```
>> syms x
>> y = 2/(x+9):
>> z = 2/(x^2+3);
>> limit(y,x,-2,'left')
ans =
2/7
>> limit(y,x,-2,'right')
ans =
2/7
>> limit(z,x,-2,'right')
ans =
2/7
```

- $x = 0$

$$f(0) = \frac{2}{(0)^2 + 3} = \frac{2}{3}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(\frac{2}{x^2 + 3} \right) = \frac{2}{(0^-)^2 + 3} = \frac{2}{0^+ + 3} = \frac{2}{3^+} = \left(\frac{2}{3} \right)^-$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(\frac{2}{3} + \frac{1}{9}x \right) = \frac{2}{3} + \frac{1}{9}(0^+) = \frac{2}{3} + 0^+ = \left(\frac{2}{3} \right)^+$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = \frac{2}{3} \quad \Rightarrow \quad \lim_{x \to 0} f(x) = \frac{2}{3} = f(0)$$

La función f (x) es continua en $x = 0$.

Con MATLAB

```
>> syms x
>> z = 2/(x^2+3);
>> w = (2/3)+((1/9)*x);
>> limit(z,x,0,'left')
ans =
2/3
>> limit(w,x,0,'right')
ans =
2/3
```

- $x = 2$

$$f(2) = \frac{(2)^2}{2} + \frac{7}{2}(2) - \frac{1}{4} = \frac{35}{4}$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \left(\frac{2}{3} + \frac{1}{9}x \right) = \frac{2}{3} + \frac{1}{9}(2^-) = \frac{2}{3} + \left(\frac{2}{9} \right)^- = \left(\frac{8}{9} \right)^-$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(\frac{x^2}{2} + \frac{7}{2}x - \frac{1}{4} \right) = \frac{(2^+)^2}{2} + \frac{7}{2}(2^+) - \frac{1}{4} = \left(\frac{35}{4} \right)^+$$

$$\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x) \quad \Rightarrow \text{La función } f(x) \text{ no es continua en } x = 2.$$

Con MATLAB

```
>> syms x
>> w = (2/3)+((1/9)*x);
>> v = ((x^2)/2)+((7/2)*x)-(1/4);
>> limit(w,x,2,'left')
ans =
8/9
>> limit(v,x,2,'right')
ans =
35/4
```

- $x = 6$

$$f(6) = \frac{(6)^2}{2} + \frac{7}{2}(6) - \frac{1}{4} = \frac{155}{4}$$

$$\lim_{x\to 6^-} f(x) = \lim_{x\to 6^-}\left(\frac{x^2}{2} + \frac{7}{2}x - \frac{1}{4}\right) = \lim_{x\to 6^-}\left[\frac{2x(x+7)-1}{4}\right] = \frac{2(6^-)(6^-+7)-1}{4}$$

$$= \frac{(12^-)(13^-)-1}{4} = \frac{156^- - 1}{4} = \left(\frac{155}{4}\right)^-$$

$$\lim_{x\to 6^+} f(x) = \lim_{x\to 6^+}\left(\frac{1}{x-8}\right) = \frac{1}{6^+ - 8} = -\frac{1}{(-2)^+} = \left(-\frac{1}{2}\right)^-$$

$$\lim_{x\to 6^+} f(x) \neq \lim_{x\to 6^-} f(x)$$

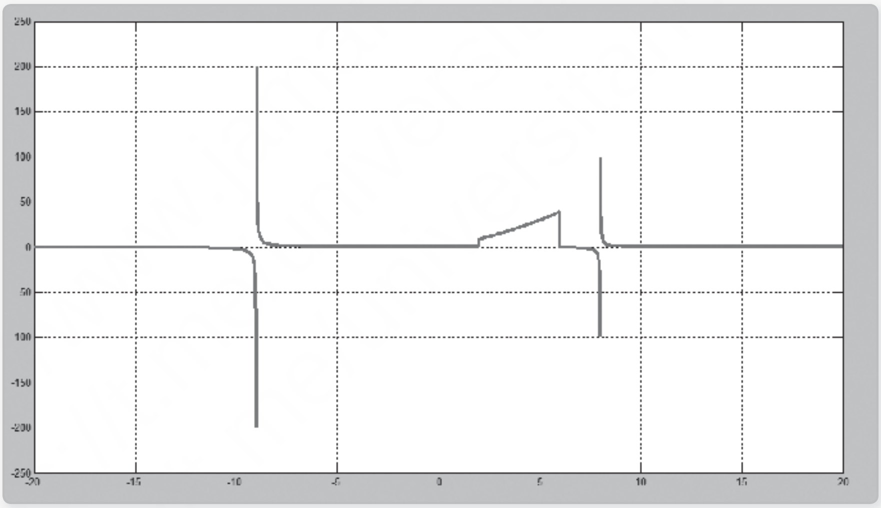
La función no es continua en $x = 6$.

Con MATLAB

```
>> syms x
>> v = ((x^2)/2)+((7/2)*x)-(1/4);
>> u = (1/(x-8));
>> limit(v,x,6,'left')
ans =
155/4
>> limit(u,x,6,'right')
ans =
-1/2
```

II. Gráfica de f (x)**:**

```
>> x1 = -20:0.01:-2;
>> y1 = 2./(x1+9);
>> x2 = -2:0.01:0;
>> y2 = 2./(x2.^2+3);
>> x3 = 0:0.01:2;
>> y3 = (2/3)+((1/9).*x3);
>> x4 = 2:0.01:6;
>> y4 = ((x4.^2)./2)+((7/2).*x4)-(1/4);
>> x5 = 6:0.01:20;
>> y5 = (1./(x5-8));
>> plot(x,y)

>> x=[x1,x2,x3,x4,x5];
>> y=[y1,y2,y3,y4,y5];
>> plot(x,y)
>> grid on
```



III. Análisis de Derivabilidad:

Calculamos la derivada de cada función que conforma "f(x)" sin considerar los extremos de los intervalos:

$$f'(x)=\begin{cases} \dfrac{-2}{(x+9)^2} & ; \quad x< -2 \wedge x\neq -9 \\ \dfrac{-4x}{(x^2+3)^2} & ; \quad -2<x<0 \\ \dfrac{1}{9} & ; \quad 0<x<2 \\ x+\dfrac{7}{2} & ; \quad 2<x<6 \\ \dfrac{-1}{(x-8)^2} & ; \quad x>6 \wedge x\neq 8 \end{cases}$$

Cálculo de las derivadas de f(x) con MATLAB

```
>> syms x
>> y = 2/(x+9);
>> diff(y,x)
ans =
-2/(x + 9)^2
>> z = 2/(x^2+3);
>> diff(z,x)
ans =
-(4*x)/(x^2 + 3)^2
>> w = (2/3)+((1/9)*x);
>> diff(w,x)
ans =
1/9
>> v = ((x^2)/2)+((7/2)*x)-(1/4);
>> diff(v,x)}
ans =
x + 7/2
>> u = (1/(x-8));
>> diff(u,x)
ans =
-1/(x - 8)^2
```

$\frac{-2}{(x+9)^2}$: No está definida en $x=-9$. Como $x=-9$ pertenece al intervalo de estudio, se analizará la derivabilidad de $f(x)$ en $x=-9$:

- $x=-9$

$$\lim_{x\to -9^+} f'(x) = \lim_{x\to -9^+} \frac{-2}{(x+9)^2} = \frac{-2}{\left(-9^+ +9\right)^2} = \frac{-2}{0^+} = -\infty$$

$$\lim_{x\to -9^-} f'(x) = \lim_{x\to -9^-} \frac{-2}{(x+9)^2} = \frac{-2}{\left(-9^- +9\right)^2} = \frac{-2}{0^+} = -\infty$$

Con MATLAB

```
>> syms x
>> limit((-2/((x+9)^2)),x,-9,'left')
ans =
-Inf
>> limit((-2/((x+9)^2)),x,-9,'right')
```

Al ser los límites laterales infinitos, "$f'(x)$" en $x=-9$ no está definida. Esto implica que en $x=-9$ tiene una asíntota vertical.

$\dfrac{-4x}{\left(x^2+3\right)^2}$: función racional que está definida en todo su intervalo de estudio ya que $\left(x^2+3\right)^2>0$.

$\dfrac{1}{9}$: función polinomial $\Rightarrow$ definida en su intervalo de estudio.

$x+\dfrac{7}{2}$: función polinomial $\Rightarrow$ definida en su intervalo de estudio.

$\dfrac{-1}{(x-8)^2}$: función racional $\Rightarrow$ no definida en $x=8$. Como $x=8$ pertenece al intervalo de estudio, se analizará la derivabilidad de $f(x)$ en $x=8$:

- $x=8$

$$\lim_{x\to 8^-} f'(x)=\lim_{x\to 8^-}\frac{-1}{(x-8)^2}=\frac{-1}{\left(8^- -8\right)^2}=\frac{-1}{\left(0^-\right)^2}=\frac{-1}{0^+}=-\infty$$

$$\lim_{x\to 8^+} f'(x)=\lim_{x\to 8^+}\frac{-1}{(x-8)^2}=\frac{-1}{\left(8^+ -8\right)^2}=\frac{-1}{\left(0^+\right)^2}=\frac{-1}{0^+}=-\infty$$

Con MATLAB

```
>> syms x
>> limit((-1/(x-8)^2),x,8,'left')
ans =
-Inf
>> limit((-1/(x-8)^2),x,8,'right')
ans =
-Inf
```

Al ser los límites laterales infinitos, $f'(x)$ en $x=8$ no está definida. Esto implica que en $x=8$ "$f'(x)$" tiene una asíntota vertical.

Ahora debemos verificar si la derivada existe en los extremos de cada intervalo, para esto calcularemos los límites laterales en torno a cada extremo de cada intervalo de $f'(x)$, si dichos límites existen la función será derivable en dichos puntos.

- $x=-2$

$$\lim_{x\to(-2)^-} f'(x)=\lim_{x\to(-2)^-}\left[\frac{-2}{(x+9)^2}\right]=\frac{-2}{\left[(-2)^-+9\right]^2}=\frac{-2}{\left(7^-\right)^2}=\frac{-2}{49^-}=\left(\frac{-2}{49}\right)^-$$

$$\lim_{x\to(-2)^+} f'(x)=\lim_{x\to(-2)^+}\left[\frac{-4x}{\left(x^2+3\right)^2}\right]=\frac{-4(-2)^+}{\left\{\left[(-2)^+\right]^2+3\right\}^2}=\frac{8^-}{\left(4^++3\right)^2}$$

$$=\frac{8^-}{\left(7^+\right)^2}=\frac{8^-}{49^+}=\left(\frac{8}{49}\right)^-$$

Con MATLAB

```
>> syms x
>> limit((-2/(x+9)^2),x,-2,'left')
ans =
-2/49
>> limit((-(4*x)/(x^2+3)^2),x,-2,'right')
ans =
8/49
```

Como $\lim\limits_{x\to(-2)^+} f'(x) \neq \lim\limits_{x\to(-2)^-} f'(x) \Rightarrow$ la función f(x) no es derivable en $x=-2$.

- $x=0$

$$\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-}\left[\frac{-4x}{\left(x^2+3\right)^2}\right] = \frac{-4\left(0^-\right)}{\left[\left(0^-\right)^2+3\right]^2} = \frac{0^+}{\left(0^++3\right)^2} = \frac{0^+}{\left(3^+\right)^2} = 0^+$$

$$\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+}\left(\frac{1}{9}\right) = \frac{1}{9}$$

Con MATLAB

```
>> syms x
>> limit((-(4*x)/(x^2+3)^2),x,0,'left')
ans =
0
>> limit((1/9),x,0,'right')
ans =
1/9
```

Como $\lim\limits_{x\to(0)^+} f'(x) \neq \lim\limits_{x\to(0)^-} f'(x) \Rightarrow$ la función f(x) no es derivable en $x=0$.

- $x=2$

$$\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^-}\left(\frac{1}{9}\right) = \frac{1}{9}$$

$$\lim_{x\to 2^+} f'(x) = \lim_{x\to 2^+}\left(x+\frac{7}{2}\right) = 2^+ + \frac{7}{2} = \left(\frac{11}{2}\right)^+$$

$$\lim_{x\to 2^+} f'(x) \neq \lim_{x\to 2^-} f'(x)$$

Como $\lim\limits_{x\to(2)^+} f'(x) \neq \lim\limits_{x\to(2)^-} f'(x) \Rightarrow$ la función f(x) no es derivable en $x=2$.

Con MATLAB

```
>> syms x
>> limit((1/9),x,2,'left')
ans =
1/9
>> limit((x+(7/2)),x,2,'right')
ans =
11/2
```

- $x = 6$

$$\lim_{x\to 6^-} f'(x) = \lim_{x\to 6^-}\left(x+\frac{7}{2}\right) = 6^- + \frac{7}{2} = \left(\frac{19}{2}\right)^-$$

$$\lim_{x\to 6^+} f'(x) = \lim_{x\to 6^+}\left[\frac{-1}{(x-8)^2}\right] = \frac{-1}{\left(6^+-8\right)^2} = \frac{-1}{\left[(-2)^+\right]^2}$$

$$\frac{-1}{4^+} = \left(-\frac{1}{4}\right)^- = (-0,25)^-$$

Como $\lim_{x\to 6^+} f'(x) \neq \lim_{x\to 6^-} f'(x) \Rightarrow$ la función f(x) no es derivable en $x = 6$.

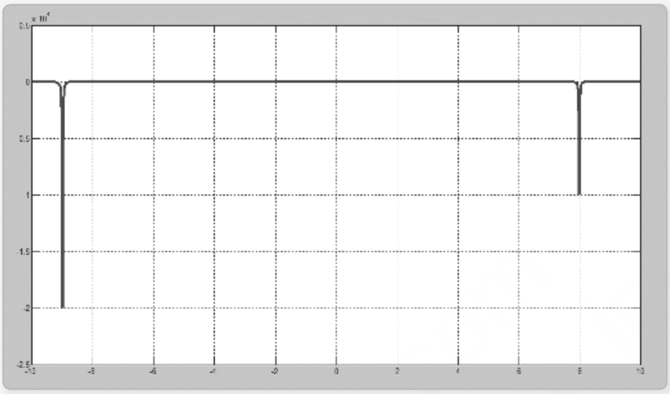
Con MATLAB

```
>> syms x
>> limit((x+(7/2)),x,6,'left')
ans =
19/2
>> limit((-1/(x-8)^2),x,6,'right')
ans =
-1/4
```

IV. Gráfica de $f'(x)$**:**

Con MATLAB

```
>> x1=-10:0.01:-2;
>> y1=-2./((x1+9).^2);
>> x2=-2:0.01:0;
>> y2=(-(4.*x2)./(x2.^2+3).^2);
>> x3=0:0.01:2;
>> y3=((2/3)+((1/9).*x3));
>> x4=2:0.01:6;
>> y4=(x4+(7/2));
>> x5=6:0.01:10;
>> y5=(-1./(x5-8).^2);
>> x=[x1,x2,x3,x4,x5];
>> y=[y1,y2,y3,y4,y5];
>> plot(x,y)
>> grid on
```



CAPÍTULO

4

APLICACIONES DE LA DERIVADA

4.1. RECTAS TANGENTES A CURVAS

EJEMPLO 1

Halla las rectas tangentes a la curva:

$$y = \frac{5x^3 + 7x^2 - 16x}{x - 2}$$

en los puntos de abscisa 0, 1, 3.

Calculamos la derivadas de la función

$$y' = \frac{(15x^2 + 14x - 16)(x-2) - (5x^3 + 7x^2 - 16)}{(x-2)^2} = \frac{10x^3 - 23x^2 - 28x + 32}{(x-2)^2}$$

Ordenadas de los puntos:

$$y(0) = 0;\ y(1) = 4;\ y(3) = 150$$

- Recta tangente en (0,0): $y'(0) = 8$

$$y = 8x$$

- Recta tangente en (1,4): $y'(1) = -9$

$$y = 4 - 9(x-1) = -9x + 13$$

- Recta tangente en (3,150): $y'(3) = 11$

$$y = 150 + 11(x-3) = 11x + 117$$

Con MATLAB

Cálculo de la derivada de la función:

```
>> syms x
>> A = 5*x^3 + 7*x^2 - 16*x
A =
5*x^3 + 7*x^2 - 16*x
>> B = x - 2
B =
x - 2
>> D = diff(A/B,x)
D =
(15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2
>> oretty(D)

      2                   3      2
  15 x  + 14 x - 16   5 x  + 7 x  - 16 x
  ----------------- - ------------------
        x - 2                     2
                           (x - 2)
>> collect(D)
ans =
(10*x^3 - 23*x^2 - 28*x + 32)/(x^2 - 4*x + 4)
>> pretty(ans)

      3       2
  10 x  - 23 x  - 28 x + 33
  -------------------------
         2
        x  - 4 x + 4
```

Ordenadas de los puntos: cuando x = 0

```
>> x = 0;
>> A = 5*x^3 + 7*x^2 - 16*x
A =
     0
>> B = x - 2
B =
    -2
>> E = (A/B)
E =
     0
```

Cuando x = 1

```
>> x = 1;
>> A = 5*x^3 + 7*x^2 - 16*x
A =
    -4
>> B = x - 2
B =
    -1
>> E = (A/B)
E =
     4
```

Cuando x = 3

```
>> x = 3;
>> A = 5*x^3 + 7*x^2 -16*x
A =
   150
>> B = x - 2
B =
     1
>> E = (A/B)
E =
   150
```

Para determinar las rectas tangentes:

Cálculo de las pendientes (m), es decir las derivadas en los puntos: x = 0; x = 1; x =11

```
>> x = 0;
>> E = (15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2
E =
     8
>> x = 1;
>> E = (15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2
E =
    -9
>> x = 3;
>> E = (15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2
E =
    11
```

Entonces las rectas tangentes en (0,0), (1,4) y (3,150), con pendientes m=8, m=-9 y m=11 son:

```
>> syms x
>> x1 = 0;
>> y1 = 0;
>> m = 8;
>> y = y1 + m*(x - x1)
y =
8*x
>> x1 = 1;
>> y1 = 4;
>> m = -9;
>> y = y1 + m*(x - x1)
y =
13 - 9*x
>> x1 = 3;
>> y1 = 150;
>> m = 11;
>> y = y1 + m*(x - x1)
y =
11*x + 117
```

Ejemplo 2

Hallar las rectas tangentes a la circunferencia:

$$x^2+y^2-2x+4y-24=0$$

en los puntos de abscisa $x_0=3$.

Obtencion de las ordenadas corrspondientes:

$$3^2+y^2-2.3+4y-24=0$$

$$9+y^2-6+4y-24=0$$

$$y^2+4y-21=0$$

$$y=\frac{-4\pm\sqrt{16+84}}{2}=\frac{-4\pm\sqrt{100}}{2}=\frac{-4\pm 10}{2}\begin{cases} y=3\rightarrow \text{Punto }(3,3)\\ y=-7\rightarrow \text{Punto }(3,-7)\end{cases}$$

Para halla la pendiente en esos puntos, derivamos implícitamente:

$$2x+2yy'-2+4y'=0$$

$$y'(2y+4)=2-2x$$

$$y'=\frac{2-2x}{2y+4}=\frac{1-x}{y+2}$$

Así: $y'(3,3)=-\frac{2}{5}$; $y'(3,-7)=\frac{2}{5}$

Recta tangente en (3,3): $y=3-\frac{2}{5}(x-3)=-\frac{2}{5}x+\frac{21}{5}$

Recta tangente en (3,–7): $y=-7+\frac{2}{5}(x-3)=\frac{2}{5}x-\frac{41}{5}$

Con MATLAB

Cálculo de las ordenadas en x = 3

```
>> syms x y
>> solve(*x^2 + y^2 - 2*x + 4*y - 24 = 0','y')
ans =
   (- x^2 + 2*x + 28)^(1/2) - 2
 - (- x^2 + 2*x + 28)^(1/2) - 2
>> clear
>> x = 3;
>> y =  (- x^2 + 2*x + 28)^(1/2) - 2
y =
     3
>> x = 3;
>> y =  -(- x^2 + 2*x + 28)^(1/2) - 2
y =
    -7
```

Cálculo de las pendientes en x = 3

```
>> syms x
>> m = diff( (- x^2 + 2*x + 28)^(1/2) - 2,x)
m =
-(2*x - 2)/(2*(- x^2 + 2*x + 28)^(1/2))
>> pretty(m)

           2 x - 2
  - ----------------------
          2            1/2
    2 (- x  + 2 x + 28)
>> n = diff( - (- x^2 + 2*x + 28)^(1/2) - 2,x)
n =
(2*x - 2)/(2*(- x^2 + 2*x + 28)^(1/2))
>> pretty(n)

         2 x - 2
  ----------------------
        2            1/2
  2 (- x  + 2 x + 28)
>> clear
>> x = 3;
>> m1 = -(2*x - 2)/(2*(- x^2 + 2*x + 28)^(1/2))
m1 =
   -0.4000
>> x = 3;
>> n1 = (2*x - 2)/(2*(- x^2 + 2*x + 28)^(1/2))
n1 =
    0.4000
```

Cálculo de las rectas tangentes

```
>> syms x
>> x1 = 3;
>> y1 = 3;
>> m1 = -0.4;
>> y = y1 + m1*(x - x1)
y =
21/5 - (2*x)/5
>> pretty(y)}

         2 x
  21/5 - ---
          5
>> x2 = 3;
>> y2 = -7;
>> n1 = 0.4;
>> y = y2 + n1*(x - x2)
y =
(2*x)/5 - 41/5
>> pretty(y)

  2 x}
  --- - 41/5
   5
```

CAPÍTULO

5

DERIVADAS Y SUS GRÁFICAS

5.1. DERIVADAS Y SUS GRÁFICAS

Hallar la gráfica de las siguientes funciones polinómicas y de su derivada:

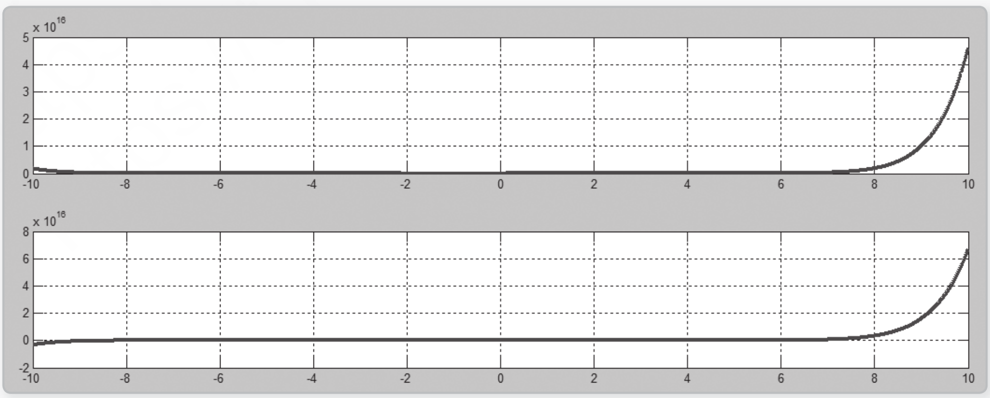
Ejemplo 1

$$y = (2x + 3)^2$$

Con MATLAB:

```
>> syms x y
>> y = (2*x + 3)^2;
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (2.*x + 3).^2;
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
```

```
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (2*x + 3)^2;
>> diff(y)
ans =
8*x + 12
>> x = -10:0.01:10;
>> y = 8.*x + 12;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```



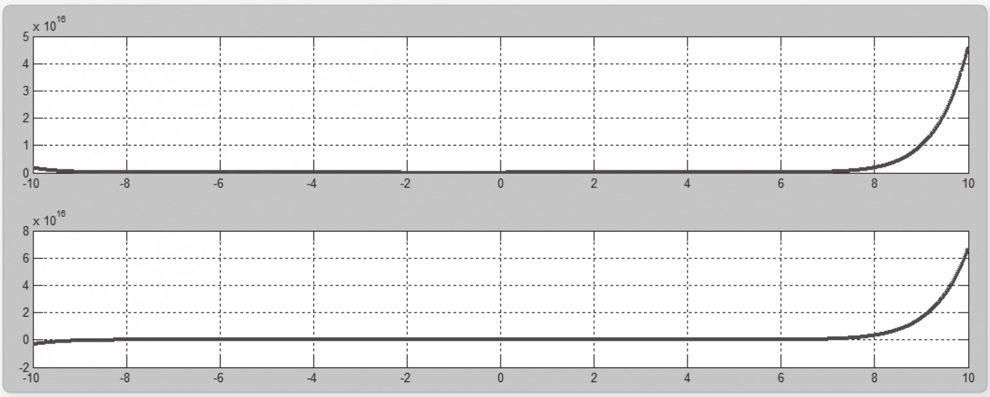
Ejemplo 2

$$y = (x^2 + 2x + 1)^8$$

Con MATLAB:

```
>> syms x y
>> y = (x^2 + 2*x +1)^8;
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (x.*2 + 2.*x +1).^8;
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
```

```
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (x^2 + 2*x +1)^8;
>> diff(y)
ans =
8*(2*x + 2)*(x^2 + 2*x + 1)^7
>> x = -10:0.01:10;
>> y = 8.*(2.*x + 2).*(x.^2 + 2.*x + 1).^7;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

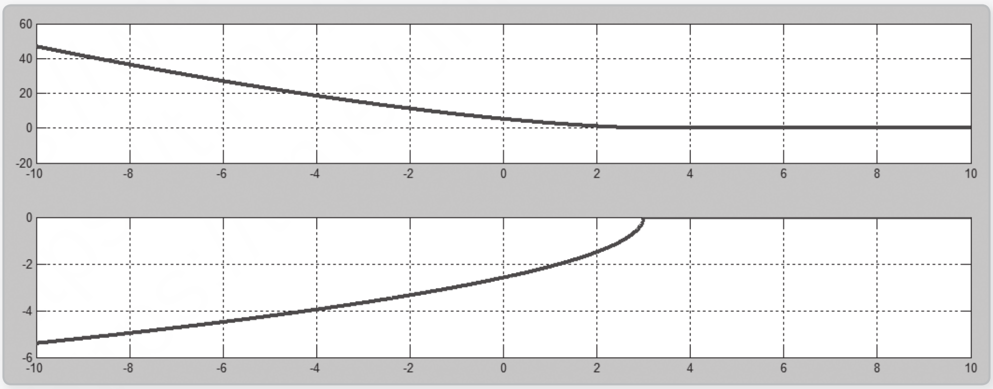


EJEMPLO 3

$$y = (3 - x)^{3/2}$$

Con MATLAB:

```
>> syms x y
>> y = (3 - x)^(3/2);
>> % HALLANDO LA GRAFICA DE LA FUNCION y :
>> x = -10:0.01:10;
>> y = (3 - x).^(3./2);
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION:
>> syms x y
>> y = (3 - x)^(3/2);
>> diff(y)
ans =
-(3*(3 - x)^(1/2))/2
>> x = -10:0.01:10;
>> y = -(3.*(3 - x).^(1./2))./2;
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

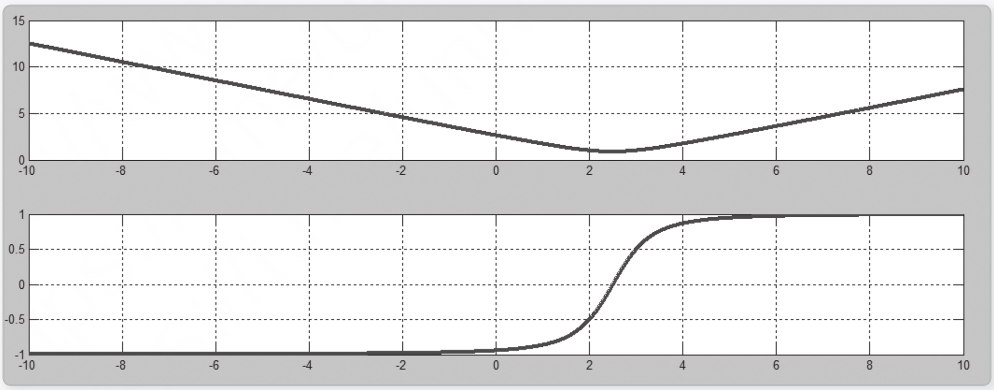


EJEMPLO 4

$$y = \sqrt{x^2 - 5x + 7}$$

Con MATLAB:

```
>> syms x y
>> y = sqrt(x^2 - 5*x +7);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = sqrt(x.^2 - 5.*x +7);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = sqrt(x^2 - 5*x +7);
>> diff(y)
ans =
(2*x - 5)/(2*(x^2 - 5*x + 7)^(1/2))
>> x = -10:0.01:10;
>> y = (2.*x - 5)./(2.*(x.^2 - 5.*x + 7).^(1./2));
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

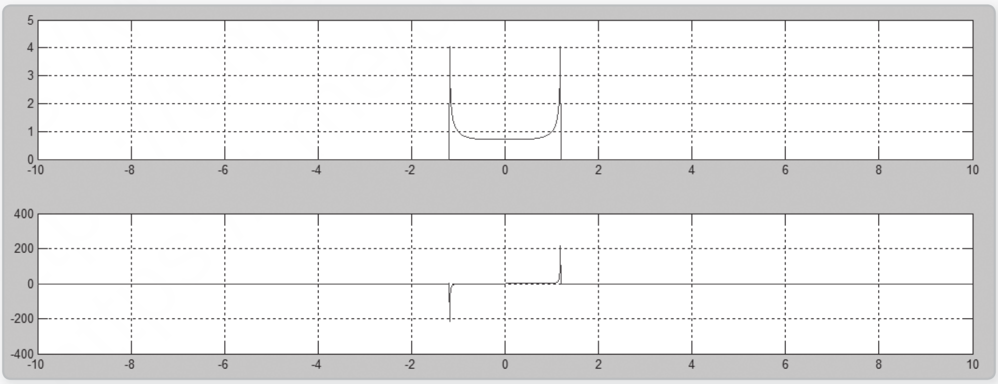


EJEMPLO 5

$$y = \frac{1}{\sqrt{2-x^4}}$$

Con MATLAB:

```
>> syms x y
>> y = 1/(sqrt(2 - x^4));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = 1./(sqrt(2 - x.^4));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = 1/(sqrt(2 - x^4));
>> diff(y)
ans =
(2*x^3)/(2 - x^4)^(3/2)
>> x = -10:0.01:10;
>> y = (2.*x.^3)./(2 - x.^4).^(3./2);
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

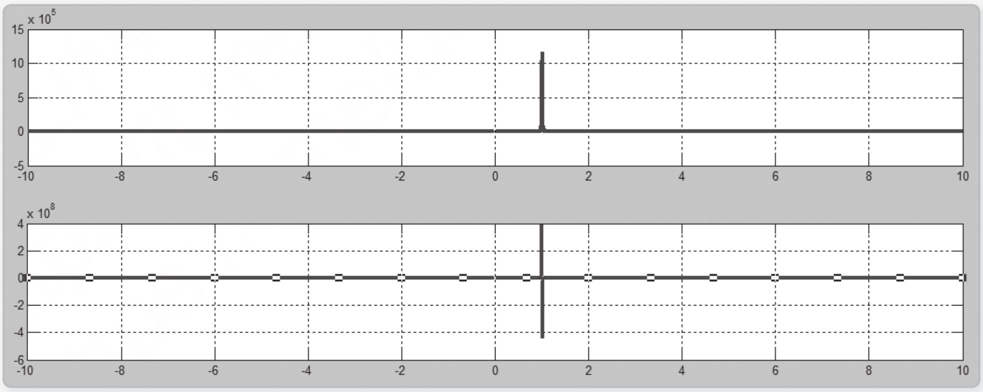


EJEMPLO 6

$$y = (x^3 - \sqrt{x})^{-3.8}$$

Con MATLAB:

```
>> syms x y
>> y = (x^3 - sqrt(x))^-3.8;
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (x.^3 - sqrt(x)).^-3.8;
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICAS DE LA DERIVADA DE LA FUNCION:
>> syms x y
>> y = (x^3 - sqrt(x))^-3.8;
>> diff(y)
ans =
-(19*(3*x^2 - 1/(2*x^(1/2))))/(5*(x^3 - x^(1/2))^(24/5))
>> x = -10:0.01:10;
>> y = -(19.*(3.*x.^2 - 1./(2.*x.^(1./2)))./(5.*(x.^3 - x.^(1./2)).^(24./5));
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

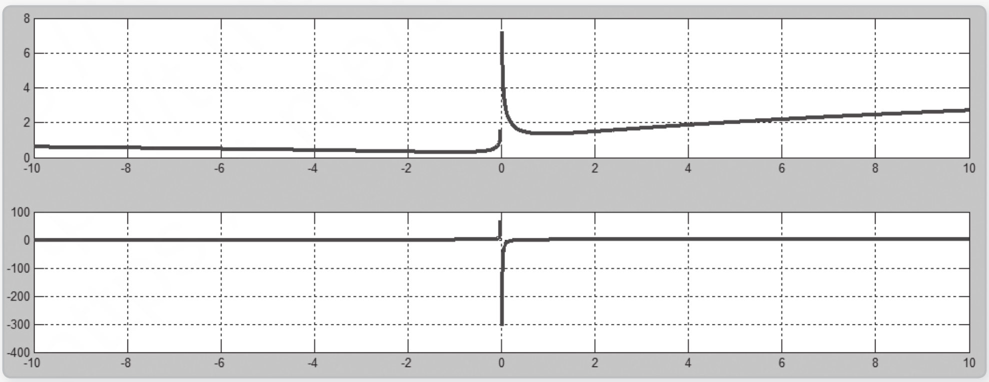


Ejemplo 7

$$y=\left(x+\frac{1}{x}\right)^{3/7}$$

Con MATLAB:

```
>> syms x y
>> y = (x + (1/x))^(3/7);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (x + (1./x)).^(3./7);
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (x + (1/x))^(3/7);
>> diff(y)
ans =
-(3*(1/x^2 - 1))/(7*(x + 1/x)^(4/7))
>> x = -10:0.01:10;
>> y = -(3.*(1./x.^2 - 1))./(7.*(x + 1./x).^(4./7));
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

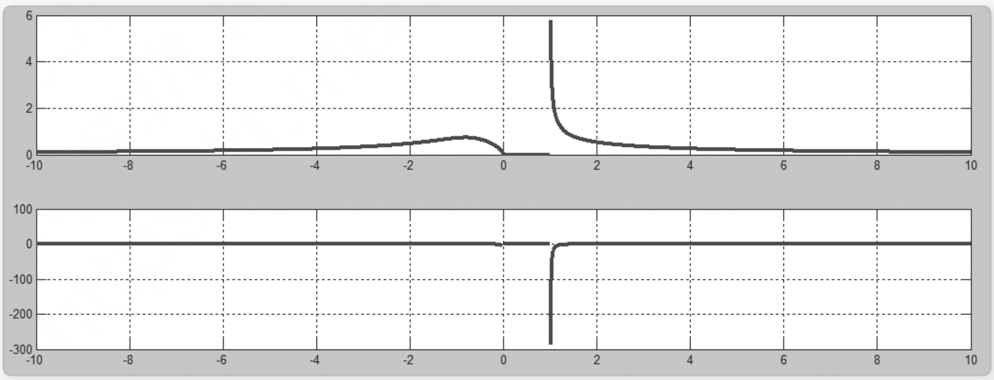


Ejemplo 8

$$y = \frac{1}{\sqrt{x^2 - \frac{1}{x}}}$$

Con MATLAB:

```
>> syms x y
>> y = 1/(sqrt(x^2 – (1/x)));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = –10:0.01:10;
>> y = 1./(sqrt(x.^2 – (1./x)));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = 1/(sqrt(x^2 – (1/x)));
>> diff(y)
ans =
–(2*x + 1/x^2)/(2*(x^2 – 1/x)^(3/2))
>> x = –10:0.01:10;
>> y = –(2.*x + 1./x.^2)./(2.*(x.^2 – 1./x).^(3./2));
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

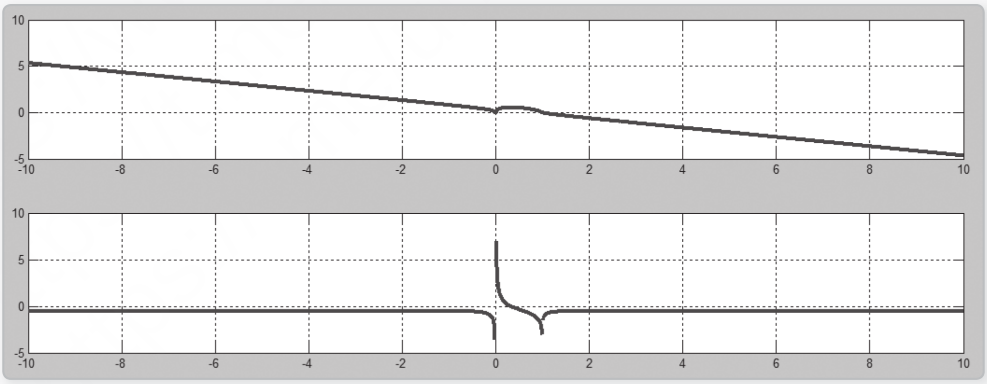


EJEMPLO 9

$$y = x^{\frac{1}{3}}(1-x)^{2/3}$$

Con MATLAB:

```
>> syms x
>> y = (x^(1/3))*((1- x)^(2/3));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (x.^(1./3)).*((1- x).^(2./3));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (x^(1/3))*((1- x)^(2/3));
>> diff(y)
ans =
(1 - x)^(2/3)/(3*x^(2/3)) - (2*x^(1/3))/(3*(1 - x)^(1/3))
>> x = -10:0.01:10;
>> y = (1 - x).^(2./3)./(3.*x.^(2./3)) - (2.*x.^(1./3))./(3.*(1 - x).^(1./3))
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

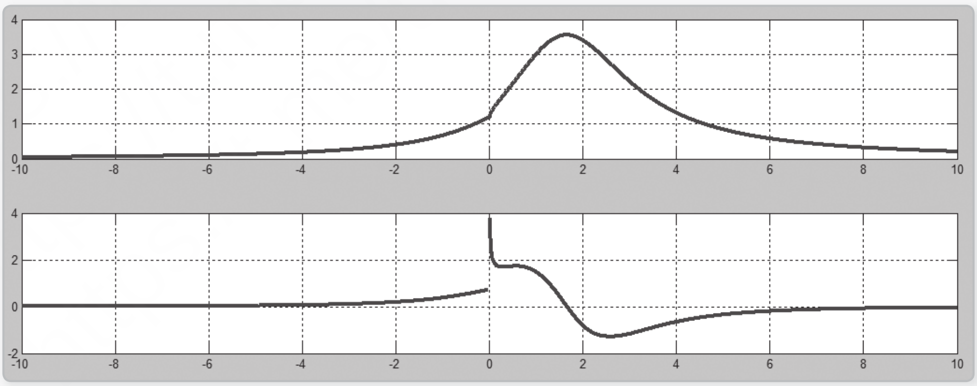


Ejemplo 10

$$y = x^{\frac{1}{3}}(1-x)^{2/3}$$

Con MATLAB:

```
>> syms x y
>> y = (sqrt(x) + 2)/((x^2 - 3*x + 5)^1/3);
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (sqrt(x) + 2)./((x.^2 - 3.*x + 5).^1./3);
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (sqrt(x) + 2)/((x^2 - 3*x + 5)^1/3);
>> diff(y)
ans =
1/(2*x^(1/2)*(x^2/3 - x + 5/3)) - (((2*x)/3 - 1)*(x^(1/2) + 2))/(x^2/3 - x + 5/3)^2
>> x = -10:0.01:10;
>> y = 1./(2.*x.^(1./2).*(x.^2./3 - x + 5./3)) - (((2.*x)./3 - 1).*(x.^(1./2) +
2))./(x.^2./3 - x + 5./3).^2;
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```



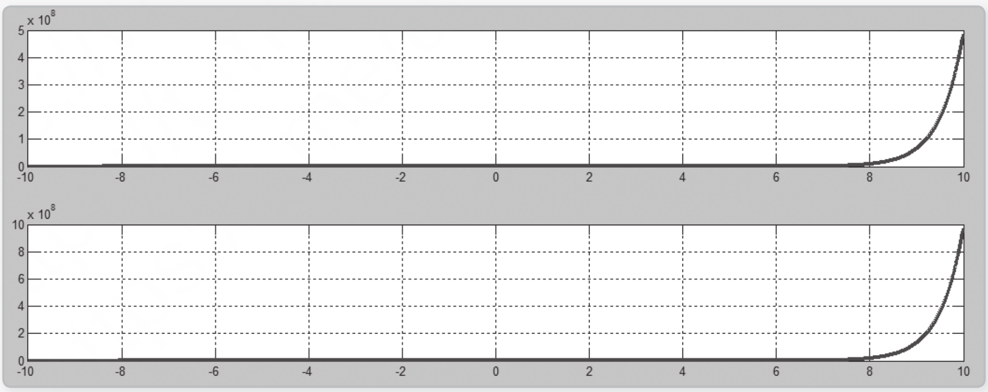
Hallar la gráfica de las siguientes funciones exponenciales y logarítmicas, además la gráfica de su derivada.

EJEMPLO 1

$$y = e^{2x}$$

Con MATLAB:

```
>> syms x y
>> y = exp(2*x) ;
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = exp(2.*x);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = exp(2*x);
>> diff(y)
ans =
2*exp(2*x)
>> x = -10:0.01:10;
>> y = 2.*exp(2.*x);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

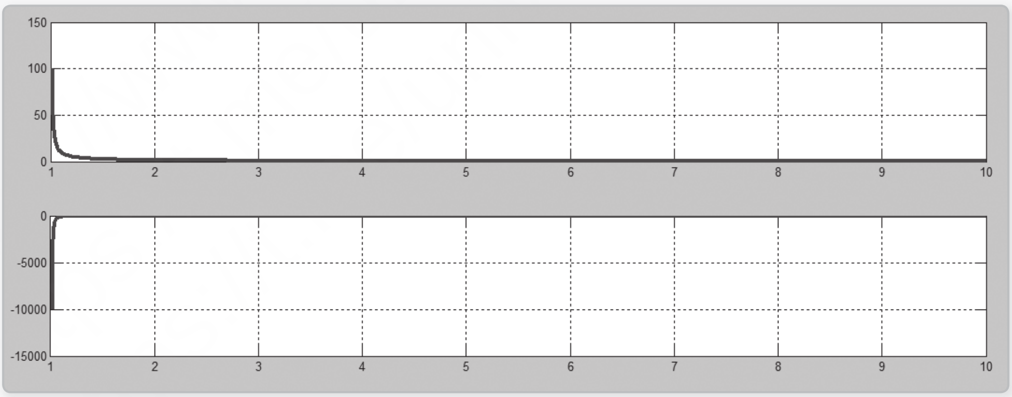


EJEMPLO 2

$$y = \frac{1}{\ln x}$$

Con MATLAB:

```
>> syms x y
>> y = 1/(log(x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = 1:0.01:10;
>> y = 1./(log(x));
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION:
>> syms x y
>> y = 1/(log(x));
>> diff(y)
ans =
-1/(x*log(x)^2)
>> x = 1:0.01:10;
>> y = -1./(x.*log(x).^2);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

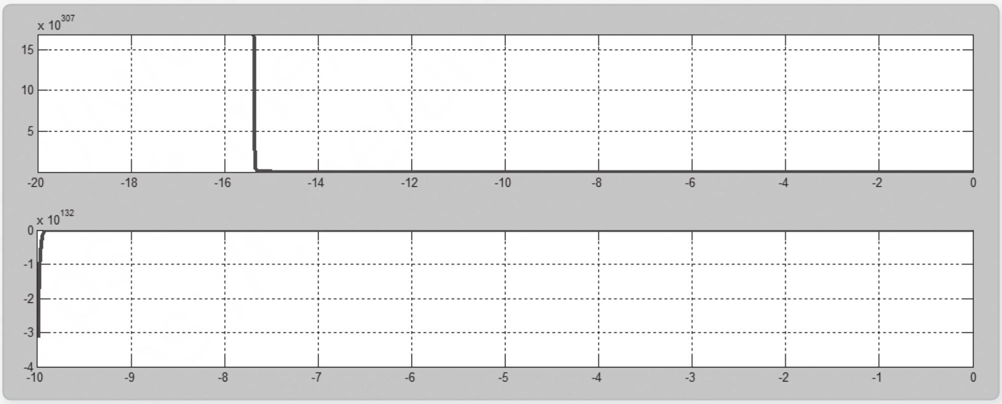


Ejemplo 3

$$y = e^{\left(3x^2+1\right)}$$

Con MATLAB:

```
>> syms x y
>> y = exp(3*x^2 + 1);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -20:0.01:0;
>> y = exp(3.*x.^2 + 1);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = exp(3*x^2 + 1);
>> diff(y)
ans =
6*x*exp(3*x^2 + 1)
>> x = -10:0.01:0;
>> y = 6.*x.*exp(3.*x.^2 + 1);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

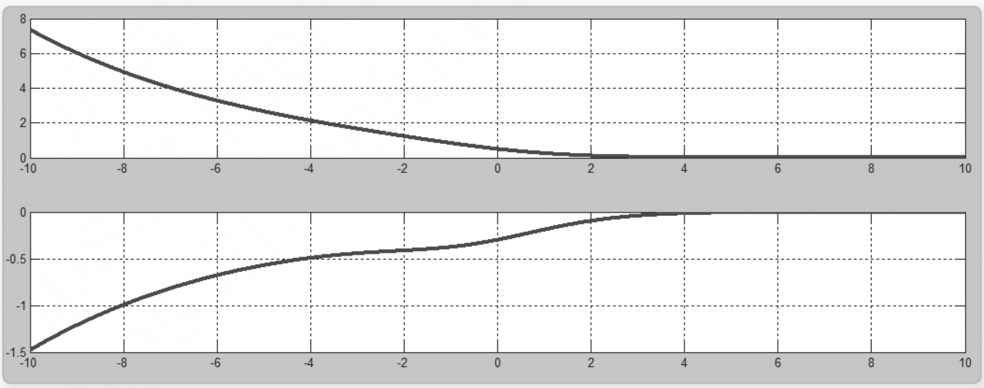


EJEMPLO 4

$$y = \frac{1y}{e^{0,2x} + e^{x}}$$

Con MATLAB:

```
>> syms x y
>> y = 1/(exp(0.2*x) + exp(x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = 1./(exp(0.2.*x) + exp(x));
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = 1/(exp(0.2*x) + exp(x));
>> diff(y)
ans =
-(exp(x/5)/5 + exp(x))/(exp(x/5) + exp(x))^2
>> x = -10:0.01:10;
>> y = -(exp(x./5)./5 + exp(x))./(exp(x./5) + exp(x)).^2;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

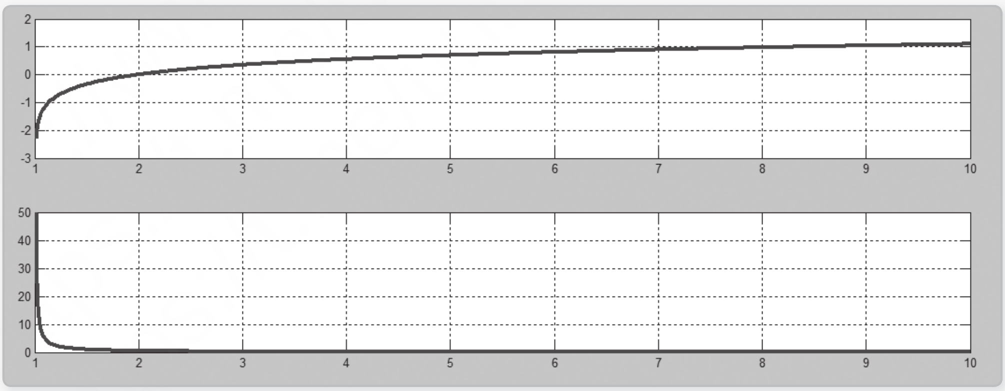


Ejemplo 5

$$y = \ln\sqrt{1-x}$$

Con MATLAB:

```
>> syms x y
>> y = log(sqrt(1-x));
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = 1:0.01:10;
>> y = 1og(sqrt(1-x));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = log(sqrt(1-x));
>> diff(y)
ans =
1/(2*(x - 1))
>> x = 1:0:01:10;
>> y = 1./(2.*(x - 1));
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

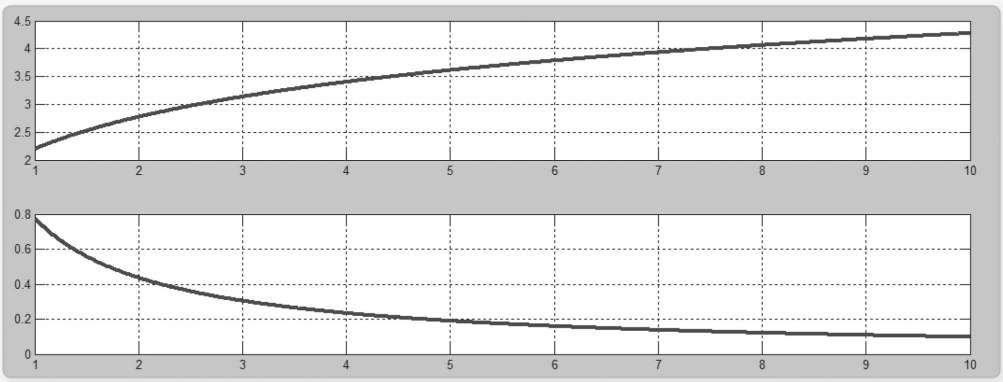


EJEMPLO 6

$$y = \ln(7x + 2)$$

Con MATLAB:

```
>> syms x y
>> y = log(7*x + 2);
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = 1:0.01:10;
>> y = log(7.*x + 2);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = log(7*x + 2);
>> diff(y)
ans =
7/(7*x + 2)
>> x = 1:0:01:10;
>> subplot(3,1,2)
>> y = 7./(7.*x + 2);
>> plot(x,y)
>> grid on
```

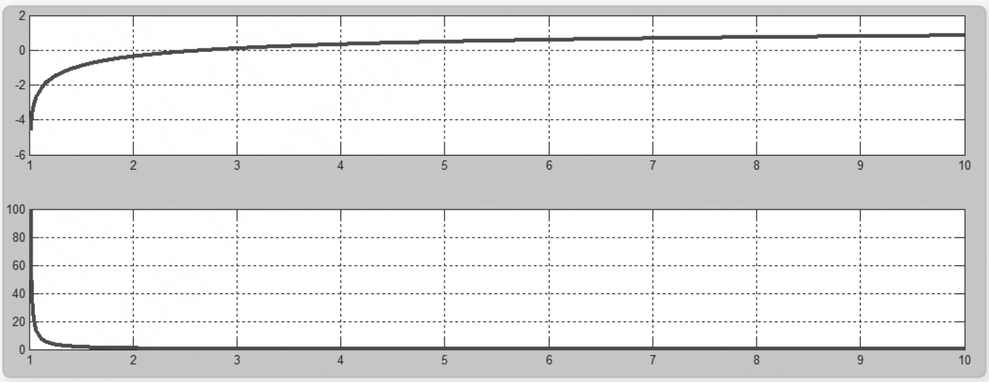


Ejemplo 7

$$y = \ln\left(\ln\left(\frac{1}{x}\right)\right)$$

Con MATLAB:

```
>> syms x y
>> y = log(log(1/x));
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = 1:0.01:10;
>> y = log(log(1./x));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = log(log(1/x));
>> diff(y)
ans =
-1/(x*log(1/x))
>> x = 1:0:01:10;
>> y = -1./(x.*log(1./x));
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

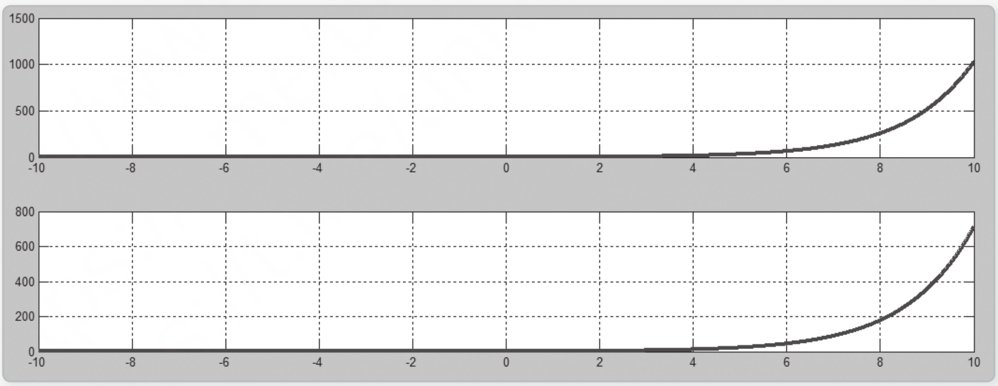


Ejemplo 8

$$y = 2^x$$

Con MATLAB:

```
>> syms x y
>> y = 2^x;
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = 2.^x;
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = 2^x;
>> diff(y)
ans =
2^x*log(2)
>> x = -10:0.01:10;
>> y = 2.^x.*log(2);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

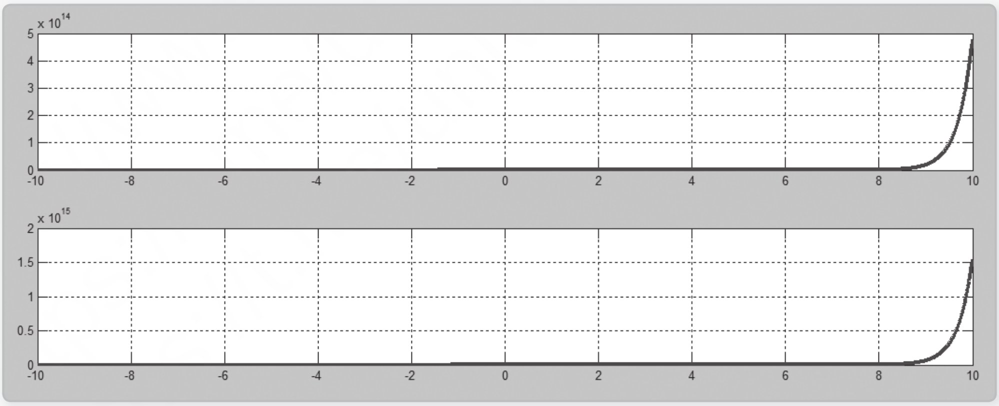


Ejemplo 9

$$y = 5^{(2x+1)}$$

Con MATLAB:

```
>> syms x y
>> y = 5^(2*x + 1);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = 5.^(2.*x + 1);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = 5^(2*x + 1);
>> diff(y)
ans =
2*5^(2*x + 1)*log(5)
>> x = -10:0.01:10;
>> y = 2.*5.^(2.*x + 1).*log(5);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

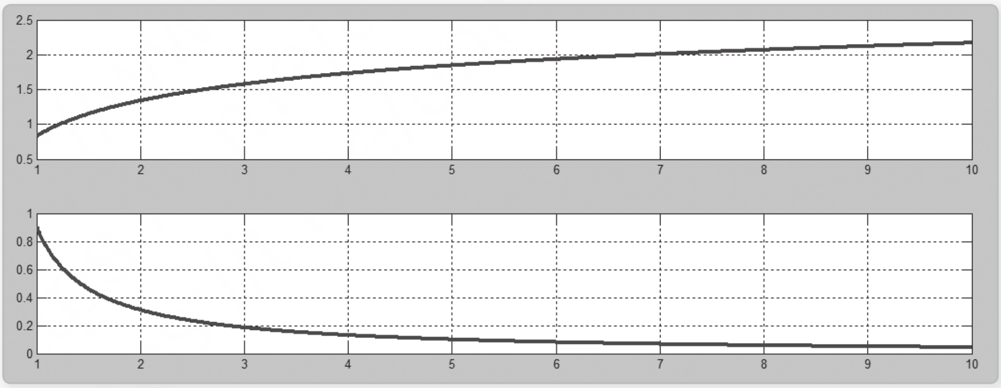


EJEMPLO 10

$$y = \sqrt{\ln\left(x^2 + x\right)}$$

Con MATLAB:

```
>> syms x y
>> y = sqrt(log(x^2 + x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = 1:0.01:10;
>> y = sqrt(log(x.^2 + x));
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> % HALLAR LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = sqrt(log(x^2 + x));
>> diff(y)
ans =
(2*x + 1)/(2*log(x^2 + x)^(1/2)*(x^2 + x))
>> x = 1:0.01:10;
>> y = (2.*x + 1)./(2.*log(x.^2 + x).^(1./2).*(x.^2 + x));
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```



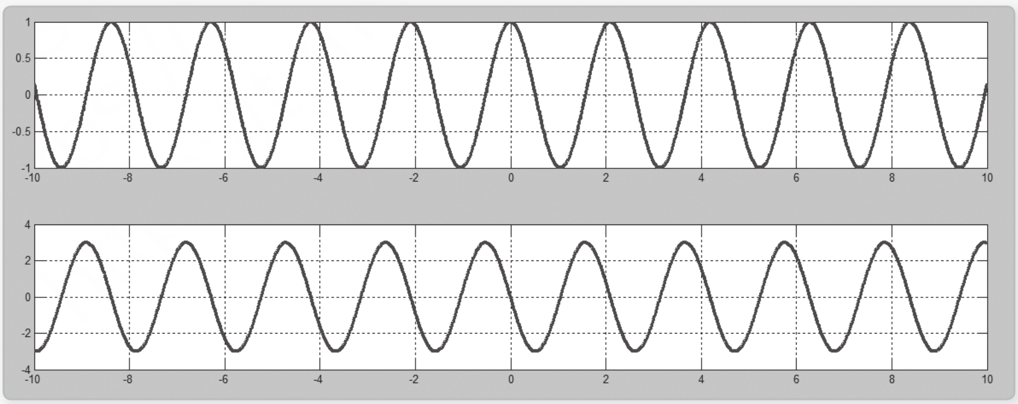
Hallar la gráfica de las siguientes funciones trigonométricas, además la gráfica de su derivada.

EJEMPLO 1

$$y = \cos 3x$$

Con MATLAB:

```
>> syms x y
>> y = cos(3*x);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = cos(3.*x);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> hold on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = cos(3*x);
>> diff(y)

ans =

(-3)*sin(3*x)

>> x = -10:0.01:10;
>> y = (-3).*sin(3.*x);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```



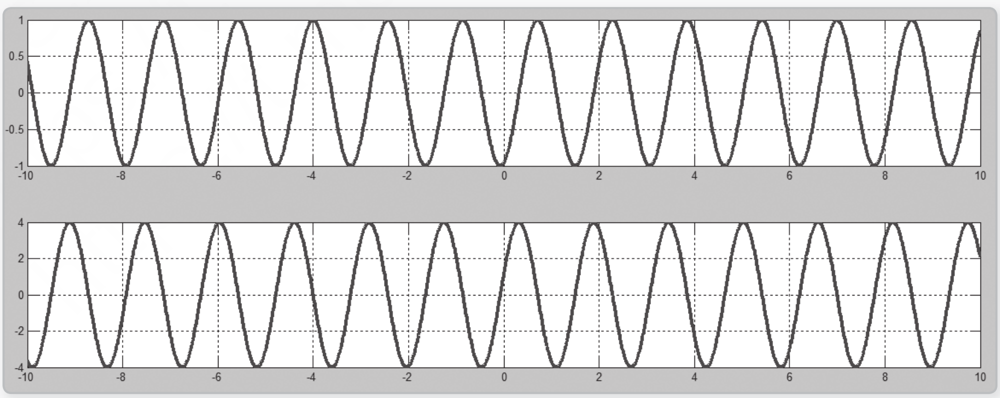
Ejemplo 2

$$y = \text{sen}(4x + 5)$$

Con MATLAB:

```
>> syms x y
>> y = sin(4*x + 5);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = sin(4.*x + 5);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> hold on
```

```
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = sin(4*x + 5);
>> diff(y)

ans =

4*cos(4*x + 5)

>> x = -10:0.01:10;
>> y = 4.*cos(4.*x + 5);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

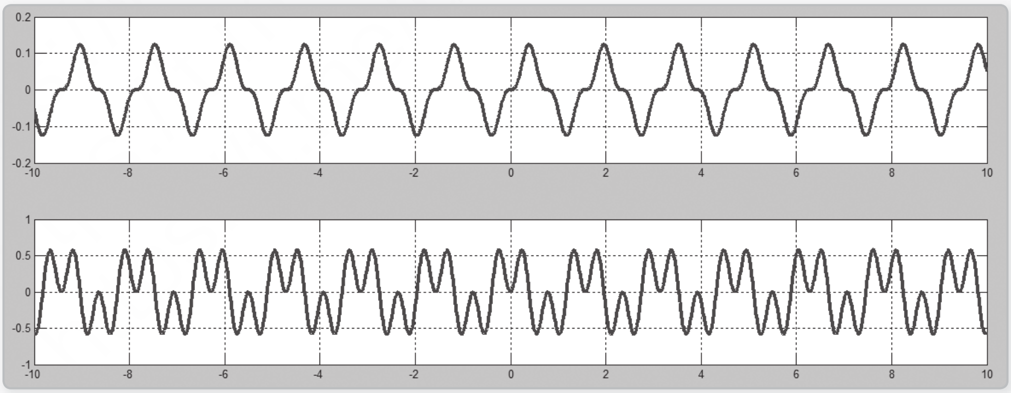


Ejemplo 3

$$y = \operatorname{sen}^3(2x).\cos^3(2x)$$

Con MATLAB:

```
>> syms x y
>> y = (sin(2*x)^3)*(cos(2*x)^3);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (sin(2.*x).^3).*(cos(2.*x).^3);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> hold on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (sin(2*x)^3)*(cos(2*x)^3);
>> diff(y)

ans =

6*cos(2*x)^4*sin(2*x)^2 - 6*cos(2*x)^2*sin(2*x)^4

>> x = -10:0.01:10;
>> y = 6.*cos(2.*x).^4.*sin(2.*x).^2 - 6.*cos(2.*x).^2.*sin(2.*x).^4;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

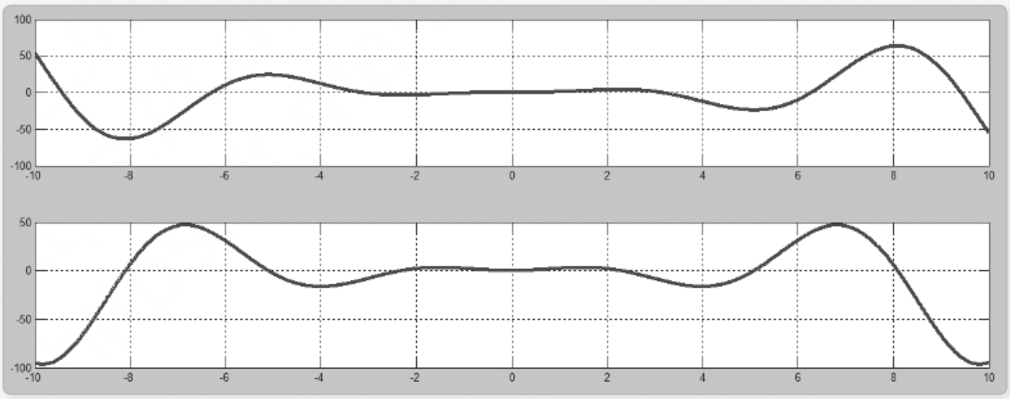


EJEMPLO 4

$$y = x^2.\text{senx}$$

Con MATLAB:

```
>> syms x y
>> y = x^2*sin(x);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = x.^2.*sin(x);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> hold on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = x^2*sin(x);
>> diff(y)

ans =

x^2*cos(x) + 2*x*sin(x)

>> x = -10:0.01:10;
>> y = x.^2.*cos(x) + 2.*x.*sin(x);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

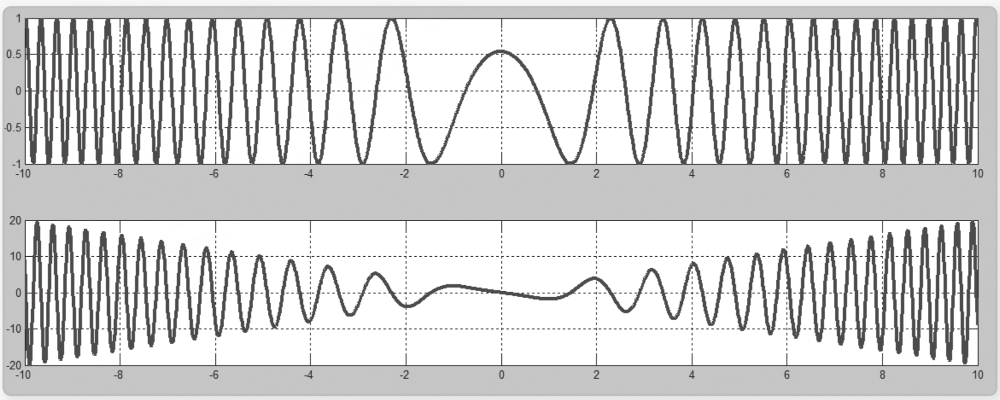


EJEMPLO 5

$$y = \cos(x^2 + 1)$$

Con MATLAB:

```
>> syms x y
>> y = cos(x^2 + 1);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = cos(x.^2 + 1);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> hold on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = cos(x^2 + 1);
>> diff(y)

ans =

(-2)*x*sin(x^2 + 1)

>> x = -10:0.01:10;
>> y = (-2).*x.*sin(x.^2 + 1);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

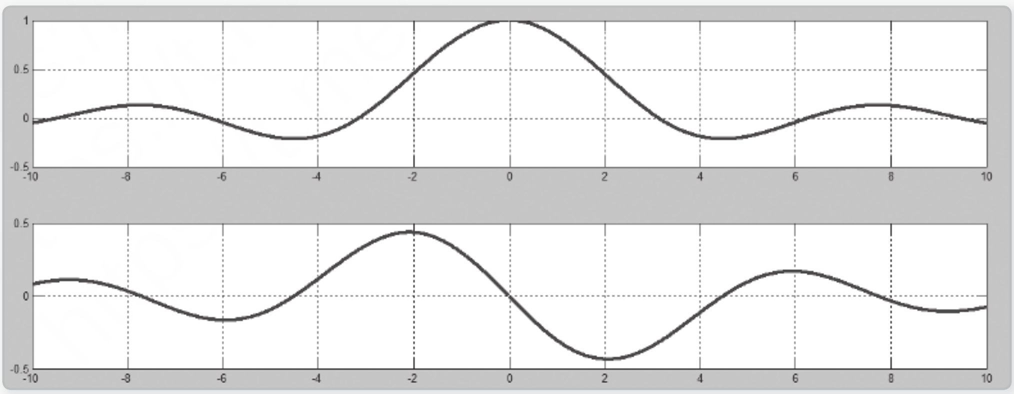


Ejemplo 6

$$y = \frac{\text{sen}x}{x}$$

Con MATLAB:

```
>> syms x y
>> y = sin(x)/x;
>> % HALLANDO LA ECUACION DE LA GRAFICA:
>> x = -10:0.01:10;
>> y = sin(x)./x;
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> hold on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = sin(x)/x;
>> diff(y)

ans =

cos(x)/x - sin(x)/x^2

>> x = -10:0.01:10;
>> y = cos(x)./x - sin(x)./x.^2;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```



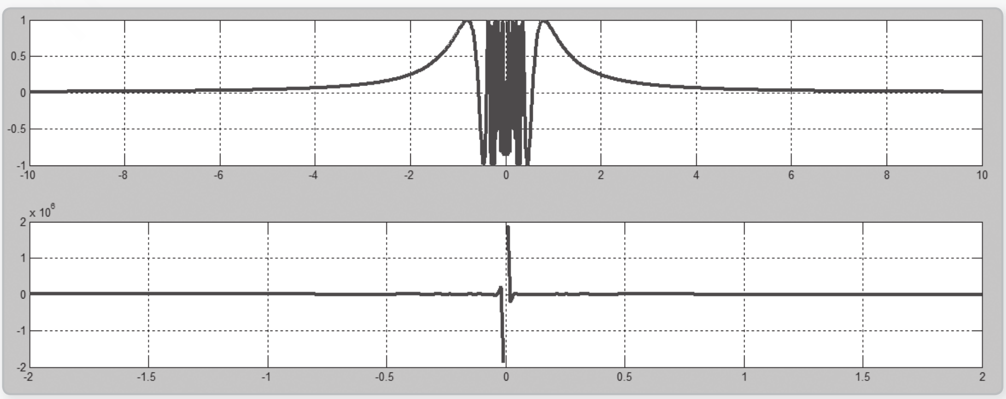
Ejemplo 7

$$y = \sin\left(\frac{1}{x^2}\right)$$

Con MATLAB:

```
>> syms x y
>> y = sin(1/x^2):
>> % HALLAMOS LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = sin(1./x.^2);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> hold on
>> % HALLAMOS LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = sin(1/x^2);
>> diff(y)

ans =

-(2*cos(1/x^2))/x^3

>> x = 0:0.01:10;
>> y = -(2.*cos(1./x.^2))./x.^3;
>> subplot(3,1,2)
>> plot(x,y)
```

```
>> % HALLAMOS LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = sin(1/x^2);
>> diff(y)

ans =

-(2*cos(1/x^2))/x^3

>> x = -2:0.01:2;
>> y = -(2.*cos(1./x.^2))./x.^3;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

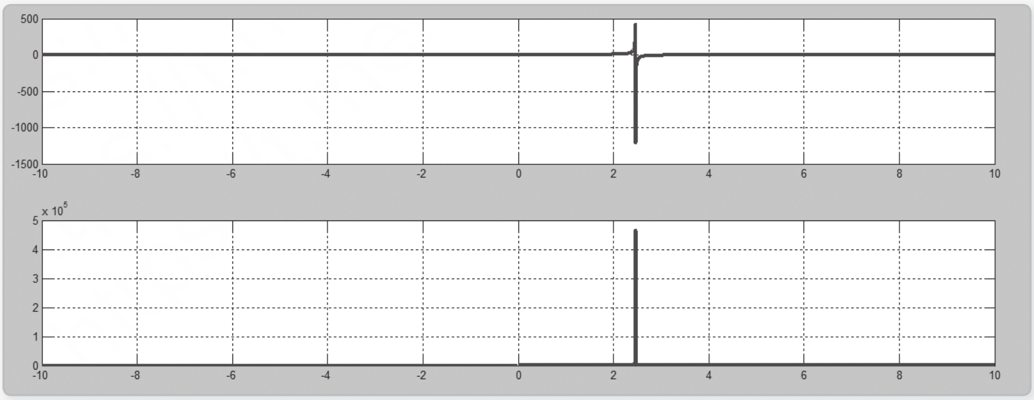


EJEMPLO 8

$$y = \tan\sqrt{x}$$

Con MATLAB:

```
>> syms x y
>> y = tan(sqrt(x));
>> %  GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = tan(sqrt(x));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = tan(sqrt(x));
>> diff(y)

ans =

(tan(x^(1/2))^2 + 1)/(2*x^(1/2))

>> x = -10:0.01:10;
>> y = (tan(x.^(1./2)).^2 + 1)./(2.*x.^(1./2));
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

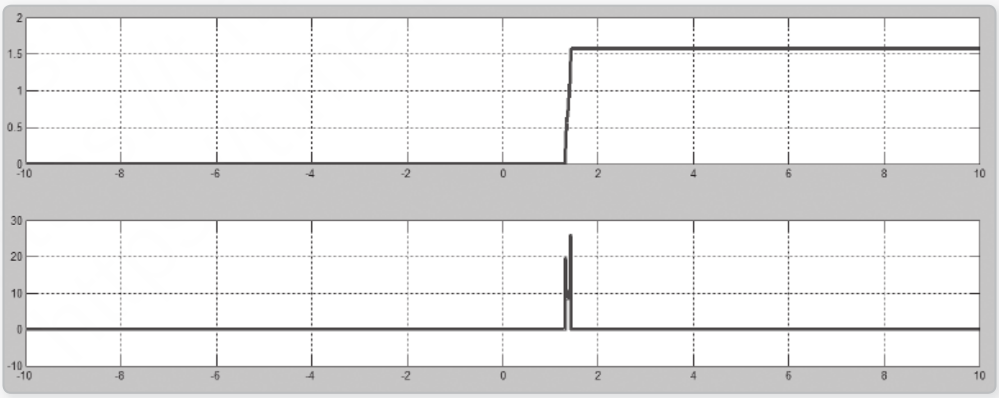


Ejemplo 9

$$y = \arcsin\left(\sqrt{x^3 + x^2 - 4}\right)$$

Con MATLAB:

```
>> syms x y
>> y = asin(sqrt(x^3 + x^2 - 4));
>> %  HALLANDO LA ECUACION DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = asin(sqrt(x.^3 + x.^2 - 4));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA ECUACION DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = asin(sqrt(x^3 + x^2 - 4));
>> diff(y)

ans =

(3*x^2 + 2*x)/(2*(x^3 + x^2 - 4)^(1/2)*(5 - x^2 - x^3)^(1/2))

>> x = -10:0.01:10;
>> y = (3.*x.^2 + 2.*x)./(2.*(x.^3 + x.^2 - 4).^(1./2).*(5 - x.^2 - x.^3).^(1./2));
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

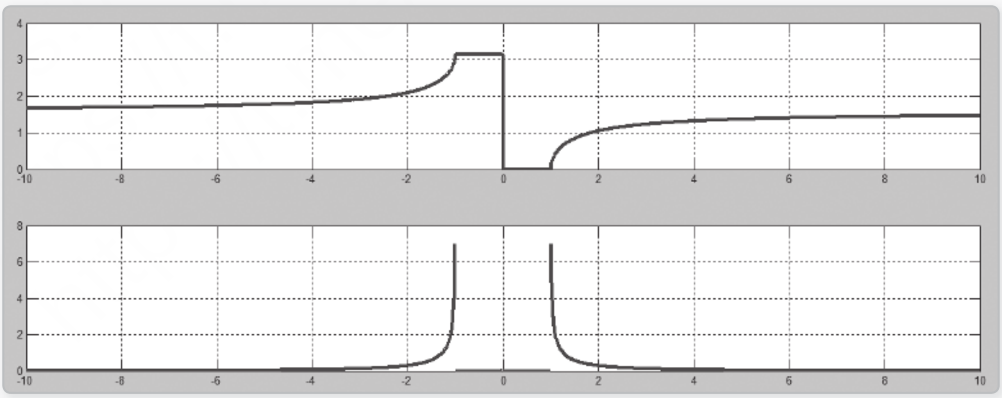


EJEMPLO 10

$$y = \arccos\left(\frac{1}{x}\right)$$

Con MATLAB:

```
>> syms x y
>> y = acos(1/x);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = acos(1./x);
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION:
>> syms x y
>> y = acos(1/x);
>> diff(y)

ans =

1/(x^2*(1 - 1/x^2)^(1/2))

>> x = -10:0.01:10;
>> y = 1./(x.^2.*(1 - 1./x.^2).^(1./2));
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```



5.2. RECTAS TANGENTES

PROBLEMA 1

Halla las rectas tangentes a la curva $y = \dfrac{5x^3 + 7x^2 - 16x}{x - 2}$ en los puntos de abcisas 0, 1, 3.

Calculamos la derivada de la función:

$$y' = \frac{(15x^2 + 14x - 16)(x-2) - (5x^3 + 7x^2 - 16x)}{(x-2)^2} = \frac{10x^3 - 23x^2 - 28x + 32}{(x-2)^2}$$

Ordenadas de los puntos:

$$y(0) = 0 \; ; \; y(1) = 4 \; ; \; y(3) = 150$$

- Recta tangente en (0,0): y'(0) = 8

$$y = 8x$$

- Recta tangente en (0,0): y'(1) = –9

$$y = 4 - 9(x - 1) = -9x + 13$$

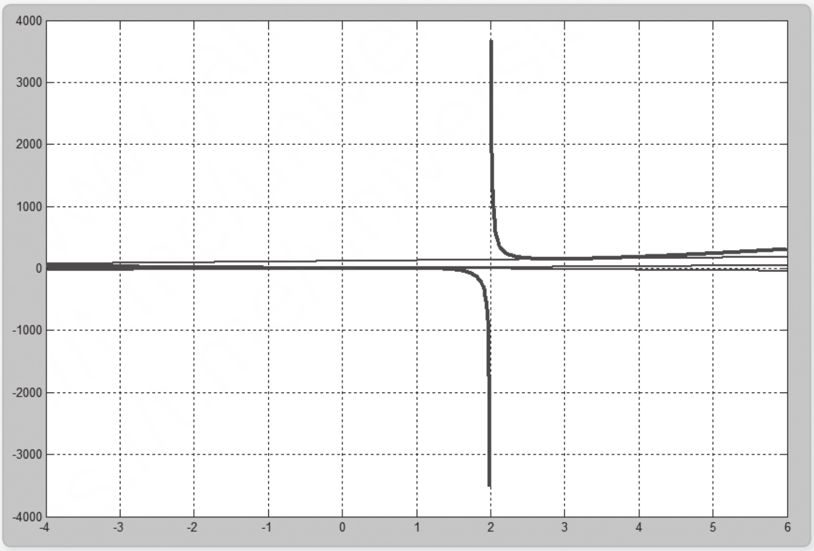
- Recta tangente en (0,0): y'(3) = 11

$$y = 150 + 11(x - 3) = 11x + 117$$

Con MATLAB:

```
>> syms x y
A=5*x^3+7*x^2-16*x;
B=x-2;
% CALCULO DE LA PENDIENTE
y= A/B;
diff(y)
 ans =
 (15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2
m=(15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2;
% CALCULO DE LAS PENDIENTES
x = 0;
m=(15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2
m =     8
m=(15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2;
% CALCULO DE LAS PENDIENTES
x = 1;
m=(15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2
m = -9
m=(15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2;
% CALCULO DE LAS PENDIENTES
x = 3;
m=(15*x^2 + 14*x - 16)/(x - 2) - (5*x^3 + 7*x^2 - 16*x)/(x - 2)^2
m =    11
% CALCULO DE LAS ORDENADAS
T = (5*x^3+7*x^2-16*x)/ (x-2),
x=0;
T = (5*x^3+7*x^2-16*x)/ (x-2)
T =     0
```

```
x=1;
T = (5*x^3+7*x^2-16*x)/ (x-2)
T =     4
x=3;
T = (5*x^3+7*x^2-16*x)/ (x-2)
T =   150
>> x= -4:0.01:6;
>> T=(5.*x.^3+7.*x.^2-16.*x)./(x-2);
>> plot(x,T)
>> grid on
>> hold on
>> a=8.*x;
>> plot(x,a)
>> hold on
>> b=-9.*(x-1)+4;
>> plot(x,b)
>> hold on
>> c=11.*(x-3)+150;
>> plot(x,c)
```



PROBLEMA 2

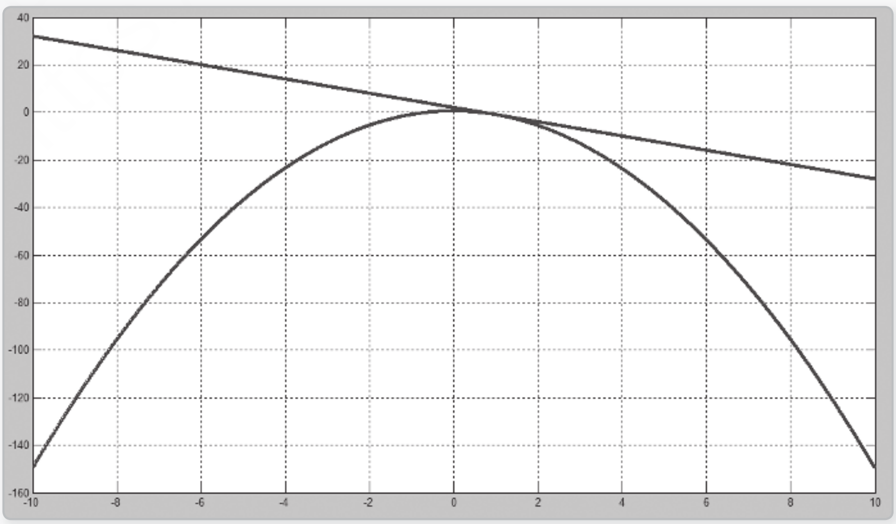
Hallar la ecuación de la recta tangente a la curva $y = \frac{1 - 3x^2}{2}$ en $x = 1$

- Ordenada en el punto: $x = 1 \rightarrow y = -1$
- Pendiente de la recta: $y' = -3x \rightarrow y'(1) = -3$

Recta tangente: $y = -1 - 3(x - 1) = -3x + 2$

Con MATLAB:

```
>>  >> syms x y
A= 1-3*x^2;
B= 2;
% CALCULO DE LA PENDIENTE
y= A/B;
diff(y)
 ans =
 -3*x
m= -3*x;
% CALCULO DE LA PENDIENTE
m= -3*x
m = -3*x
 >> m= -3*x;
% CALCULO DE LA PENDIENTE
x=1;
m= -3*x
m =    -3
% CALCULO DE LAS COORDENADAS
T=(1-3*x^2)/2;
x=1;
T=(1-3*x^2)/2;
T =  -1
>> x=-10:0.01:10;
T=(1-3.*x.^2)./2;
plot(x,T)
grid on
hold on
a=-3.*(x-1)-1;
plot(x,a)
```



PROBLEMA 3

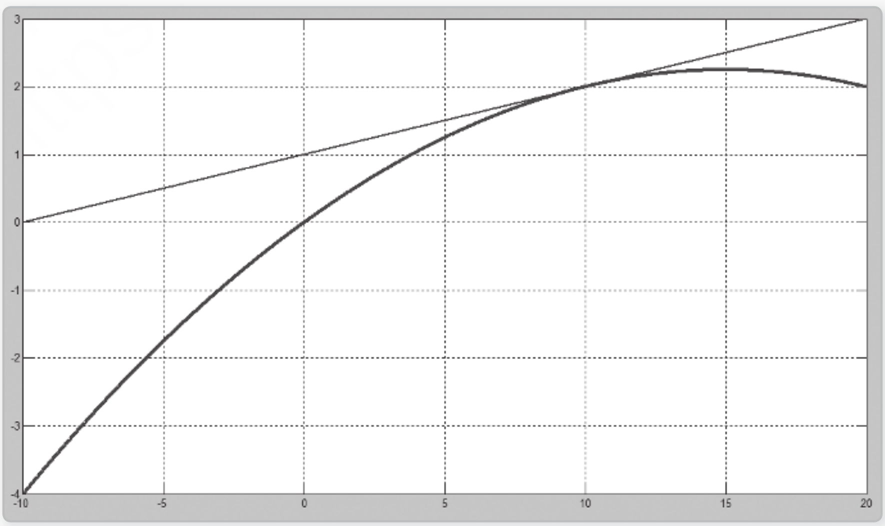
Hallar la ecuación de la recta tangente a la curva $y = 0{,}3x - 0{,}01x^2$ en $x = 10$

- Ordenada en el punto: $x = 10 \rightarrow y = 2$
- Pendiente de la recta: $y' = 0{,}3 - 0{,}02x \rightarrow y'(10) = 0{,}3 - 0{,}2 = 0{,}1$

Recta tangente: $y = 2 + 0{,}1(x - 10) = 0{,}1x + 1$

Con MATLAB:

```
>> syms x y
>> A = 0.3*x - 0.01*x^2;
>> % CALCULO DE LA PENDIENTE:
>> diff(A)
ans =
3/10 - x/50
>> m = 3/10 - x/50;
>> % VALOR DE LA PENDIENTE:
>> x = 10;
>> m = 3/10 - x/50
m =
    0.1000
>> % CALCULO DE LA ORDENADA:
>> x = 10;
>> A = 0.3*x - 0.01*x^2
A =
     2
>> % ENTONCES LA ECUACION DE LA RECTA ES:
>> y =  0.1000*(x-10)+2;

>> % GRAFICA DE LA FUNCION:
>> x = -10:0.01:20;
>> A = 0.3.*x - 0.01.*x.^2;
>> plot(x,A)
>> grid on
>> hold on
>> y =  0.1000.*(x-10)+2;
>> plot(x,y)
```



Problema 4

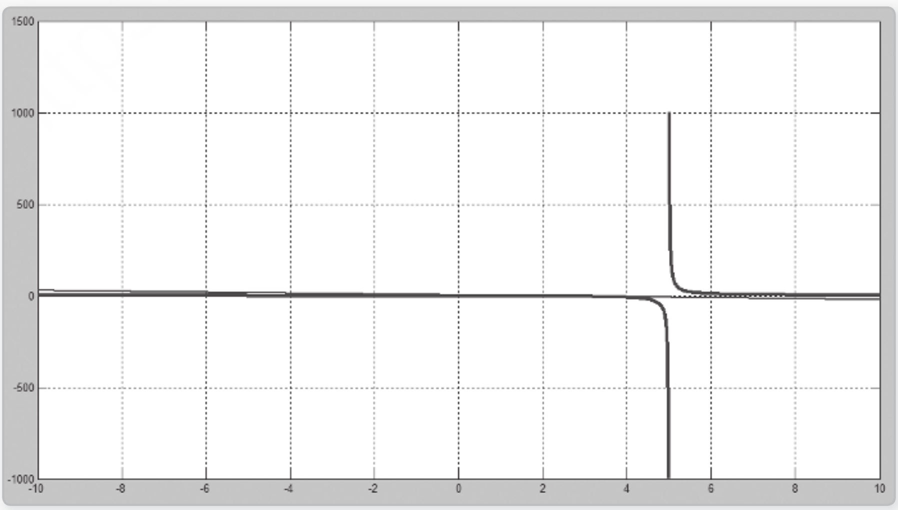
Hallar la ecuación de la recta tangente a la curva $y = \dfrac{x+5}{x-5}$ en x = 3

- Ordenada en el punto: $x = 3 \rightarrow y = -4$
- Pendiente de la recta: $y' = \dfrac{-10}{(x-5)^2} \rightarrow y'(3) = \dfrac{-10}{4} = \dfrac{-5}{2}$

Recta tangente: $y = -4 - \dfrac{5}{2}(x-3) = \dfrac{-5}{2}x + \dfrac{7}{2}$

Con MATLAB:

```
>> syms x y
A= x+5;
B= x-5;
% CALCULO DE LA PENDIENTE
y= A/B;
diff(y)
 ans = 1/(x - 5) - (x + 5)/(x - 5)^2
 >> m= 1/(x - 5) - (x + 5)/(x - 5)^2;
% CALCULO DE LA PENDIENTE
x=3;
m= 1/(x - 5) - (x + 5)/(x - 5)^2
m =   -2.5000
% CALCULO DE LA COORDENADA
T= (x+5)/(x-5);
x=3;
T= (x+5)/(x-5)
T =    -4
ans =
1/(x - 5) - (x + 5)/(x - 5)^2
ans =
1/(x - 5) - (x + 5)/(x - 5)^2
>> x= -10:0.01:10;
T= (x+5)./(x-5);
plot(x,T)
grid on
hold on
a=-2.5000.*(x-3)-4;
plot(x,a)
```



PROBLEMA 5

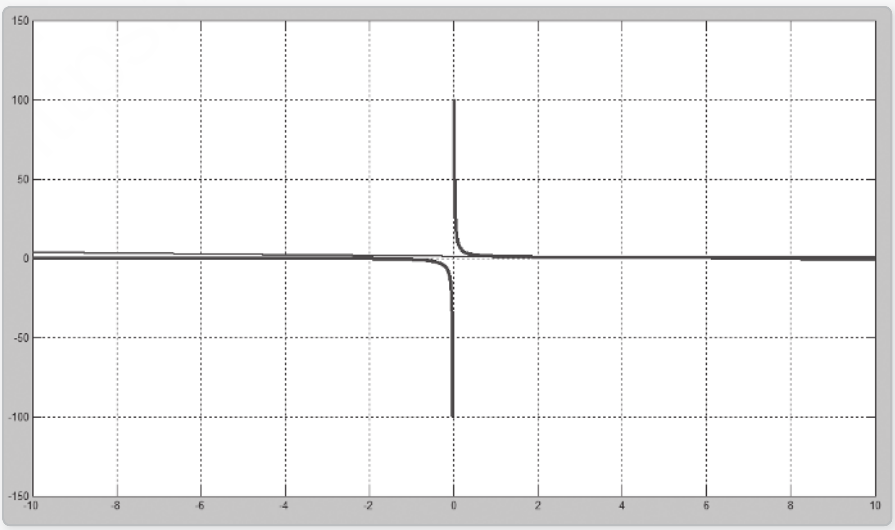
Hallar la ecuación de la recta tangente a la curva $y = \frac{1}{x}$ en $x = 2$

- Ordenada en el punto: $x = 2 \rightarrow y = \frac{1}{2}$
- Pendiente de la recta: $y' = \frac{-1}{x^2} \rightarrow y'(2) = \frac{-1}{4}$

Recta tangente: $y = \frac{1}{2} - \frac{1}{4}(x - 2) = \frac{-1}{4}x + 1$

Con MATLAB:

```
>> syms x y
>> A = (1/x);
>> % CALCULO DE LA PENDIENTE:
>> diff(A)
ans =
-1/x^2
>> m = -1/x^2;
>> % VALOR DE LA PENDIENTE:
>> x = 2;
>> m = -1/x^2
m =
   -0.2500
>> % CALCULO DE LA ORDENADA:
>> x = 2;
>> A = (1/x)
A =
    0.5000
>> % ENTONCES LA ECUACION DE LA RECTA ES:
>> y = -0.25*(x-2)+0.5;
>> % GRAFICA DE LA FUNCION:
>> x = -10:0.01:10;
>> A = (1./x);
>> plot(x,A)
>> grid on
>> hold on
>> y = -0.25.*(x-2)+0.5;
>> plot(x,y)
```



PROBLEMA 6

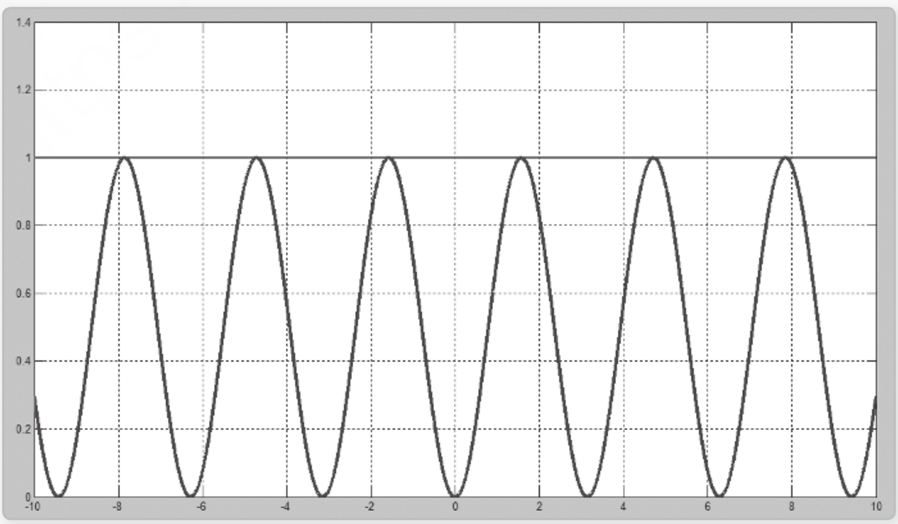
Hallar la ecuación de la recta tangente a la curva $y = sen^2 x$ en $x = \frac{\pi}{2}$

- Ordenada en el punto: $x = \frac{\pi}{2} \rightarrow y = 1$
- Pendiente de la recta: $y' = 2senxcosx \rightarrow y'\left(\frac{\pi}{2}\right) = 0$

Recta tangente: $y = 1$

Con MATLAB:

```
>> syms x y
>> A = sin(x)^2;
>> % CALCULO DE LA PENDIENTE:
>> diff(A)
ans =
2*cos(x)*sin(x)
>> m = 2*cos(x)*sin(x);
>> % VALOR DE LA PENDIENTE:
>> x = pi/2;
>> m = 2*cos(x)*sin(x)
m =
   1.2246e-16
>> % CALCULO DE LA ORDENADA:
>> x = pi/2;;
>> A = sin(x)^2
A =
     1
>> % ENTONCES LA ECUACION DE LA RECTA ES:
>> y = 1.2246e-16*(x-(pi/2))+1;
>> % GRAFICA DE LA FUNCION:
>> x = -10:0.01:10;
>> A = sin(x).^2;
>> plot(x,A)
>> grid on
>> hold on
>> y = 1.2246e–16*(x-(pi/2))+1;
>> plot(x,y)
```



PROBLEMA 7

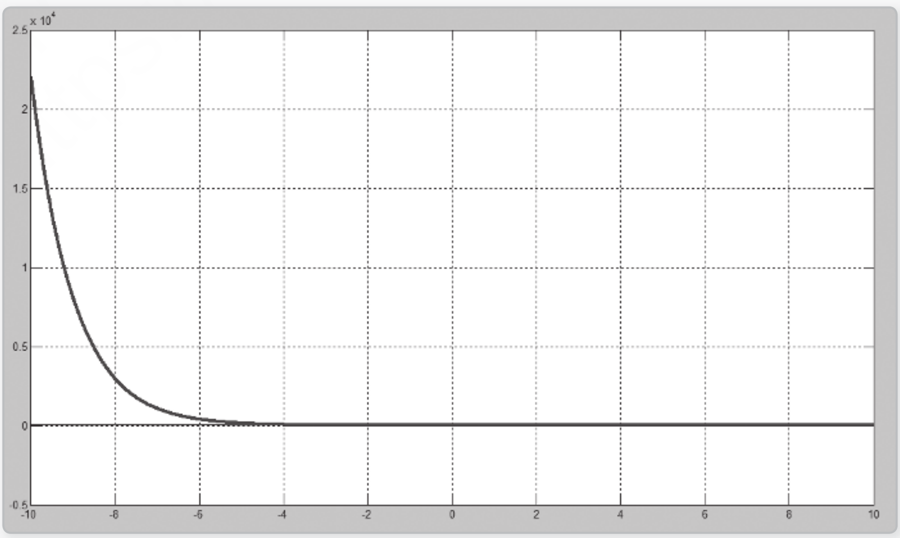
Hallar la ecuación de la recta tangente a la curva $y = e^{-x}$ en $x = 0$

- Ordenada en el punto: $x = 0 \rightarrow y = 1$
- Pendiente de la recta: $y' = -e^{-x} \rightarrow y'(0) = -1$

Recta tangente: $y = 1 - 1x = -x + 1$

Con MATLAB:

```
>> syms x y
>> A = exp(-x);
>> % CALCULO DE LA PENDIENTE:
>> diff(A)
ans =
-exp(-x)
>> m = -exp(-x);
>> % VALOR DE LA PENDIENTE:
>> x = 0;
>> m = -exp(-x)
m =
    -1
>> % CALCULO DE LA ORDENADA:
>> x = 0;
>> A = exp(-x)
A =
     1
>> % ENTONCES LA ECUACION DE LA RECTA:
>> y =  -1*(x-0)+1;
>> % GRAFICA DE LA FUNCION:
>> x = -10:0.01:10;
>> A = exp(-x);
>> plot(x,A)
>> grid on
>> hold on
>> y =  -1.*(x-0)+1;
>> plot(x,y)
```



PROBLEMA 8

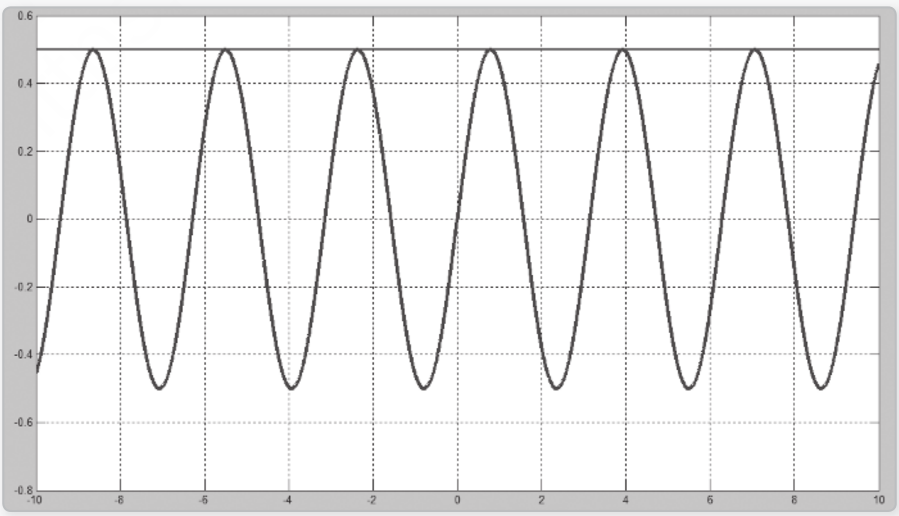
Hallar la ecuación de la recta tangente a la curva $y = \operatorname{sen}x\cos x$ en $x = \frac{\pi}{4}$

- Ordenada en el punto: $x = \frac{\pi}{4} \longrightarrow y = \frac{1}{2}$
- Pendiente de la recta: $y' = \cos^2 x - \operatorname{sen}^2 x \longrightarrow y'\left(\frac{\pi}{4}\right) = 0$

Recta tangente: $y = \frac{1}{2}$

Con MATLAB:

```
>> syms x y
>> A = sin(x)*cos(x);
>> % CALCULO DE LA PENDIENTE:
>> diff(A)
ans =
cos(x)^2 - sin(x)^2
>> m = cos(x)^2 - sin(x)^2;
>> % VALOR DE LA PENDIENTE:
>> x = pi/4;
>> m = cos(x)^2 - sin(x)^2
m =
   2.2204e-16
>> % CALCULO DE LA ORDENADA:
>> x = pi/4;
>> A = sin(x)*cos(x)
A =
    0.5000
>> % ENTONCES LA ECUACION DE LA RECTA:
>> y = 2.2204e-16*(x-pi/4)+0.5;
>> % GRAFICA DE LA FUNCION:
>> x = -10:0.01:10;
>> A = sin(x).*cos(x);
>> plot(x,A)
>> grid on
>> hold on
>> y = 2.2204e-16.*(x-pi./4)+0.5;
>> plot(x,y)
```



PROBLEMA 9

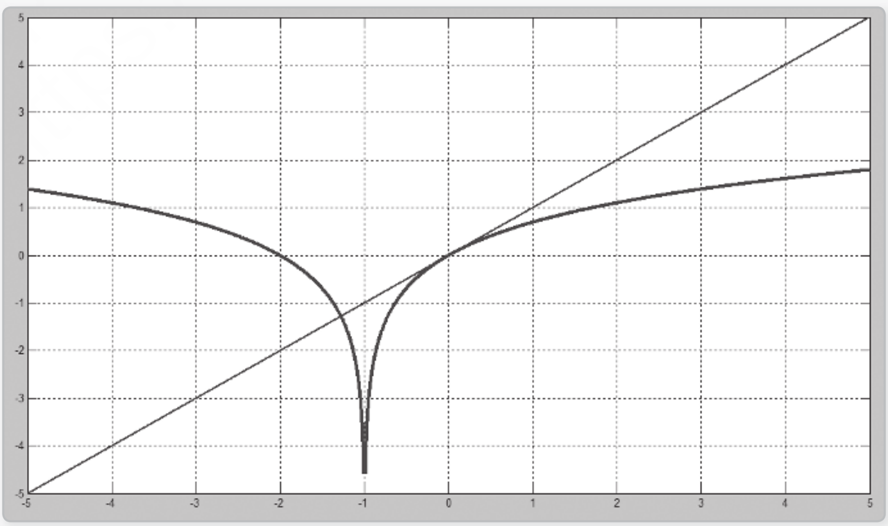
Hallar la ecuación de la recta tangente a la curva $y = \ln(x + 1)$ en $x = 0$

- Ordenada en el punto: $x = 0 \rightarrow y = 0$
- Pendiente de la recta: $y' = \dfrac{1}{x+1} \rightarrow y'(0) = 1$

Recta tangente: $y = x$

Con MATLAB:

```
>> syms x y
>> A = log(x+1);
>> % CALCULO DE LA PENDIENTE:
>> diff(A)
ans =
1/(x + 1)
>> m = 1/(x + 1);
>> % VALOR DE LA PENDIENTE:
>> x = 0;
>> m = 1/(x + 1)
m =
     1
>> % CALCULO DE LA ORDENADA:
>> x = 0;
>> A = log(x+1)
A =
     0
>> % ENTONCES LA ECUACION DE LA RECTA ES:
>> y = 1*(x-0)+0;

>> % GRAFICA DE LA FUNCION:
>> x = -5:0.01:5;
>> A = log(x+1);
>> plot (x,A)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> hold on
>> y = 1.*(x-0)+0;
>> plot(x,y)
```



5.3. MÁXIMOS Y MÍNIMOS

PROBLEMA 1

Hallar los extremos absolutos de la función

$$f(x) = 2x^3 - 3x^2 - 12x + 15$$

en el intervalo [0,3].

Solución:

1. Hallamos los puntos críticos:
   a) Puntos en los que la derivada no está definida: No existen ya que $f'(x) = 6x^2 - 6x - 12$ está definida en todo $\mathbb{R}$.
   b) Puntos en los que la derivada vale cero:

$$6x^2 - 6x - 12 = 0 \rightarrow x^2 - x - 2 = 0 \rightarrow x$$

$$= \frac{1 \pm \sqrt{1+8}}{2} = \frac{1+3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

2. Comparamos los valores de la función en los puntos críticos y en los extremos del intervalo:

$f(0) = 15$ Máximo

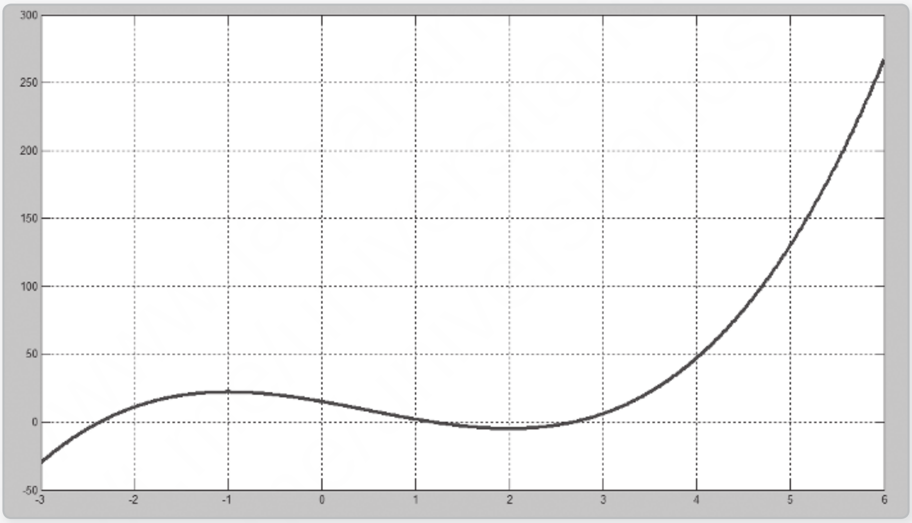
$f(2) = 16 - 12 - 24 + 15 = -5$ Mínimo

$f(3) = 54 - 27 - 36 + 15 = 0$

Con MATLAB:

```
>> syms x y
>> y = 2*x^3 - 3*x^2 - 12*x + 15;
>> % HALLANDO LOS PUNTOS CRITICOS, PARA ELLO SE DERIVA LA FUNCION:
>> diff(y)
ans =
6*x^2 - 6*x - 12
>> % IGUALANDO A CERO EL VALOR DE LA DERIVADA
>> % CREANDO UN POLINOMIO p
>> p = [6 -6 -12];
>> % CALCULO DE LAS RAICES
>> roots(p)
ans =
     2
    -1
>> % CALCULO DEL MAXIMO Y MINIMO DE LA FUNCION
>> % NO SE CONSIDERA -1 PORQUE NO ESTA EN EL INTERVALO SOLICITADO
>> % SE EVALUARA 2 Y LOS EXTREMOS DEL INTERVALO: 0 Y 3
>> X = 0;
>> y = 2*x^3 - 3*x^2 - 12*x + 15
y =
    15
>> x = 2;
```

```
>> y = 2*x^3 - 3*x^2 - 12*x + 15
y =
    -5
>> x = 3;
>> y = 2*x^3 - 3*x^2 - 12*x + 15
y =
     6
>> %OBSERVAMOS QUE EL MAXIMO ES 15 Y EL MINIMO ES -5
>> X = -3:0.01:6;
>> Y = 2.*X.^3 - 3.*X.^2 - 12.*z + 15;
>> plot(x,y)
>> grid on
```



PROBLEMA 2

Hallar los extremos absolutos de la función:

$$f(x) = x^5 - x$$

en el intervalo [2,4].

Solución:

1. Hallamos los puntos críticos:
   a) Puntos en los que la derivada no está definida: No existen ya que $f'(x) = 5x^4 - 1$ está definida en todo $\mathbb{R}$.
   b) Puntos en los que la derivada vale cero:

$$5x^4 - 1 = 0 \longrightarrow x^4 = \frac{1}{5} \longrightarrow x = \pm\sqrt{\frac{1}{5}} \notin [2,4]$$

   Luego no existe ningún punto crítico dentro del intervalo, por tanto:

2. Comparamos los valores de la función en los extremos del intervalo:

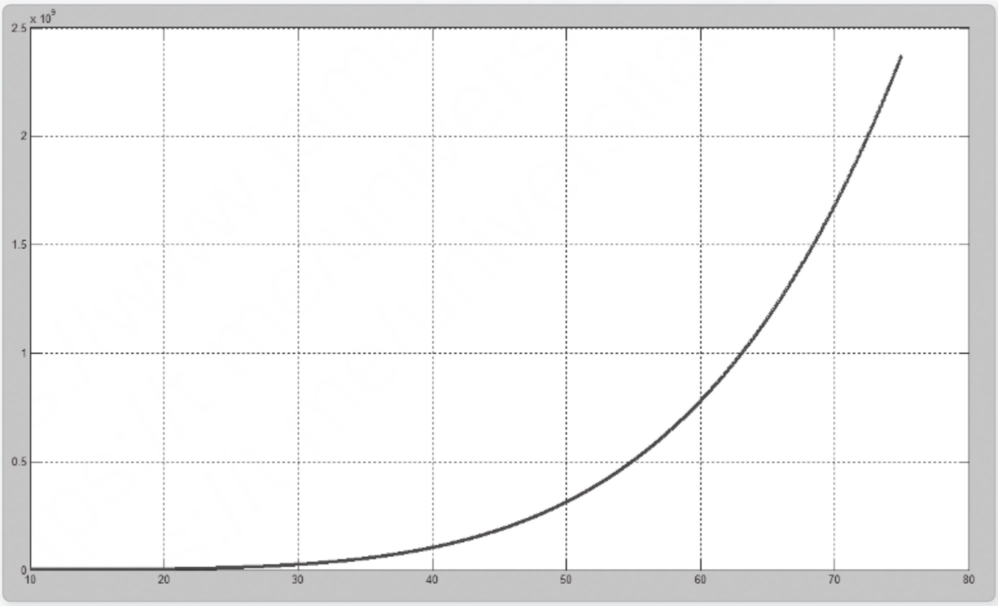
$f(2) = 30$ Mínimo

$f(4) = 1020$ Máximo

Con MATLAB:

```
>> syms x y
>> y = x^5 - x;
>> % HALLANDO LOS PUNTOS CRITICOS, PARA ELLO SE DERIVA LA FUNCION:
>> diff(y)

ans =

5*x^4 - 1

>> % IGUALANDO A CERO EL VALOR DE LA DERIVADA:
>> % CREANDO UN POLINOMIO p:
>> p = [5 0 0 0 -1];
>> % CALCULO DE LAS RAICES:
>> roots(p)

ans =

  -0.6687
   0.0000 + 0.6687i
   0.0000 - 0.6687i
   0.6687

>> % CALCULO DEL MAXIMO Y MINIMO DE LA FUNCION:
>> % NO SE CONSIDERA NINGUNA RAIZ PORQUE NO ESTAN EN EL INTERVALO SOLICITADO
>> % SE EVALUARA LOS EXTREMOS DEL INTERVALO: 2 Y 4
>> X = 2;
>> y = x^5 - x
```

```
y =

    30

>> x = 4;
>> y = x^5 – x

y =

        1020

>> % OBSERVAMOS QUE EL MAXIMO ES 1020 Y EL MINIMO ES 30
>> x = 10:0.01:75;
>> y = x.^5 0 x;
>> plot(x,y)
>> grid on
```



PROBLEMA 3

Hallar los extremos absolutos de la función:

$$f(x) = 3 - |x - 2|$$

en el intervalo [1,4].

Solución:

Para hallar la derivada de la función eliminamos el valor absoluto.

$$f(x) = 3 - |x-2| = \begin{cases} 3-(x-2) & \text{si } x \leq 2 \\ 3-(-x+1) & \text{si } x < 2 \end{cases} = \begin{cases} 5-x & \text{si } x \geq 2 \\ 1+x & \text{si } x < 2 \end{cases}$$

Con lo cual, la función derivada es:

$$f'(x) = \begin{cases} -1 & \text{si } x > 2 \\ 1 & \text{si } x < 2 \end{cases}$$

1. Hallamos los puntos críticos:
   a) Puntos en los que la derivada no está definida: x = 2
   b) Puntos en los que la derivada vale cero: No existen
2. Comparamos los valores de la función en los puntos críticos y en los extremos del intervalo:

f(1) = 2

f(2) = 3 Máximo

f(4) = 1 Mínimo

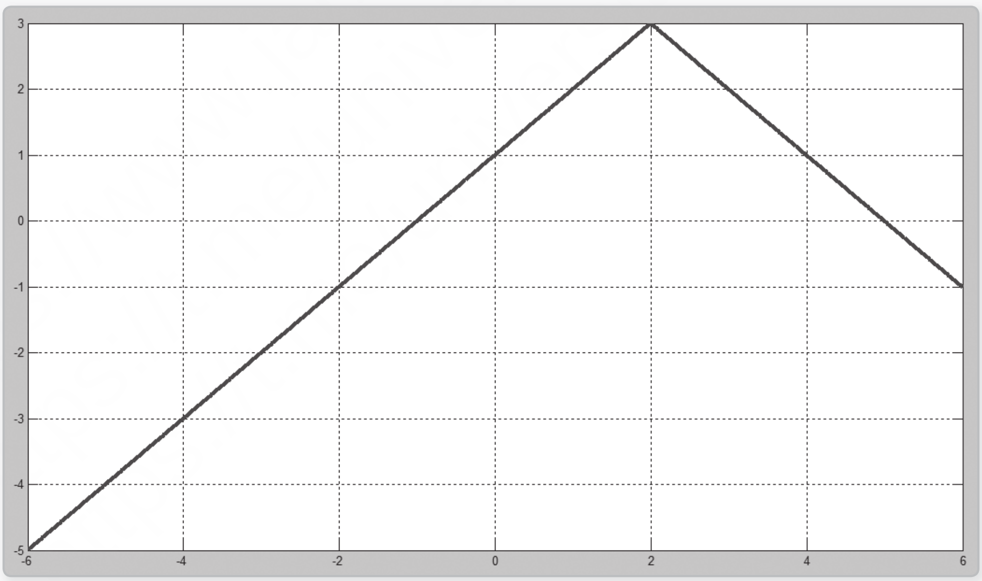
Máximos y mínimos absolutos en intervalos abiertos

Para hallar el máximo y el mínimo de una función continua en un intervalo abierto se "cierra" el intervlo hallando los límites de la función en los extremos del mismo.

Con MATLAB:

```
>> syms x y
>> y1 = 3 - (x - 2);
>> y2 = 3 - (-x + 1);
>> % HALLANDO LOS PUNTOS CRITICOS, PARA ELLO SE DERIVA LA FUNCION:
>> diff(y)
ans =
-1
>> diff(y2)
ans =
1
>> % SE OBSERVA QUE LA DERIVADA NO ESTA DEFINIDA EN x = 2:
>> % NO EXISTEN PUNTOS EN DONDE LA DERIVADA TENGA VALOR 0.
>> % ANALIZANDO LOS PUNTOS CRITICOS Y LOS EXTREMOS DEL INTERVALO:
>> x = 1;
```

```
>> y = 3 - abs(x - 2)
y =
     2
>> x = 2;
>> y = 3 - abs(x - 2)
y =
     3
>> x = 4;
>> y = 3 - abs(x - 2)
y =
     1
>> % ENTONCES EL MINIMO ES 1 Y EL MAXIMO ES 4.
>> x = -6:0.01:6;
>> y = 3 - abs(x - 2);
>> plot(x,y)
>> grid on
```



Halla los máximos, los mínimos y los puntos de inflexión de las siguientes funciones:

PROBLEMA 1

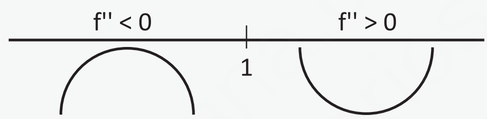
$$y = x^3 - 3x^2 + 9x + 22$$

$$f'(x) = 3x^2 - 6x + 9$$

$$f'(x) = 0 \rightarrow 3x^2 - 6x + 9 = 0 \rightarrow \text{No tiene solución}$$

No tiene ni máxios ni mínimos

$$f''(x) = 6x - 6 = 0 \quad x = 1$$

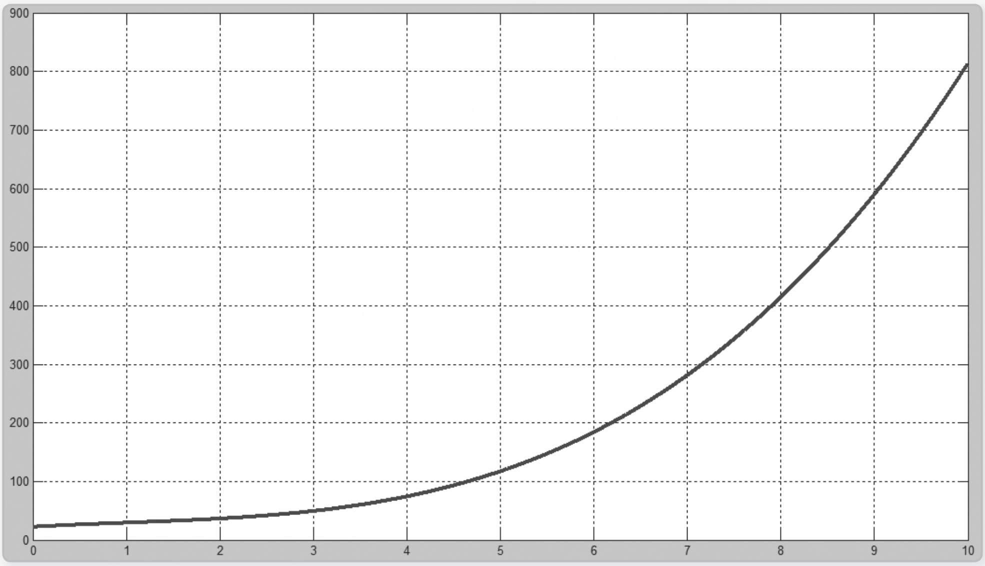


Hay un punto de inflexión en (1, 29).

Con MATLAB:

```
>> syms x y
>> y = x^3 - 3*x^2 + 9*x +22;
>> % HALLANDO LOS PUNTOS CRITICOS, PARA ELLO SE DERIVA LA FUNCION:
>> diff(y)
ans =
3*x^2 - 6*x + 9
>> % HALLANDO LAS RAICES, PARA ELLO CREAMOS UN POLINOMIO p:
>> p = [3 -6 9]
p =
     3    -6     9
>> roots(p)
ans =
   1.0000 + 1.4142i
   1.0000 - 1.4142i
>> % NO TIENE SOLUCIONES REALES, POR LO TANTO NO TIENE MAXIMOS NI MINIMOS.
>> % DERIVAMOS POR SEGUNDA VEZ.
>> diff(y,2)
ans =
6*x - 6
>> % HALLANDO SUS RAICES:
>> p = [6 -6]
p =
     6    -6
>> roots(p)
ans =
     1
```

```
>> % HALLANDO LA ORDENADA:
>> x = 1;
>> y = x^3 - 3*x^2 + 9*x +22
y =
    29
>> % POR LO TANTO HAY UN PUNTO DE INFLEXION EN (1,29).
>> x = 0:0.01:10;
>> y = x.^3 - 3.*x.^2 + 9.*x +22;
>> plot(x,y)
```

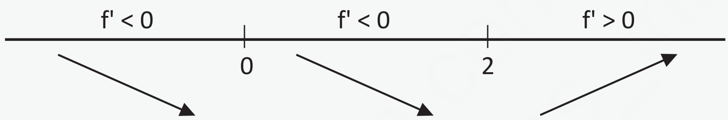


PROBLEMA 2

$$y = \frac{3x^4 - 8x^3}{12}$$

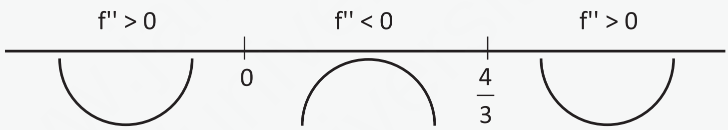
$$f'(x) = \frac{12x^3 - 24x^2}{12} = x^3 - 2x^2$$

$$f'(x) = 0 \rightarrow x^2(x-2) = 0 \begin{cases} x = 0 \rightarrow y = 0 \\ x = 2 \rightarrow y = -4/3 \end{cases}$$



Hay un mínimo en $\left(2, \frac{-4}{3}\right)$.

$$f''(x) = 3x^2 - 4x = 0 \rightarrow x(3x-4) = 0 \begin{cases} x = 0 \rightarrow y = 0 \\ x = 4/3 \rightarrow y = -64/81 \end{cases}$$

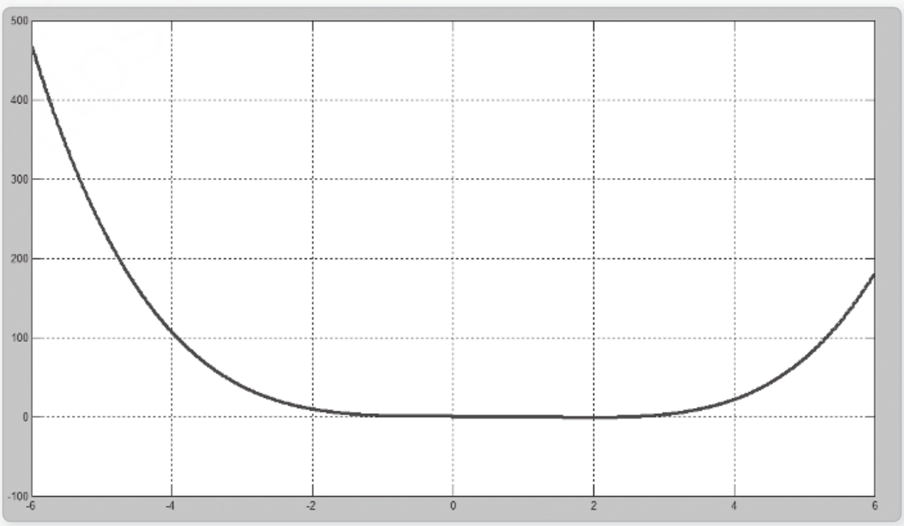


Hay un punto de inflexión en (0,0) y otro en $\left(\frac{4}{3}, \frac{-64}{81}\right)$

Con MATLAB:

```
>> syms x y
>> y = (3*x^4 - 8*x^3)/12;
>> % HALLANDO LOS PUNTOS CRITICOS, PARA ESTO SE DERIVA LA FUNCION:
>> diff(y)
ans =
X^3 - 2*X^2
>> % HALLANDO LAS RAICES, PARA ELLO CREAMOS UN POLINOMIO p:
>> p = [1 -2 0 0]
p =
     1    -2     0     0
>> roots(p)
ans =
     0
     0
     2
>> % HALLANDO LAS ORDENADAS:
>> x = 0;
>> y = (3*x^4 - 8*x^3)/12
y =
     0
>> x = 2;
>> y = (3*x^4 - 8*x^3)/12
y =
   -1.3333
```

```
>> format rar
>> y = (3*x^4 - 8*x^3)/12
y =
      -4/3
>> % ENTONCES HAY UN MINIMO EN (2,-4/3).
>> syms x y
>> y = (3*x^4 - 8*x^3)/12;
>> diff(y,2)
ans =
3*x^2 - 4*x
>> % HALLANDO LAS RAICES:
>> p = [3 -4 0]
p =
          3.00         -4.00             0
>> roots(p)
ans =
             0
          1.33
>> format rat
>> p = [3 -4 0]
p =
       3             -4              0
>> roots(p)
ans =
       0
       4/3
>> % CALCULO DE LAS ORDENADAS:
>> x = 0;
>> z = 3*x^2 - 4*x
z =
       0
>> x = 4/3;
>> z = 3*x^2 - 4*x
z =
       0
>> % GRAFICANDO:
>> x = -6:0.01:6;
>> y = (3.*x.^4 - 8.*x.^3)./12;
>> plot(x,y)
>> grid on
```

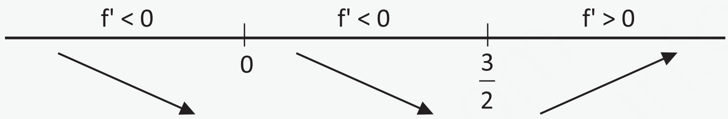


PROBLEMA 3

$$y = x^4 - 2x^3$$

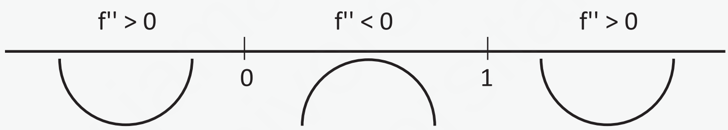
$$f'(x) = 4x^3 - 6x^2$$

$$f'(x) = 0 \rightarrow x^2(4x-6) = 0 \begin{cases} x = 0 \rightarrow y = 0 \\ x = 3/2 \rightarrow y = -27/16 \end{cases}$$



Hay un mínimo en $\left(\frac{3}{2}, \frac{-27}{16}\right)$.

$$f''(x) = 12x^2 - 12x = 12x(x-1) = 0 \begin{cases} x = 0 \rightarrow y = 0 \\ x = 1 \rightarrow y = -1 \end{cases}$$

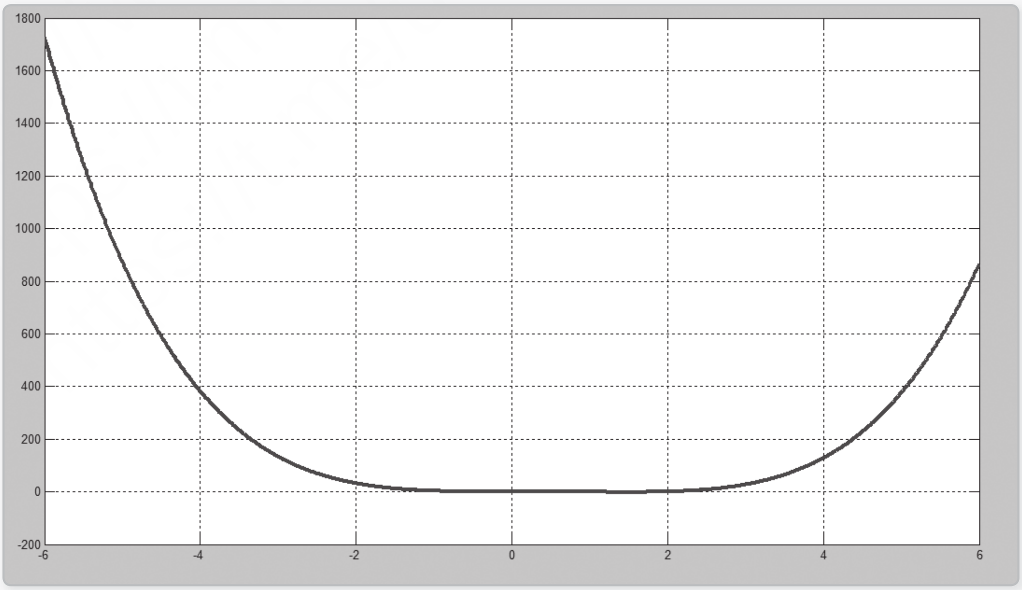


Hay un punto de inflexión en (0,0) y otro en (1,–1).

Con MATLAB:

```
>> syms x y
>> y = x^4 - 2*x^3;
>> % HALLANDO LOS PUNTOS CRITICOS, PARA ESTO SE DERIVA LA FUNCION:
>> diff(y)
ans =
4*x^3 - 6*x^2
>> % HALLANDO LAS RAICES, PARA ELLO CREAMOS UN POLINOMIO p:
>> p = [4 -6 0 0]
p =
       4            -6              0              0
>> roots(p)
ans =
       0
       0
       3/2
>> % HALLANDO LAS ORDENADAS:
>> x = 0;
>> y = x^4 - 2*x^3
y =
       0
>> x = 3/2;
>> y = x^4 - 2*x^3
y =
     -27/16
```

```
>> % ENTONCES HAY UN MINIMO EN (3/2,-27/16).
>> syms x y
>> y = x^4 - 2*x^3;
>> diff(y,2)
ans =
12*x^2 - 12*x
>> % HALLANDO LAS RAICES:
>> p = [12 -12 0]
p =
     12           -12             0
>> roots(p)
ans =
      0
      1
>> % CALCULO DE LAS ORDENADAS:
>> x = 0;
>> z = 12*x^2 - 12*x
z =
      0
>> x = 1;
>> z = 12*x^2 - 12*x
z =
      0
>> % GRAFICANDO:
>> x = -6:0.01:6;
>> y = x.^4 - 2.*x.^3;
>> plot(x,y)
>> grid on
```

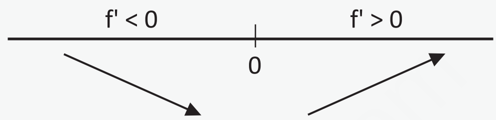


PROBLEMA 4

$$y = x^4 + 2x^2$$

$$f'(x) = 4x^3 + 4x$$

$$f'(x) = 0 \longrightarrow 4x(x^2 + 1) = 0 \longrightarrow x = 0 \longrightarrow y = 0$$



Hay un mínimo en (0,0).

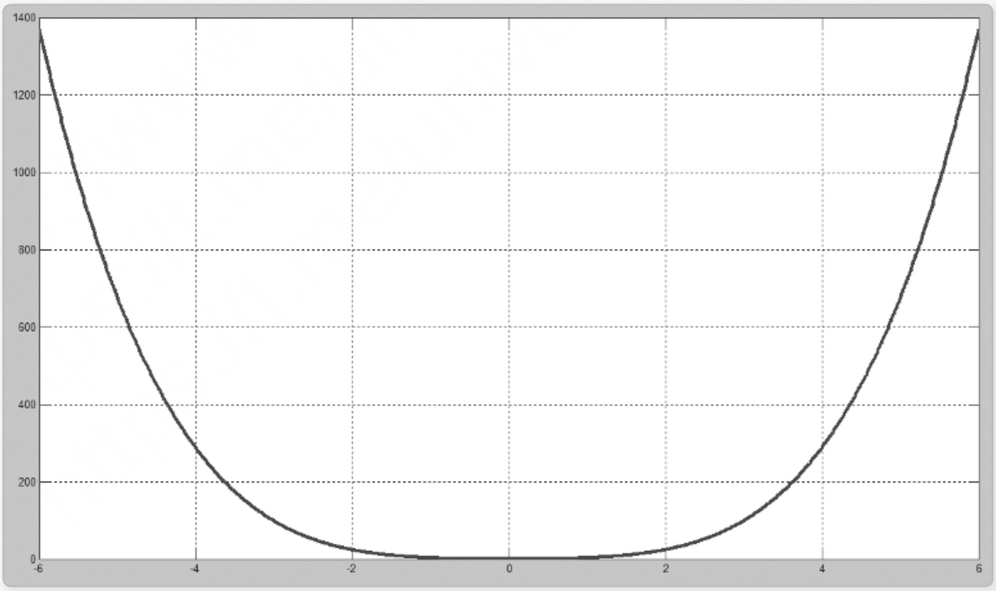
$$f''(x) = 12x^2 + 4 \neq 0 \text{ para todo x.}$$

No hay punto de inflexión.

Con MATLAB:

```
>> syms x y
>> y = x^4 + 2*x^2;
>> % HALLANDO LOS PUNTOS CRITICOS, PARA ESTO SE DERIVA LA FUNCION:
>> diff(y)

ans =

4*x^3 + 4*x

>> % HALLANDO LAS RAICES, PARA ELLO CREAMOS UN POLINOMIO:
>> p = [4 0 4 0]

p =

     4     0     4     0

>> roots(p)

ans =

        0
        0 + 1.0000i
        0 - 1.0000i

>> % HALLANDO LAS ORDENADAS:
>> x = 0;
>> y = x^4 + 2*x^2

y =

     0

>> % ENTONCES HAY UN MINIMO EN (0,0).
>> % CALCULANDO LA SEGUNDA DERIVADA
>> syms x y
>> y = x^4 + 2*x^2;
>> diff(y,2)
```

```
ans =

12*x^2 +4

>> % HALLANDO LAS RAICES:
>> p = [12 0 4]

p =

    12     0     4

>> roots(p)

ans =

        0 + 0.5774i
        0 - 0.5774i

>> % POR LO TANTO NO EXISTEN PUNTOW DE INFLEXION:
>> % GRAFICANDO:
>> x = -6:0.01:6;
>> y = x.^4 - 2.*x.^2;
>> plot(x,y)
>> grid on
```



PROBLEMA 5

El movimiento de una partícula está definido por la ecuación:

$$x = 2t^3 - 6t^2 + 28t - 10$$

Donde x se expresa en metros y t en segundos.

a) Calcular la posición, velocidad y aceleración cuando t = 10s.

b) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

• Cálculo de la posición, velocidad y aceleración para t = 10s.

Con MATLAB:

```
>> syms x t y
>> % LA ECUACION DE LA POSICION ES:
>> x = 2*t^3 - 6*t^2 + 28*t - 10;
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
6*t^2 - 12*t + 28
>> v = 6*t^2 - 12*t + 28;
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
12*t - 12
>> a = 12*t - 12;
>> % PARA t = 10 segundos SE TIENE:
>> t = 10;
>> x = 2*t^3 - 6*t^2 + 28*t - 10
x =
        1670
>> t = 10;
>> v = 6*t^2 - 12*t + 28
v =
   508
>> t = 10;
>> a = 12*t - 12
a =
   108
```

Se observa que los valores de la posición, velocidad y aceleración a los 10s son:

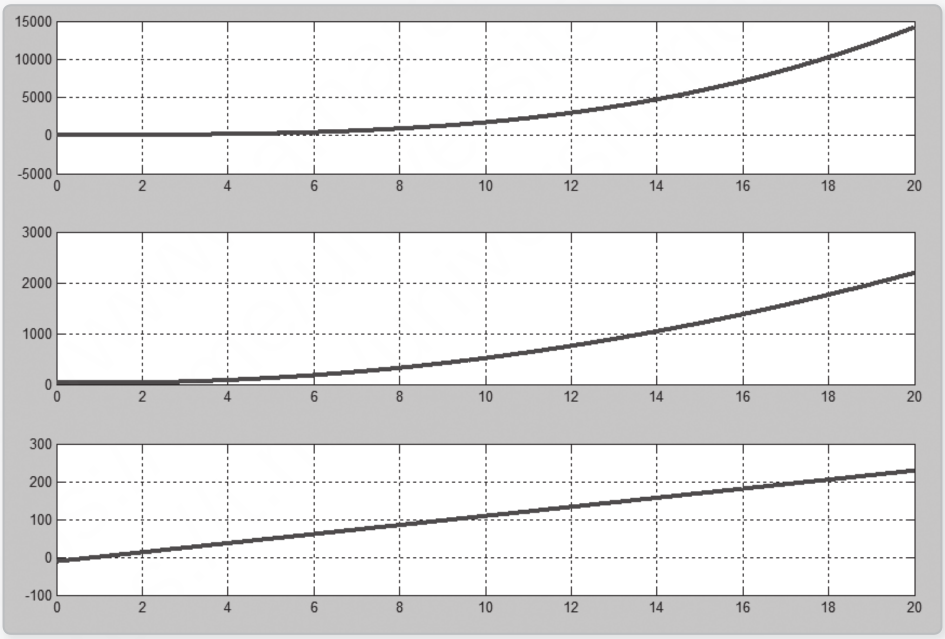
$$x = 1670\,m$$

$$v = 508\frac{m}{s}$$

$$a = 108\frac{m}{s^2}$$

- Gráficas de la posición, velocidad y aceleración.

```
>> t = 0:0.01:20;
>> x = 2.*t.^3 – 6.*t.^2 + 28.*t – 10;
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = 6.*t.^2 – 12.*t +28;
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> v = 12.*t – 12;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 6

El movimiento de una partícula está definido por la ecuación:

$$x = t^3 - 10t^2 + 20t - 16$$

Donde x se expresa en metros y t en segundos.

a) Calcular la posición, velocidad y aceleración cuando t = 12s.

b) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

• Cálculo de la posición, velocidad y aceleración para t = 12s.

Con MATLAB:

```
>> syms x t y
>> % LA ECUACION DE LA POSICION ES:
>> x = t^3 – 10*t^2 + 20*t –16;
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
3*t^2 – 20*t + 20
>> v = 3*t^2 – 20*t + 20;
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
6*t – 20
>> a = 6*t – 20;
>> % PARA t = 12 segundos SE TIENE:
>> t = 12;
>> x = t^3 – 10*t^2 + 20*t –16
x =
   512
>> t = 12;
>> v = 3*t^2 – 20*t + 20
v =
   212
>> t = 12;
>> a = 6*t – 20
a =
    52
```

Se observa que los valores de la posición, velocidad y aceleración a los 10s son:

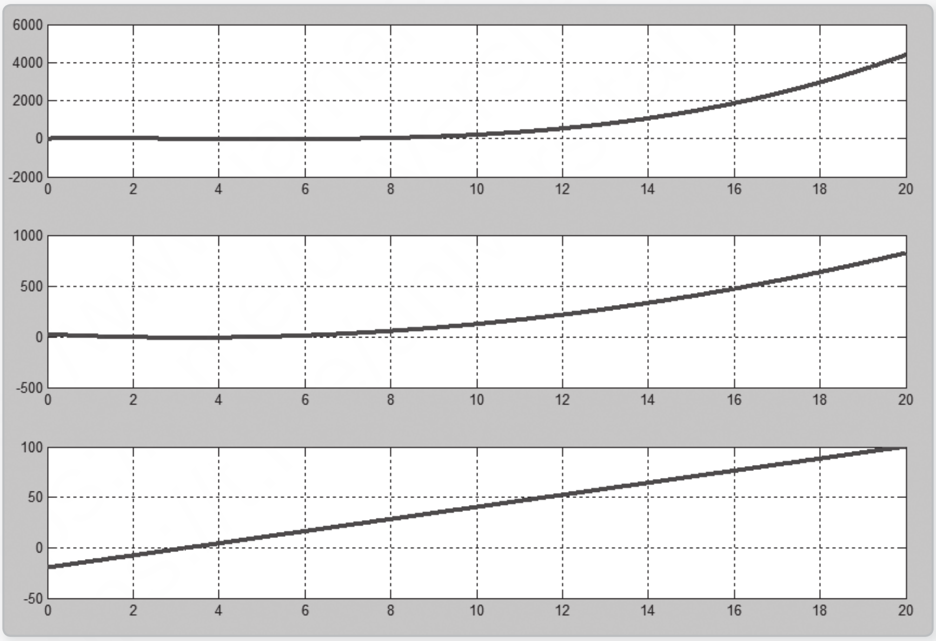
$$x = 512\,m$$

$$v = 212\,\frac{m}{s}$$

$$a = 52\,\frac{m}{s^2}$$

- Gráficas de la posición, velocidad y aceleración.

```
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:20;
>> x = t.^3 - 10.*t.^2 + 20.*t -16;
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = 3.*t.^2 - 20.*t + 20;
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = 6.*t - 20;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 7

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = 2\text{sen}\left(0.40t + \frac{\pi}{6}\right)$$

a) Determinar la elongación, velocidad y aceleración cuando t = 18s.
b) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

• Cálculo de la posición, velocidad y aceleración para t = 18s.

Con MATLAB:

```
>> syms x t y
>> % LA ECUACION DE LA POSICION ES:
>> x = 2*sin(0.4*t+pi/6);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
(4*cos(pi/6 + (2*t)/5))/5
>> v = (4*cos(pi/6 + (2*t)/5))/5;
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
-(8*sin(pi/6 + (2*t)/5))/25
>> a = -(8*sin(pi/6 + (2*t)/5))/25;
>> % PARA t = 18 segundos  SE TIENE:
>> t = 18;
>> x = 2*sin(0.4*t+pi/6)
x =
    1.9830
>> t = 18;
>> v = (4*cos(pi/6 + (2*t)/5))/5
v =
    0.1040
>> t = 18;
>> a = -(8*sin(pi/6 + (2*t)/5))/25
a =
   -0.3173
```

Se observa que los valores de la posición, velocidad y aceleración a los 10s son:

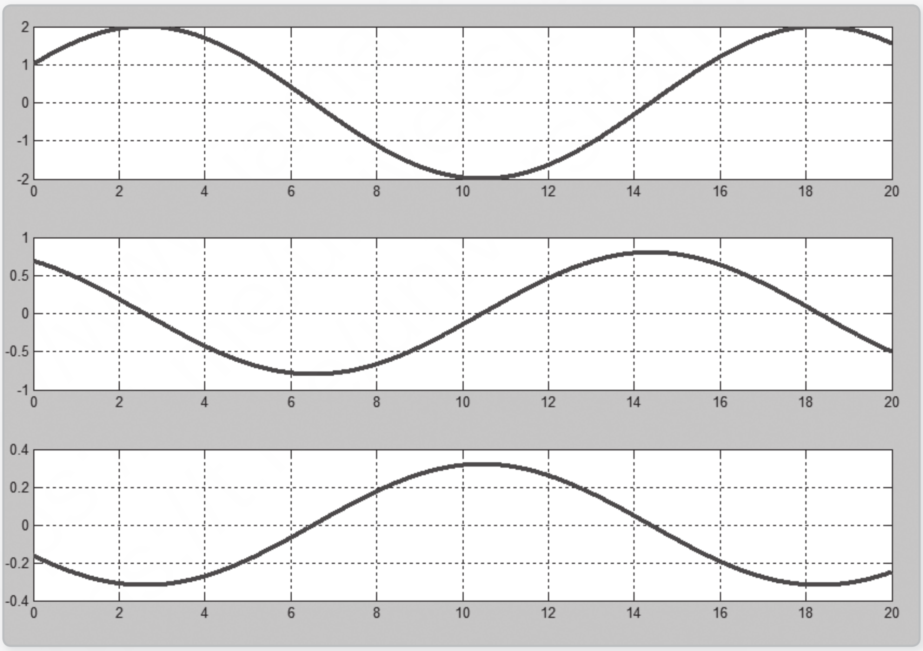
$$x = 1.9830\text{m}$$

$$v = 0.1040\frac{\text{m}}{\text{s}}$$

$$a = -0.3173\frac{\text{m}}{\text{s}^2}$$

- Gráficas de la posición, velocidad y aceleración.

```
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:20;
>> x = 2.*sin(0.4.*t+pi./6);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = (4.*cos(pi./6 + (2.*t)./5))./5;
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = -(8.*sin(pi./6 + (2.*t)./5))./25;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 8

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = 2\cos\left(0{,}20t + \frac{\pi}{4}\right)$$

a) Determinar la elongación, velocidad y aceleración cuando t = 15s.
b) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

• Cálculo de la posición, velocidad y aceleración para t = 15s.

Con MATLAB:

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = 2*cos(0.20*t+pi/4);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
-(2*sin(pi/4 + t/5))/5
>> v = -(2*sin(pi/4 + t/5))/5;
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
-(2*cos(pi/4 + t/5))/25
>> a = -(2*cos(pi/4 + t/5))/25
>> % PARA t = 15 segundos SE TIENE:
>> t = 15;
>> x = 2*cos(0.20*t+pi/4)
x =
   -1.5996
>> t = 15;
>> v = -(2*sin(pi/4 + t/5))/5
v =
    0.2401
>> t = 15;
>> a = -(2*cos(pi/4 + t/5))/25
a =
    0.0640
```

Se observa que los valores de la posición, velocidad y aceleración a los 15s son:

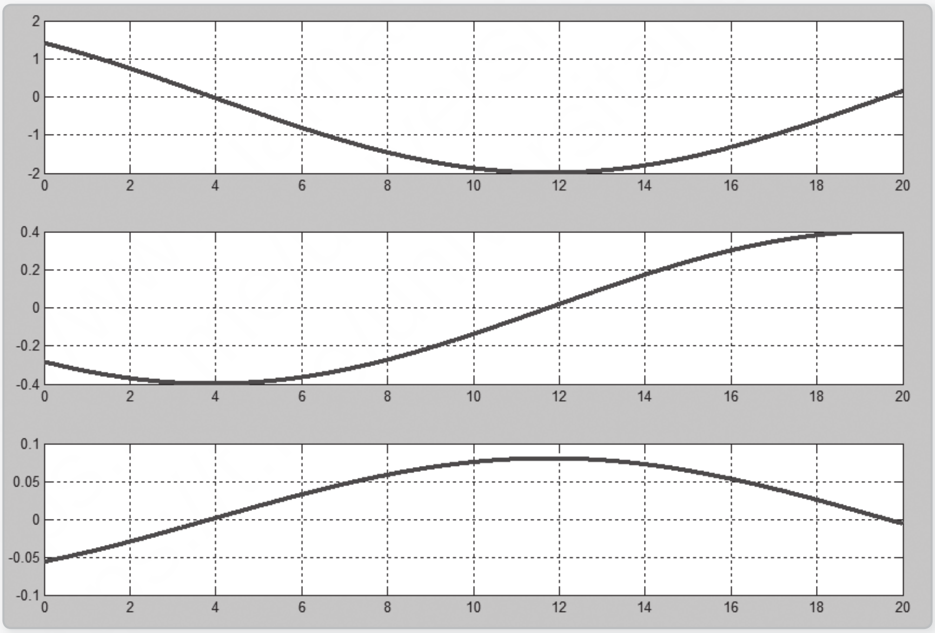
$$x = -1.5996\,m$$

$$v = 0.2401\frac{m}{s}$$

$$a = 0.0640\frac{m}{s^2}$$

- Gráficas de la posición, velocidad y aceleración.

```
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:20;
>> x = 2.*cos(0.20.*t+pi./4);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = -(2.*sin(pi./4 + t./5))./5;
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = -(2.*oos(pi./4 + t./5))./25;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 9

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = e^{2t}$$

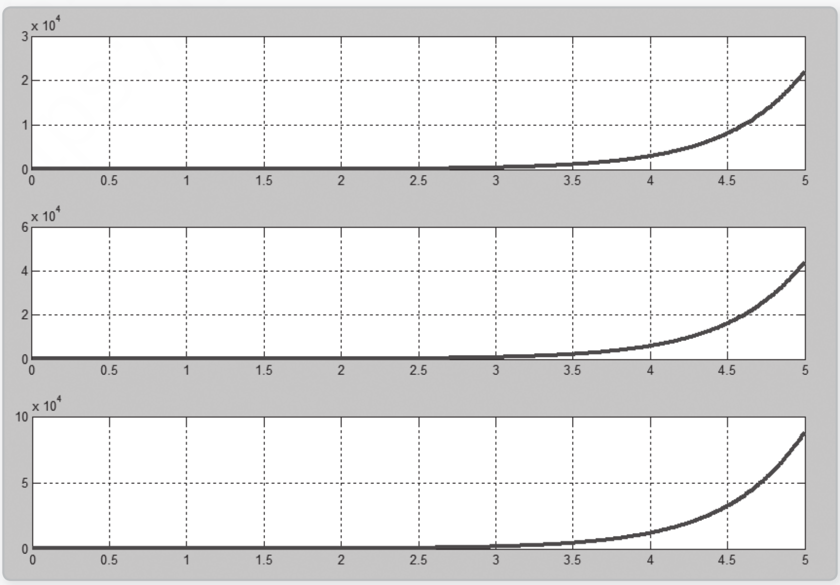
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = exp(2*t);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
2*exp(2*t)
>> v = 2*exp(2*t);
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
4*exp(2*t)
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = exp(2.*t);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = 2.*exp(2.*t);
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = 4.*exp(2.*t);
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 10

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = \ln(2t)$$

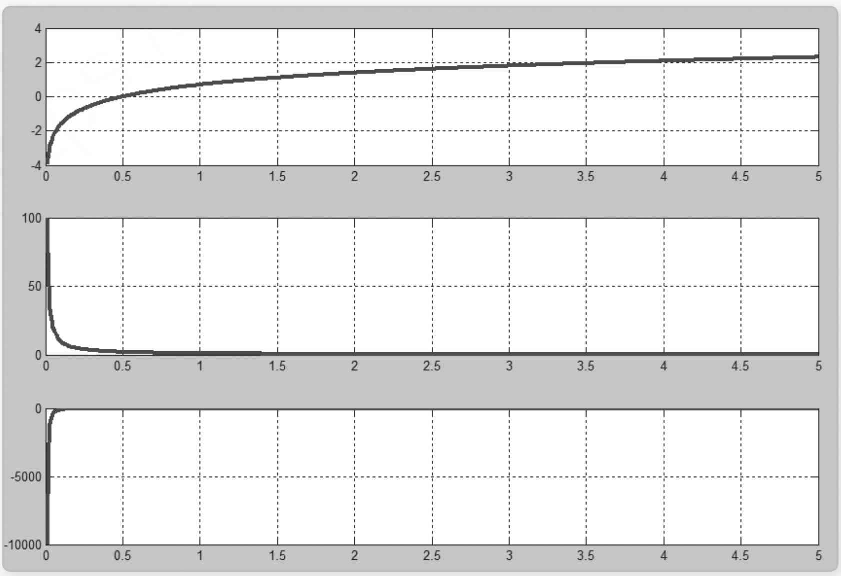
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = log(2*t);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
1/t
>> v = 1/t;
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
-1/t^2
>> a = -1/t^2;
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = log(2.*t);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = 1./t;
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = -1./t.^2;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 11

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = \frac{1}{\ln\left(t^2\right)}$$

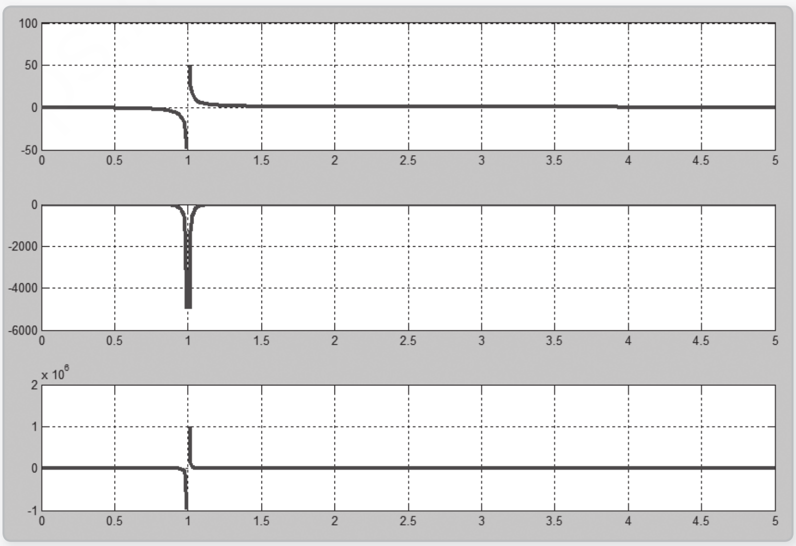
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = 1/(log(t^2));
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
-2/(t*log(t^2)^2)
>> v = -2/(t*log(t^2)^2);
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
2/(t^2*log(t^2)^2) + 8/(t^2*log(t^2)^3)
>> a = 2/(t^2*log(t^2)^2) + 8/(t^2*log(t^2)^3);
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = 1./(log(t.^2));
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = -2./(t.*log(t.^2).^2);
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = 2./(t.^2.*log(t.^2).^2) + 8./(t.^2.*log(t.^2).^3);
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 12

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = e^{0,2t}\ \mathrm{sen}\left(2t - \frac{\pi}{4}\right)$$

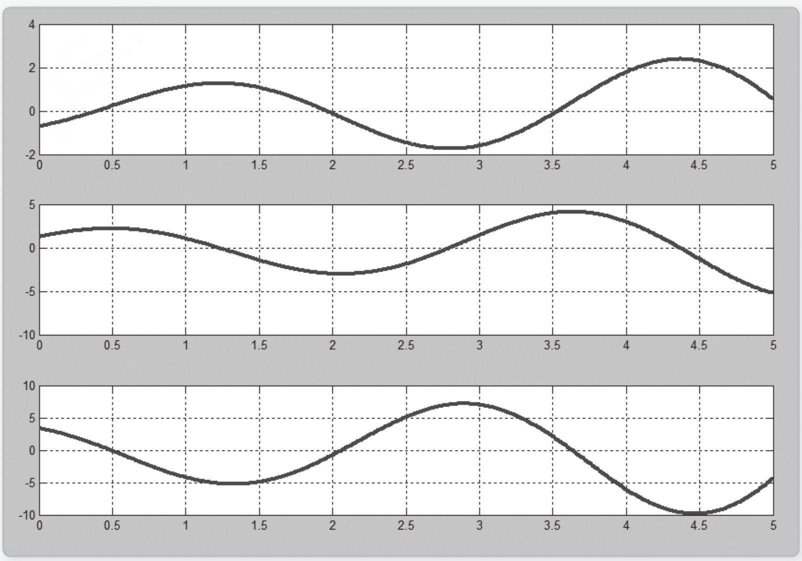
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = exp(0.2*t)*sin(2*t-pi/4);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
2*cos(2*t - pi/4)*exp(t/5) + (sin(2*t - pi/4)*exp(t/5))/5
>> v = 2*cos(2*t - pi/4)*exp(t/5) + (sin(2*t - pi/4)*exp(t/5))/5;
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
(4*cos(2*t - pi/4)*exp(t/5))/5 - (99*sin(2*t - pi/4)*exp(t/5))/25
>> a = (4*cos(2*t - pi/4)*exp(t/5))/5 - (99*sin(2*t - pi/4)*exp(t/5))/25;
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = exp(0.2.*t).*sin(2.*t-pi./4);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = 2.*cos(2.*t - pi./4).*exp(t./5) + (sin(2.*t - pi./4).*exp(t./5))./5;
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = (4.*cos(2.*t - pi./4).*exp(t./5))./5 - (99.*sin(2.*t - pi./4).*exp(t./5))./25;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 13

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = e^{-0,4t}\cos\left(5t + \frac{\pi}{3}\right)$$

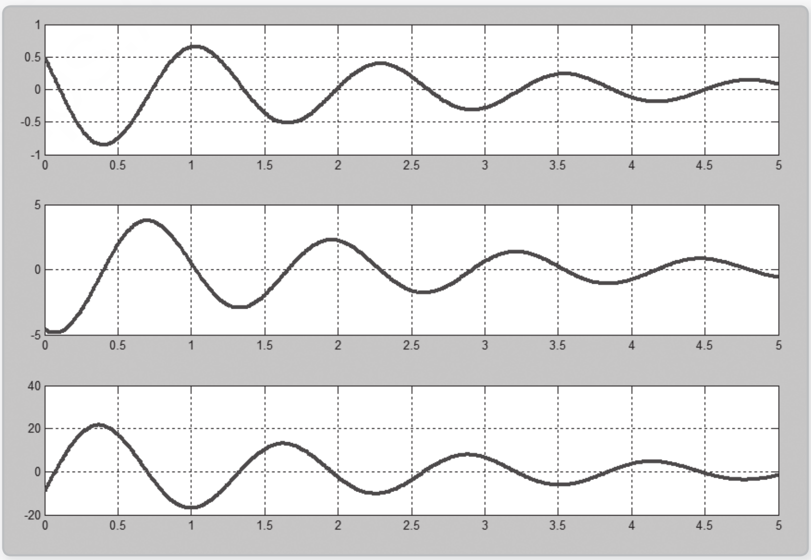
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = exp(-0.4*t)*cos(5*t+pi/3);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
- (2*cos(pi/3 + 5*t)*exp(-(2*t)/5))/5 - 5*sin(pi/3 + 5*t)*exp(-(2*t)/5)
>> v = - (2*cos(pi/3 + 5*t)*exp(-(2*t)/5))/5 - 5*sin(pi/3 + 5*t)*exp(-(2*t)/5);
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
4*sin(pi/3 + 5*t)*exp(-(2*t)/5) - (621*cos(pi/3 + 5*t)*exp(-(2*t)/5))/25
>> a = 4*sin(pi/3 + 5*t)*exp(-(2*t)/5) - (621*cos(pi/3 + 5*t)*exp(-(2*t)/5))/25;
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = exp(-0.4.*t).*cos(5.*t+pi./3);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = -(2.*cos(pi./3 + 5.*t).*exp(-(2.*t)./5))./5 - 5.*sin(pi./3 + 5.*t).*exp(-(2.*t)./5);
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = 4.*sin(pi./3 + 5.*t).*exp(-(2.*t)./5) - (621.*cos(pi./3 + 5.*t).*exp(-(2.*t)./5))./25;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



Problema 14

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = 4\operatorname{sen}(2t) - 3\cos(4t)$$

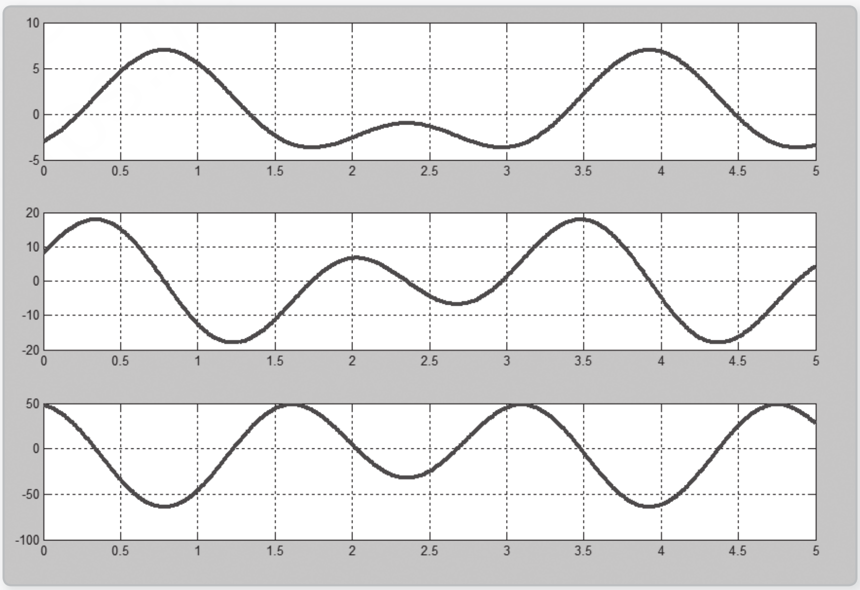
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = 4*sin(2*t)-3*cos(4*t);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
8*cos(2*t) + 12*sin(4*t)
>> v = 8*cos(2*t) + 12*sin(4*t);
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
48*cos(4*t) - 16*sin(2*t)
>> a = 48*cos(4*t) - 16*sin(2*t);
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = 4.*sin(2.*t)-3.*cos(4.*t);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = 8.*cos(2.*t) + 12.*sin(4.*t);
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = 48.*cos(4.*t) - 16.*sin(2.*t);
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 15

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = 5\ \text{sen}\left(2t + \frac{\pi}{3}\right)$$

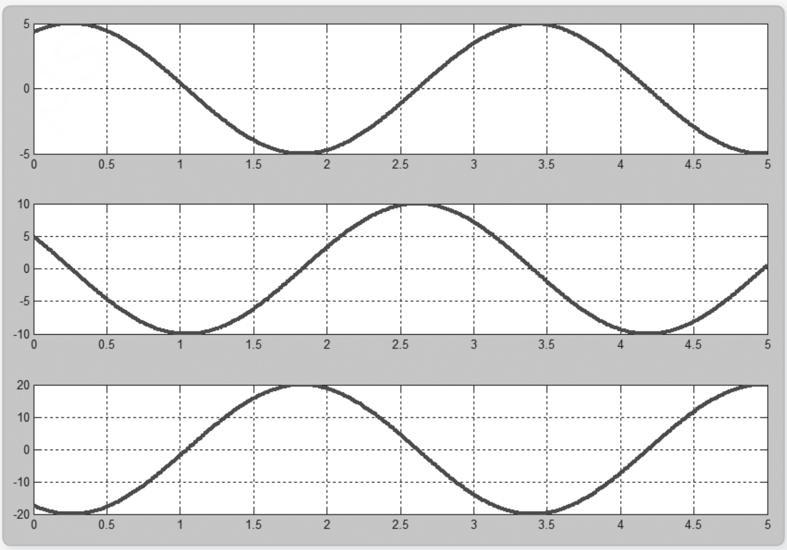
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = 5*sin(2*t+pi/3);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
10*cos(pi/3 + 2*t)
>> v = 10*cos(pi/3 + 2*t);
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
-20*sin(pi/3 + 2*t)
>> a = -20*sin(pi/3 + 2*t);
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = 5.*sin(2.*t+pi./3);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = 10.*cos(pi./3 + 2.*t);
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = -20.*sin(pi./3 + 2.*t);
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



Problema 16

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = 4^t$$

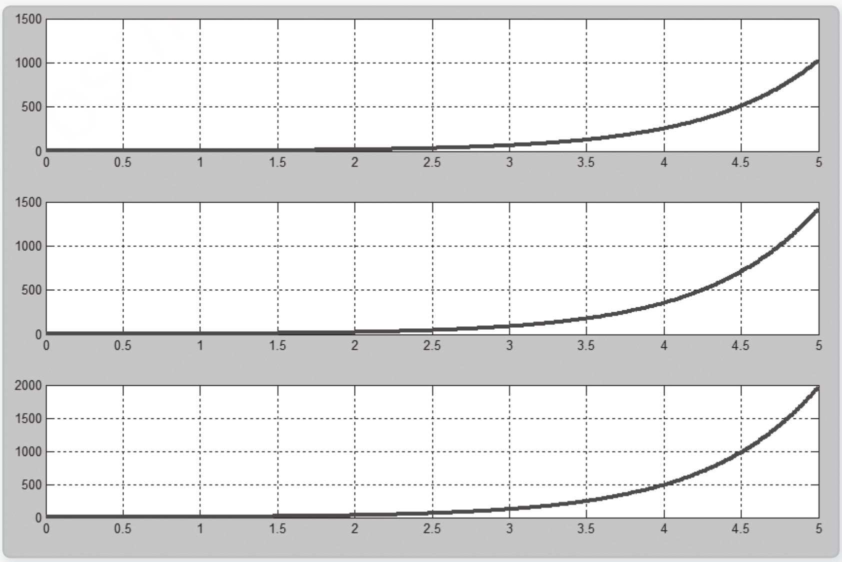
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = 4^t;
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
4^t*log(4)
>> v = 4^t*log(4);
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
(6243314768165359*4^t*log(4))/4503599627370496
>> a = (6243314768165359*4^t*log(4))/4503599627370496;
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = 4.^t;
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = 4.^t.*log(4);
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = (6243314768165359.*4.^t.*log(4))./4503599627370496;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



Problema 17

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = 5\arcsin(5t)$$

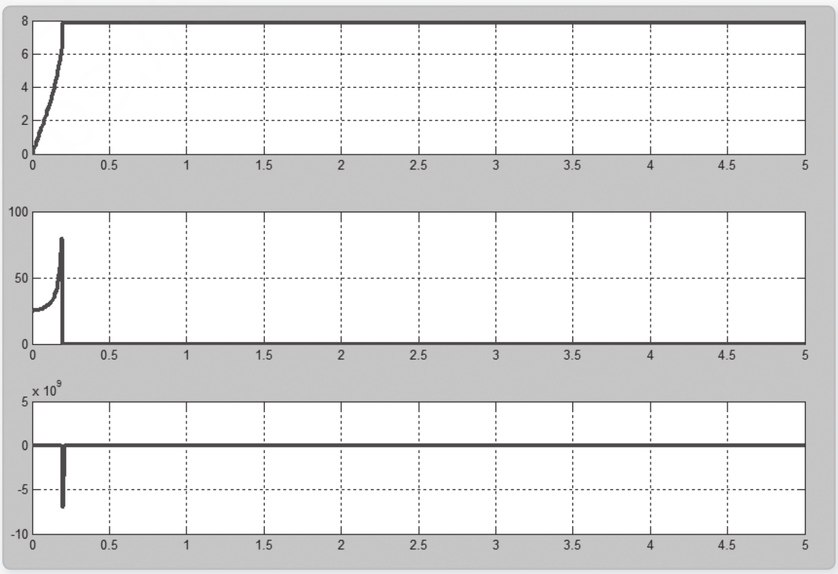
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = 5*asin(5*t);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
25/(1 - 25*t^2)^(1/2)
>> v = 25/(1 - 25*t^2)^(1/2);
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
(625*t)/(1 - 25*t^2)^(3/2)
>> a = (625*t)/(1 - 25*t^2)^(3/2);
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = 0:0.01:5;
>> x = 5.*asin(5.*t);
>> subplot(3,1,1)
>> plot(t,x)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> v = 25./(1 - 25.*t.^2).^(1./2);
>> subplot(3,1,2)
>> plot(t,v)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> a = (625.*t)./(1 - 25.*t.^2).^(3./2);
>> subplot(3,1,3)
>> plot(t,a)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```



PROBLEMA 18

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = 6\ln\left(\sqrt{3t}\right)$$

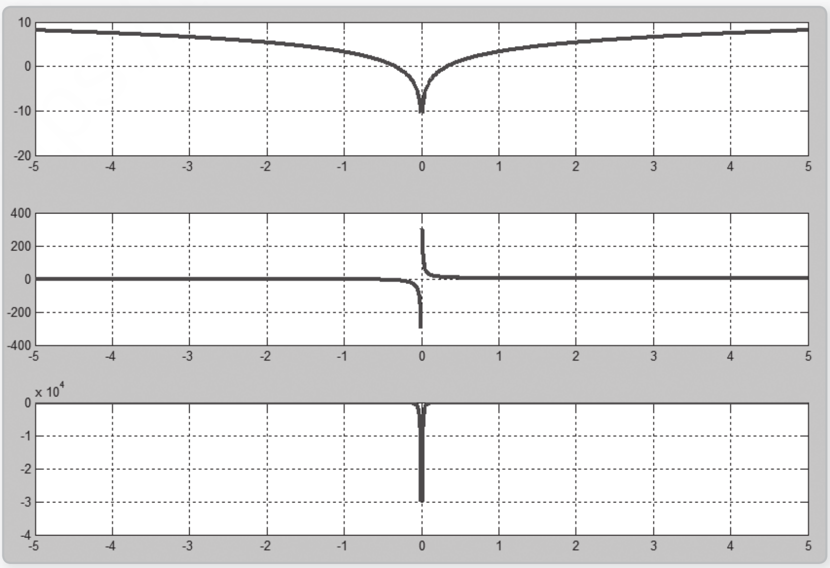
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = 6*log(sqrt(3*t));
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
3/t
>> v = 3/t;
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
-3/t^2
>> a = -3/t^2;
>> % GRAFICAS DE LA POSICION, VELOCIDAD Y ACELERACION:
>> t = -5:0.01:5;
>> x = 6.*log(sqrt(3.*t));
>> subplot(3,1,1)
>> plot(t,x)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> v = 3./t;
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = -3./t.^2;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 19

Una partícula efectúa un movimiento vibratorio armónico definido por la ecuación:

$$x = \operatorname{arctg}\left(t^2\right)$$

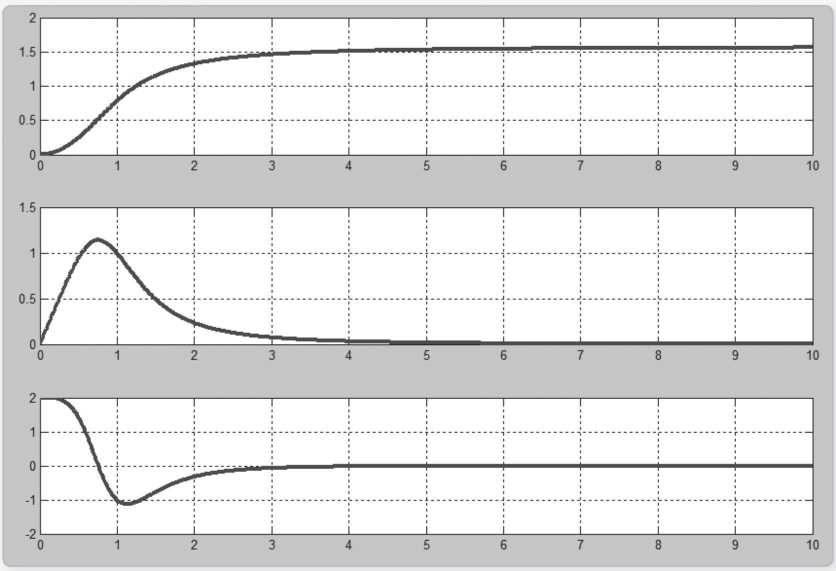
a) Realizar la gráfica para la posición, velocidad y aceleración.

Resolución:

Con MATLAB:

- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION ES:
>> x = atan(t^2);
>> % LA VELOCIDAD DE LA PARTICULA ES LA DERIVADA DE LA POSICION:
>> diff(x,t)
ans =
(2*t)/(t^4 + 1)
>> v = (2*t)/(t^4 + 1);
>> % LA ACELERACION DE LA PARTICULA ES LA DERIVADA DE LA VELOCIDAD:
>> diff(v,t)
ans =
2/(t^4 + 1) – (8*t^4)/(t^4 + 1)^2
>> a = 2/(t^4 + 1) – (8*t^4)/(t^4 + 1)^2;
>> % GRAFICAS DE LA POSICION, LA VELOCIDAD Y LA ACELERACION:
>> t = 0:0.01:10;
>> x = atam(t.^2);
>> subplot(3,1,1)
>> plot(t,x)
>> grid on
>> v = (2.*t)./(t.^4 + 1);
>> subplot(3,1,2)
>> plot(t,v)
>> grid on
>> a = 2./(t.^4 + 1) – (8.*t.^4)./(t.^4 + 1).^2;
>> subplot(3,1,3)
>> plot(t,a)
>> grid on
```



PROBLEMA 20

El movimiento de una partícula está definido por la ecuación:

$$\begin{cases} x_1 = t^3 + 2t + 1 \ , & \text{para el intervalo } 0 \le t \le 6 \\ x_2 = \sqrt{t} \ , & \text{para el intervalo } 6 \le t \le 12 \\ x_3 = -t^2 \ , & \text{para el intervalo } 12 \le t \le 20 \end{cases}$$

Donde x se expresa en metros y t en segundos

a) Realizar las gráficas para la posición, velocidad y aceleración.

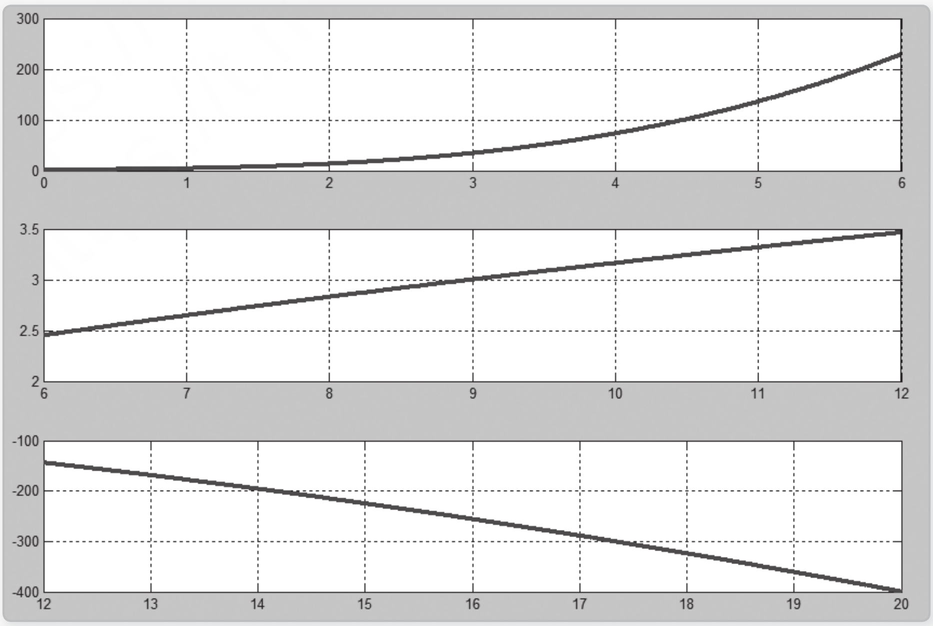
Resolución:

Con MATLAB:

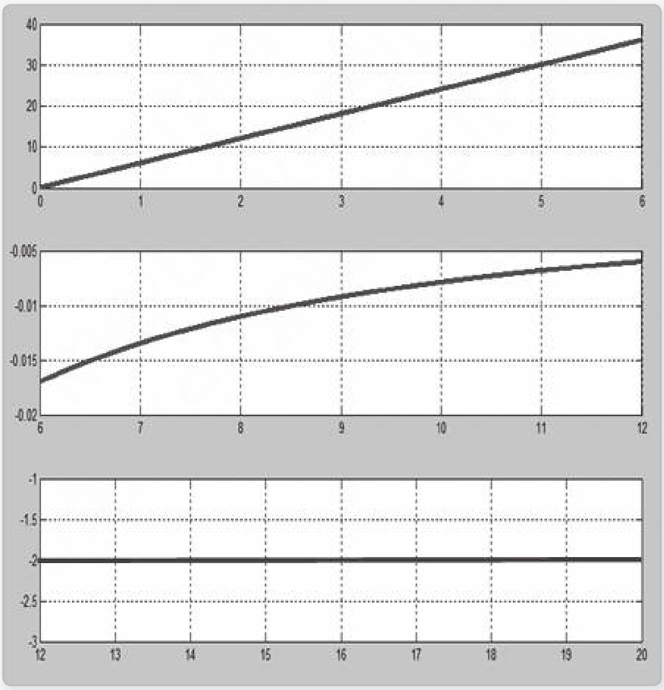
- Gráficas de la posición, velocidad y aceleración.

```
>> syms x t
>> % LA ECUACION DE LA POSICION DEL PRIMER TRAMO ES:
>> x1 = t^3 + 2*t + 1;
>> % LA VELOCIDAD DE LA PARTICULA EN EL PRIMER TRAMO ES:
>> diff(x1,t)

ans =

3*t^2 + 2

>> v1 = 3*t^2 + 2;
>> % LA ACELERACION EN EL PRIMER TRAMO ES:
>> diff(v1,t)

ans =

6*t

>> a1 = 6*t;

>> % LA ECUACION DE LA POSICION DEL SEGUNDO TRAMO ES:
>> x2 = sqrt(t);
>> % LA VELOCIDAD DE LA PARTICULA EN EL SEGUNDO TRAMO ES:
>> diff(x2,t)

ans =

1/(2*t^(1/2))

>> v2 = 1/(2*t^(1/2));

>> % LA ACELERACION EN EL SEGUNDO TRAMO ES:
>> diff(v2,t)

ans =

-1/(4*t^(3/2))

>> a2 = -1/(4*t^(3/2));
>> % LA ECUACION DE LA POSICION DEL TERCER TRAMO ES:
>> x3 = -t^2;
```

```
>> % LA VELOCIDAD DE LA PARTICULA EN EL TERCER TRAMO ES:
>> diff(x3,t)

ans =

(-2)*t

>> v3 = (-2)*t;
>> % LA ACELERACION EN EL TERCER TRAMO ES:
>> diff(v3,t)

ans =

-2

>> a3 = -2;
>> % GRAFICAS DE LA POSICION
>> t1 = 0:0.01:6;
>> x1 = t1.^3 + 2.*t1 + 1;
>> subplot(3,1,1)
>> plot(t1,x1)
>> grid on
>> t2 = 6:0.01:12;
>> x2 = sqrt(t2);
>> subplot(3,1,2)
>> plot(t2,x2)
>> grid on
>> t3 = 12:0.01:20;
>> x3 = -t3.^2;
>> subplot(3,1,3)
>> plot(t3,x3)
>> grid on
```



```
>> % GRAFICAS DE LA ACELERACION
>> t1 = 0:0.01:6;
>> a = 6.*t1;
>> subplot(3,1,1)
>> plot(t1,a)
>> grid on
>> t2 = 6:0.01:12;
>> a2 = -1./(4.*t2.^(3./2));
>> subplot(3,1,2)
>> plot(t2,a2)
>> grid on
>> t3 = 12:0.01:20;
>> a3 = -2;
>> subplot(3,1,3)
>> plot(t3,a3)
>> grid on
```



PROBLEMA 21

El movimiento de una partícula está definido por la ecuación:

$$\begin{cases} x_1 = \ln t & , \text{ para el intervalo } 0 \le t \le 6 \\ x_2 = -\sqrt{t-1} & , \text{ para el intervalo } 6 \le t \le 12 \\ x_3 = \dfrac{1}{t} & , \text{ para el intervalo } 12 \le t \le 20 \end{cases}$$

Donde x se expresa en metros y t en segundos

a) Realizar las gráficas para la posición, velocidad y aceleración.

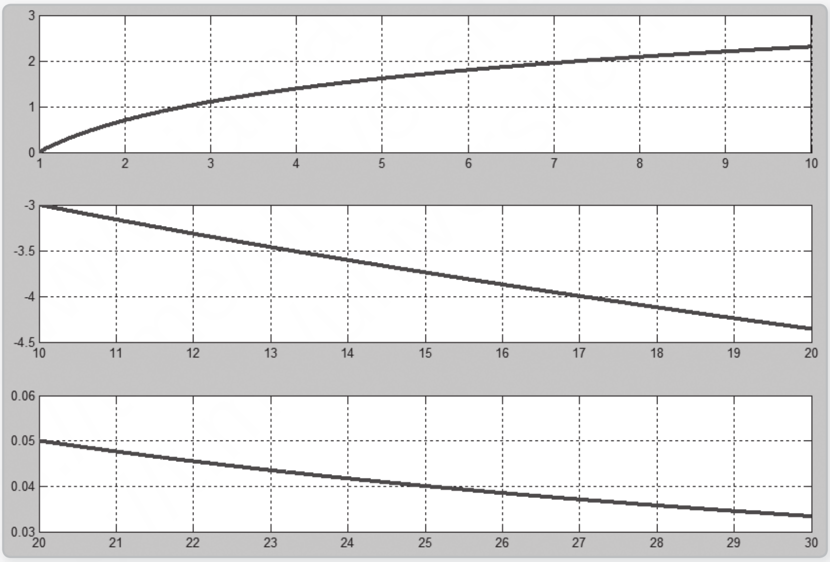
Resolución:

Con MATLAB:

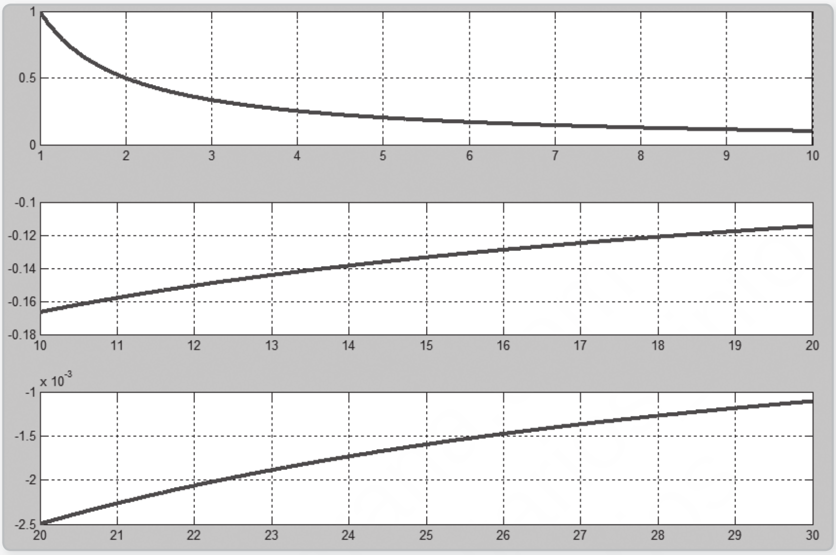
```
>> syms x t
>> % LA ECUACION DE LA POSICION EN EL PRIMER TRAMO ES:
>> x1 = log(t);
>> % LA VELOCIDAD DE LA PARTICULA EN EL PRIMER TRAMO ES:
>> diff(x1,t)

ans =

1/t

>> v1= 1/t;
>> % LA ACELERACION DE LA PARTICULA EN EL PRIMEE TRAMO ES:
>> diff(v,t)

ans =

-1/t^2

>> a1 = -1/t^2;
>> % LA ECUACION DE LA POSICION EN EL SEGUNDO TRAMO ES:
>> x2 = -sqrt(t-1);
>> % LA VELOCIDAD DE LA PARTICULA EN EL SEGUNDO TRAMO ES:
>> diff(x2,t)

ans =

-1/(2*(t - 1)^(1/2))

>> v2 = -1/(2*(t - 1)^(1/2));

>> % LA ACELERACION DE LA PARTICULA EN EL TERCER TRAMO ES:
>> diff(v3,t)

ans =

2/t^3

>> a3 = 2/t^3;
```

- Gráficas con MATLAB:

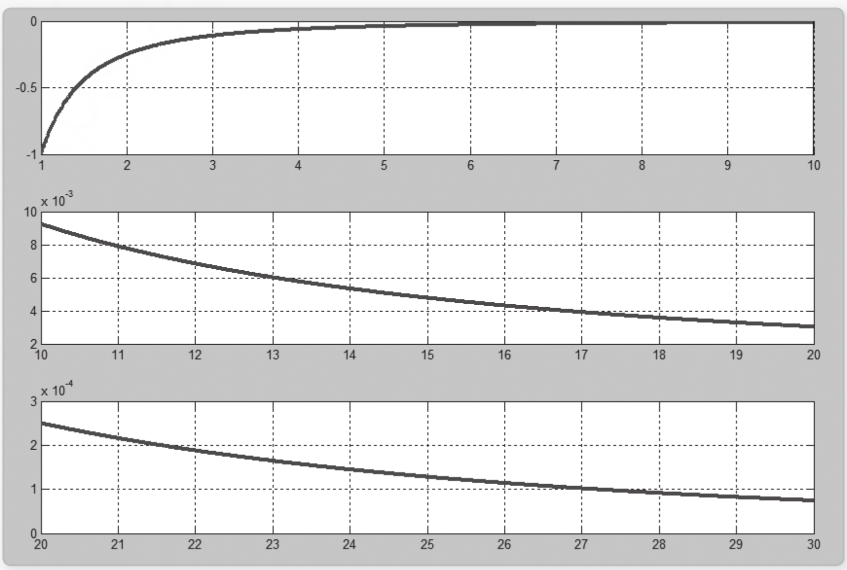
```
>> % GRAFICAS DE LA POSICION:
>> t1 = 1:0.01:10;
>> x1 = log(t1);
>> subplot(3,1,1)
>> plot(t1,x1)
>> grid on
>> t2 = 10:0.01:20;
>> x2 = -sqrt(t2-1);
>> subplot(3,1,2)
>> plot(t2,x2)
>> grid on
>> t3 = 20:0.01:30;
>> x3 = 1./t3;
>> subplot(3,1,3)
>> plot(t3,x3)
>> grid on
```




```
>> % GRAFICAS DE LA VELOCIDAD:
>> t1 = 1:0.01:10;
>> v = 1./t1;
>> subplot(3,1,1)
>> plot(t1,v)
>> grid on
>> t2 = 10:0.01:20;
>> v2 = -1./(2.*(t2 - 1).^(1./2));
>> subplot(3,1,2)
>> plot(t2,v2)
>> grid on
>> t3 = 20:0.01:30;
>> v3 = -1./t3.^2;
>> subplot(3,1,3)
>> plot(t3,v3)
>> grid on
```




```
>> % GRAFICAS DE LA ACELERACION:
>> t1 = 1:0.01:10;
>> a1 = -1./t1.^2;
>> subplot(3,1,1)
>> plot(t1,a1)
>> grid on
>> t2 = 10:0.01:20;
>> a2 = 1./(4.*(t2 - 1).^(3./2));
>> subplot(3,1,2)
>> plot(t2,a2)
>> grid on
>> t3 = 20:0.01:30;
>> a3 = 2./t3.^3;
>> subplot(3,1,3)
>> plot(t3,a3)
>> grid on
```



Problema 22

El movimiento de una partícula está definido por la ecuación:

$$\begin{cases} x_1 = 3\operatorname{sen}(2t) & , \quad \text{para el intervalo } 0 \le t \le 6 \\ x_2 = 4t^2 & , \quad \text{para el intervalo } 6 \le t \le 12 \\ x_3 = e^{0.02t} & , \quad \text{para el intervalo } 12 \le t \le 20 \end{cases}$$

Donde x se expresa en metros y t en segundos

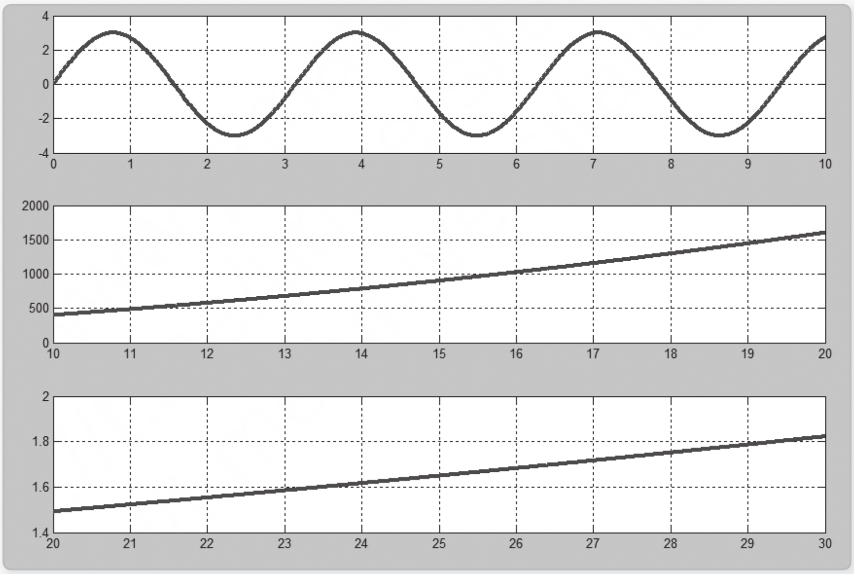
a) Realizar las gráficas para la posición, velocidad y aceleración.

Resolución:

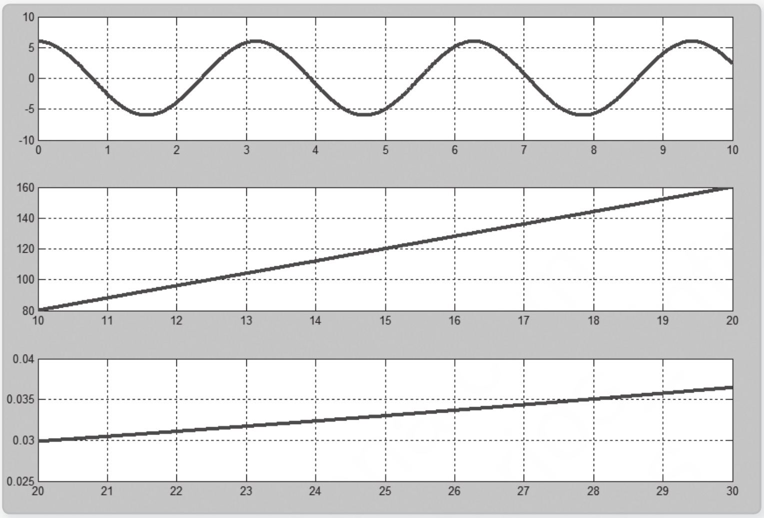
Con MATLAB:

```
>> syms x t
>> % LA ECUACION DE LA POSICION EN EL PRIMER TRAMO ES:
>> x1 = 3*sin(2*t);
>> % LA VELOCIDAD DE LA PARTICULA EN EL PRIMER TRAMO ES:
>> diff(x1,t)
ans =
6*cos(2*t)
>> v1 = 6*cos(2*t);
>> % LA ACELERACION DE LA PARTICULA EN EL PRIMEE TRAMO ES:
>> diff(v1,t)
ans =
(-12)*sin(2*t)
>> a1 =(-12)*sin(2*t);
>> % LA ECUACION DE LA POSICION EN EL SEGUNDO TRAMO ES:
>> x2 = 4*t^2;
>> % LA VELOCIDAD DE LA PARTICULA EN EL SEGUNDO TRAMO ES:
>> diff(x2,t)
ans =
8*t
>> v2 = 8*t;
>> % LA ACELERACION DE LA PARTICULA EN EL SEGUNDO TRAMO ES:
>> diff(v2,t)
ans =
8
>> a2 = 8;
>> % LA ECUACION DE LA POSICION EN EL TERCER TRAMO ES:
>> x3 = exp(0.02*t);
>> % LA VELOCIDAD DE LA PARTICULA EN EL TERCER TRAMO ES:
>> diff(x3,t)
ans =
exp(t/50)/50
>> v3 = exp(t/50)/50;
>> % LA ACELERACION DE LA PARTICULA EN EL SEGUNDO TRAMO ES:
>> diff(v3,t)
ans =
exp(t/2500)
>> a3 = exp(t/50)/2500;
```

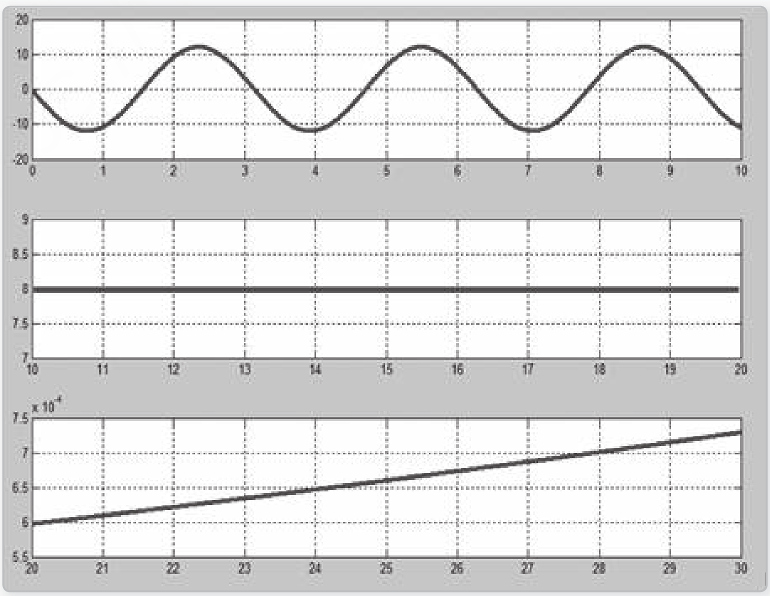
```
>> % GRAFICAS DE LA POSICION:
>> t1 = 0:0.01:10;
>> x1 = 3.*sin(2.*t1);
>> subplot(3,1,1)
>> plot(t1,x1)
>> grid on
>> t2 = 10:0.01:20;
>> x2 = 4.*t2.^2;
>> subplot(3,1,2)
>> plot(t2,x2)
>> grid on
>> t3 = 20:0.01:30;
>> x3 = exp(0.02.*t3);
>> subplot(3,1,3)
>> plot(t3,x3)
>> grid on
```




```
>> % GRAFICA DE LA VELOCIDAD:
>> t1 = 0:0.01:10;
>> v1 = 6.*cos(2.*t1);
>> subplot(3,1,1)
>> plot(t1,v1)
>> grid on
>> t2 = 10:0.01:20;
>> v2 = 8.*t2;
>> subplot(3,1,2)
>> plot(t2,v2)
>> grid on
>> t3 = 20:0.01:30;
>> v3 = exp(t3./50)./50;
>> subplot(3,1,3)
>> plot(t3,v3)
>> grid on
```



```
>> % GRAFICAS DE LA ACELERACION:
>> t1 = 0:0.01:10;
>> a1 =(-12).*sin(2.*t1);
>> subplot(3,1,1)
>> plot(t1,a1)
>> grid on
>> t2 = 10:0.01:20;
>> a2 = 8;
>> subplot(3,1,2)
>> plot(t2,a2)
>> grid on
>> t3 = 20:0.01:30;
>> a3 = exp(t3./50)./2500;
>> subplot(3,1,3)
>> plot(t3,a3)
```



CAPÍTULO

6

MISCELÁNEA CON MATLAB

Resolver los siguientes límites.

EJERCICIO 1

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

Con MATLAB:

```
>> syms x
>> limit(((exp(x)-1)/x),x,0)

ans =

1
```

EJERCICIO 2

$$\lim_{x \to 0} \frac{7^x - 1}{x}$$

Con MATLAB:

```
>> syms x
>> limit((((7^x)-1)/x),x,0)

ans =

log(7)

>> a = log(7)

a =

    1.9459
```

Ejercicio 3

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$$

Con MATLAB:

```
>> syms x
>> A = x^2 - 2*x + 1;
>> B = x^2 - 4*x + 2;
>> limit(((A/B)^x),x,inf)

ans =

exp(2)

>> a = exp(2)

a =

    7.3891
```

Ejercicio 4

$$\lim_{x\to 0}\frac{2(1-\cos x)}{x\tan(x)}$$

Con MATLAB:

```
>> syms x
>> A = 2*(1 - cos(x));
>> B = x * tan(x);
>> limit((A/B),x,0)

ans =

1
```

EJERCICIO 5

$$\lim_{x \to 1} \frac{\text{sen}(1-x)}{\sqrt{x}-1}$$

Con MATLAB:

```
>> syms x
>> A = sin(1 - x);
>> B = sqrt(x) – 1;
>> limit((A/B),x,1)

ans =

-2
```

EJERCICIO 6

$$\lim_{x \to 0} \frac{\ln(\cos x)}{\ln(1+x^2)}$$

Con MATLAB:

```
>> syms x
>> A = log(cos(x));
>> B = log(1 + (x^2));
>> limit((A/B),x,0)

ans =

-1/2
```

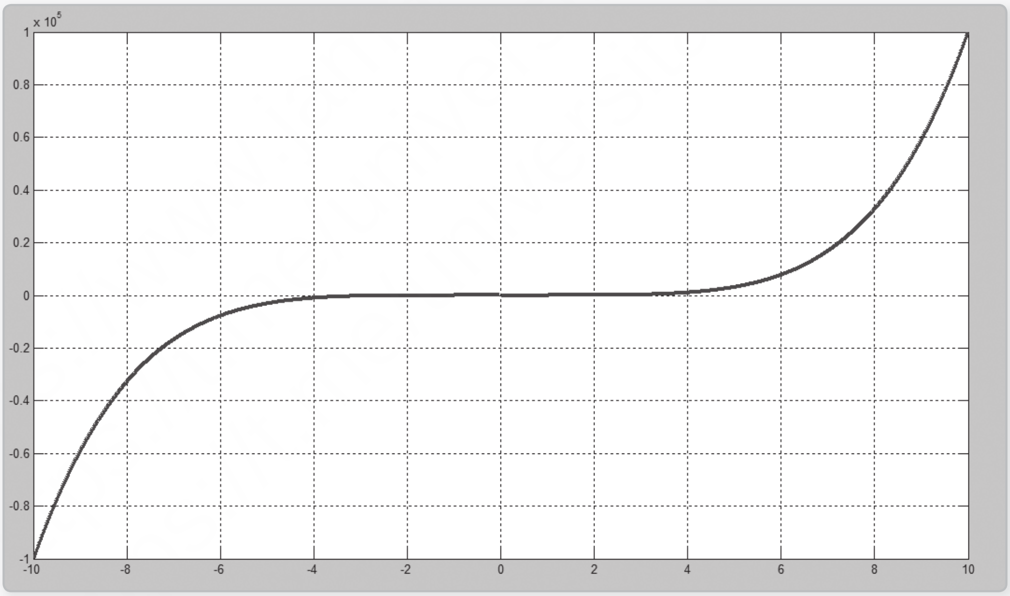
Graficar las siguientes funciones.

EJERCICIO 1

$$y = x^5 - \frac{1}{x}$$

Con MATLAB:

```
>> syms x y
>> y = x^5 - (1/x);
>> % GRAFICANDO LA FUNCION y:
>> x = -10:0.01:10;
>> Y = x.^5 - (1./x);
>> plot(x,y)
>> grid on
```

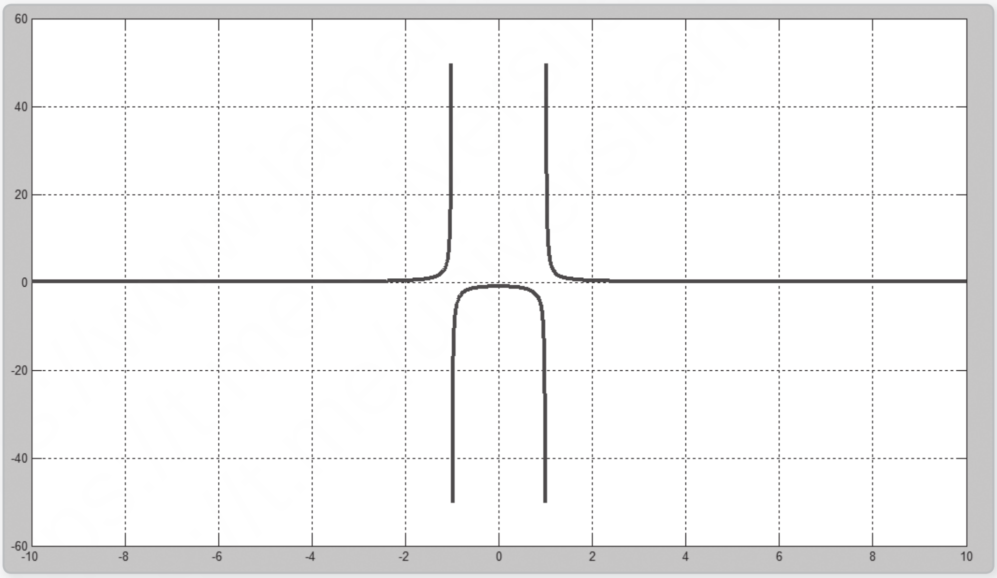


EJERCICIO 2

$$y = \frac{1}{x^2 - 1}$$

Con MATLAB:

```
>> syms x y
>> y = 1/(x^2 - 1);
>> % GRAFICANDO LA FUNCION y:
>> x = -10:0.01:10;
>> Y = 1./(x.^2 - 1);
>> plot(x,y)
>> grid on
```

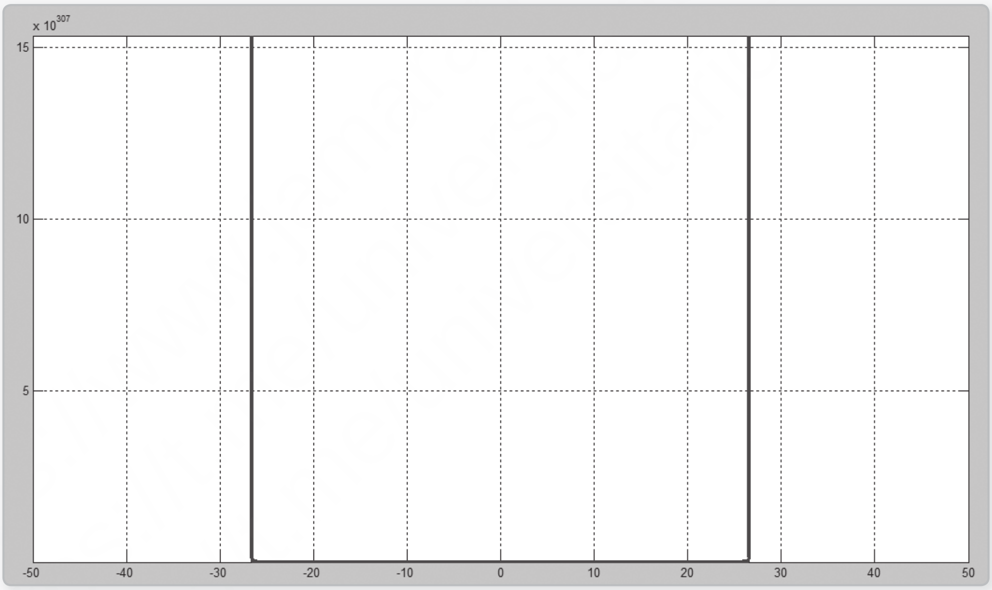


EJERCICIO 3

$$y = e^{x^2+1}$$

Con MATLAB:

```
>> syms x y
>> y = exp(x^2 + 1);
>> %  GRAFICANDO LA FUNCION y:
>> x = -50:0.01:50;
>> y = exp(x.^2 + 1);
>> plot(x,y)
>> grid on
```

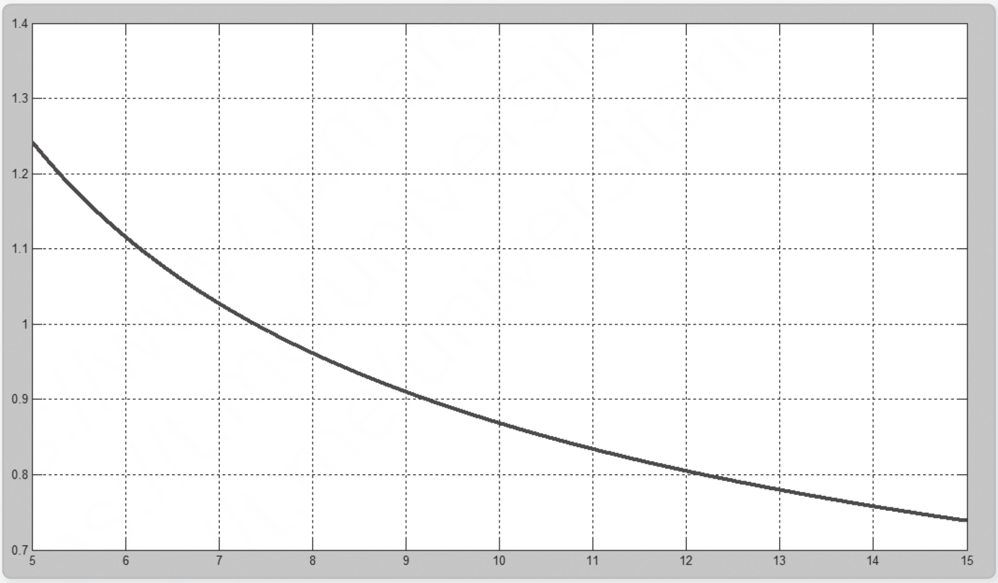


EJERCICIO 4

$$y = \frac{1}{\ln\sqrt{x}}$$

Con MATLAB:

```
>> syms x y
>> y = 1/(log(sqrt(x)));
>> %  GRAFICANDO LA FUNCION y:
>> x = 5:0.01:15;
>> y = 1./(log(sqrt(x)));
>> plot(x,y)
>> grid on
```

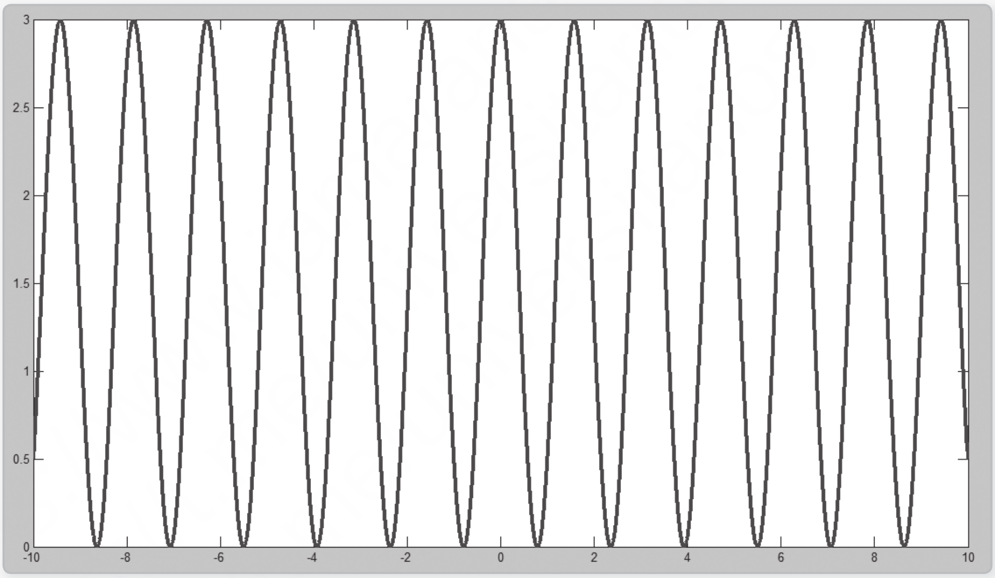


Ejercicio 5

$$y = 3\cos^2(2x)$$

Con MATLAB:

```
>> syms x
>> y = 3*((cos(2*x))^2);
>> %  GRAFICANDO LA FUNCION y:
>> x = -10:0.01:10;
>> y = 3.*((cos(2.*x)).^2);
>> plot(x,y)
```

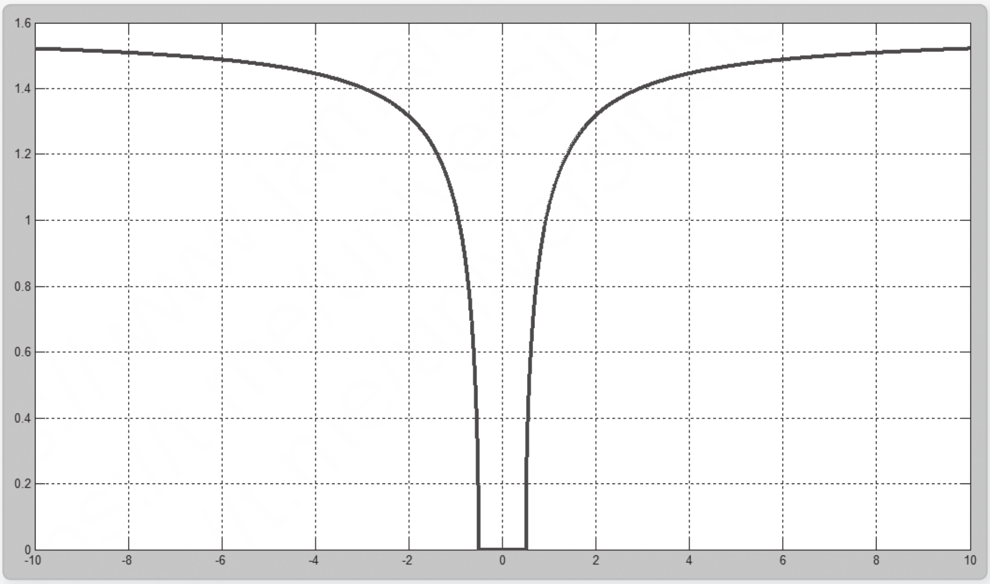


EJERCICIO 6

$$y = \arctan\left(\sqrt{4x^2 - 1}\right)$$

Con MATLAB:

```
>> syms x y
>> y = atan(sqrt(4*x^2 - 1));
>> %  GRAFICANDO LA FUNCION y:
>> x = -10:0.01:10;
>> y = atan(sqrt(4.*x.^2 - 1));
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments
>> grid on
```



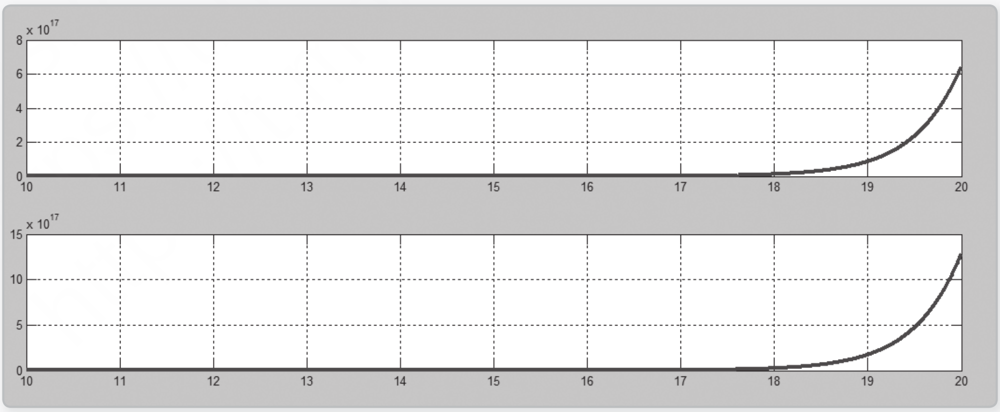
Hallar la derivada de las siguientes funciones y graficarlas.

EJERCICIO 1

$$y = e^{2x+1}$$

Con MATLAB:

```
>> syms x y
>> y = exp(2*x + 1);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = 10:0.01:20;
>> y = exp(2.*x + 1);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = exp(2*x + 1);
>> diff(y)

ans =

2*exp(2*x + 1)

>> x = 10:0.01:20;
>> y = 2.*exp(2.*x + 1);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

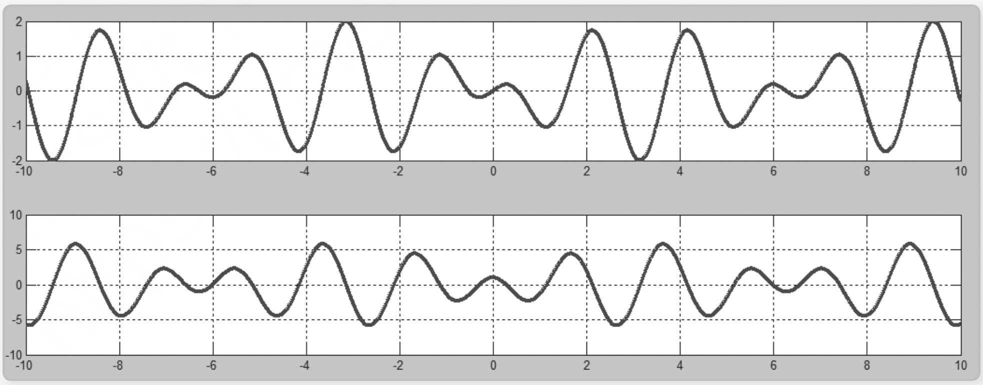


Ejercicio 2

$$y = 2\cos(3x).\text{sen}\left(\frac{x}{2}\right)$$

Con MATLAB:

```
>> syms x y
>> y = 2*cos(3*x)*sin(x/2);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = 2.*cos(3.*x).*sin(x./2);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = 2*cos(3*x)*sin(x/2);
>> diff(y)

ans =

cos(x/2)*cos(3*x) - 6*sin(x/2)*sin(3*x)

>> x = -10:0.01:10;
>> y = cos(x./2).*cos(3.*x) - 6.*sin(x./2).*sin(3.*x);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

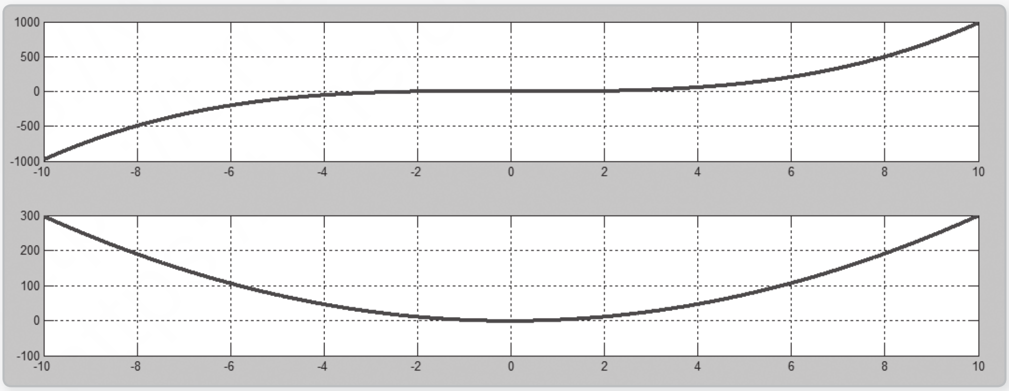


EJERCICIO 3

$$y = x^3 - 2x - 1$$

Con MATLAB:

```
>> syms x y
>> y = x^3 - 2*x - 1;
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = x.^3 - 2.*x - 1;
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = x^3 - 2*x - 1;
>> diff(y)

ans =

3*x^2 - 2

>> x = -10:0.01:10;
>> y = 3.*x.^2 - 2;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

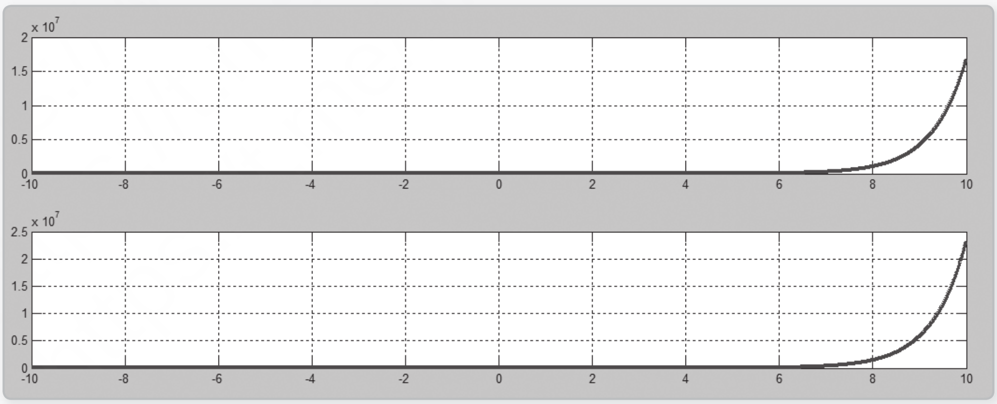


EJERCICIO 4

$$y = 4^{x+2}$$

Con MATLAB:

```
>> syms x y
>> y = 4^(x + 2);
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = 4.^(x + 2);
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = 4^(x + 2);
>> diff(y)

ans =

4^(x + 2)*log(4)

>> x = -10:0.01:10;
>> y = 4.^(x + 2).*log(4);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

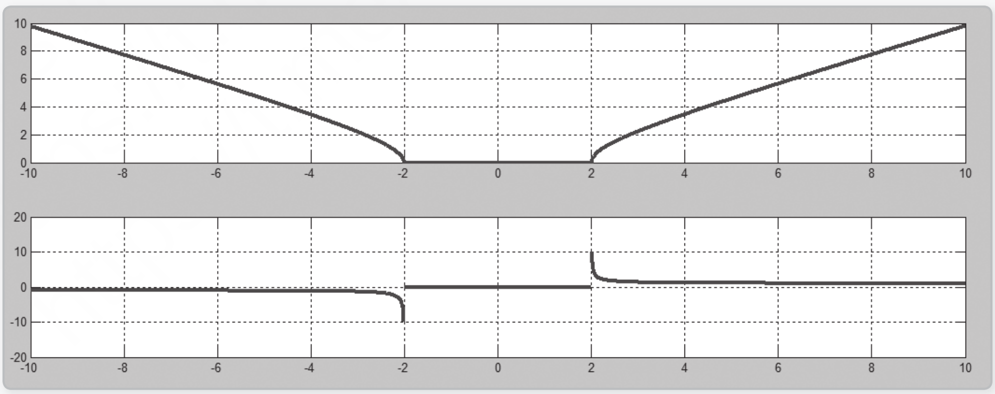


EJERCICIO 5

$$y = \sqrt{x^2 - 4}$$

Con MATLAB:

```
>> syms x y
>> y = sqrt(x^2 - 4);
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = sqrt(x.^2 - 4);
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = sqrt(x^2 - 4);
>> diff(y)

ans =

x/(x^2 - 4)^(1/2)

>> x = -10:0.01:10;
>> y = x./(x.^2 - 4).^(1./2);
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

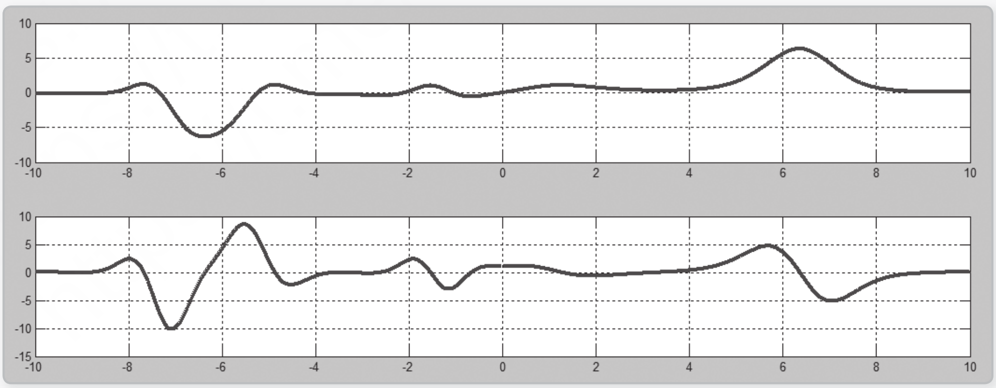


EJERCICIO 6

$$y = x^{\cos x}$$

Con MATLAB:

```
>> syms x y
>> y = x^(cos(x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = x.^(cos(x));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = x^(cos(x));
>> diff(y)

ans =

x^(cos(x) - 1)*cos(x) - x^cos(x)*log(x)*sin(x)

>> x = -10:0.01:10;
>> y = x.^(cos(x) - 1).*cos(x) - x.^cos(x).*log(x).*sin(x);
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

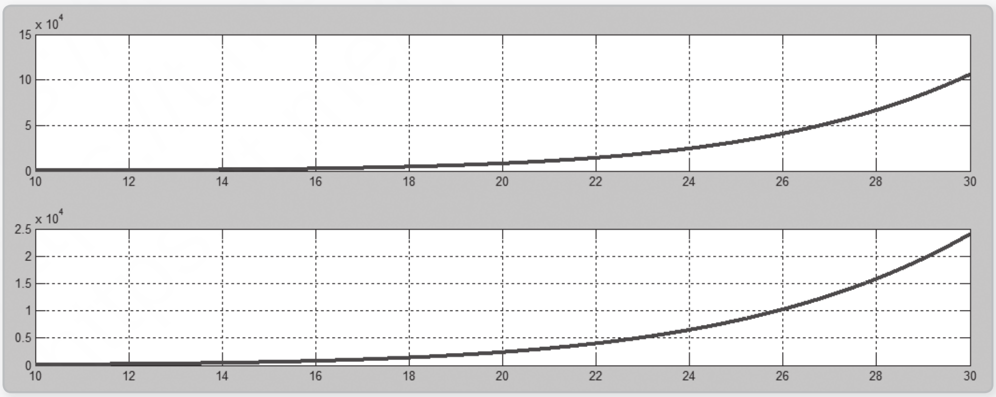


EJERCICIO 7

$$y = x^{\ln x}$$

Con MATLAB:

```
>> syms x y
>> y = x^(log(x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = 10:0.01:30;
>> y = x.^(log(x));
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = x^(log(x));
>> diff(y)

ans =

x^(log(x) - 1)*log(x) + (x^log(x)*log(x))/x

>> x = 10:0.01:30;
>> y = x.^(log(x) - 1).*log(x) + (x.^log(x).*log(x))./x;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

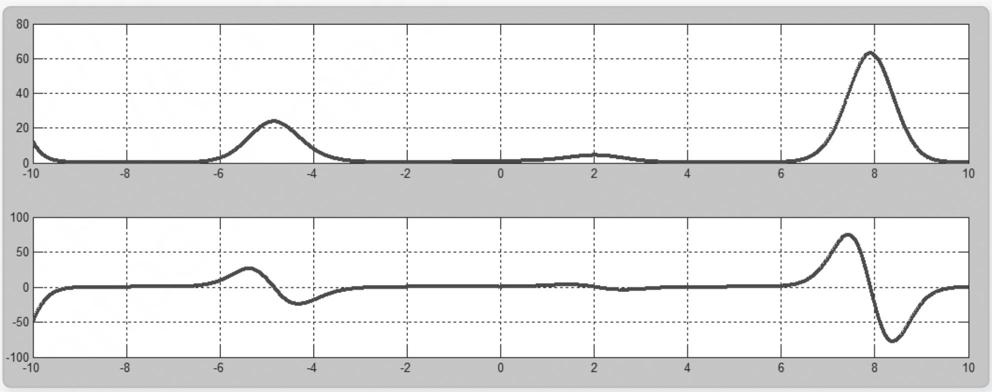


EJERCICIO 8

$$y = \left(x^2 + 1\right)^{\operatorname{sen} x}$$

Con MATLAB:

```
>> syms x y
>> y = (x^2 + 1)^(sin(x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (x.^2 + 1).^(sin(x));
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (x^2 + 1)^(sin(x));
>> diff(y)

ans =

log(x^2 + 1)*cos(x)*(x^2 + 1)^sin(x) + 2*x*sin(x)*(x^2 + 1)^(sin(x) - 1)

>> x = -10:0.01:10;
>> y = log(x.^2 + 1).*cos(x).*(x.^2 + 1).^sin(x) + 2.*x.*sin(x).*(x.^2 + 1).^(sin(x) - 1);
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

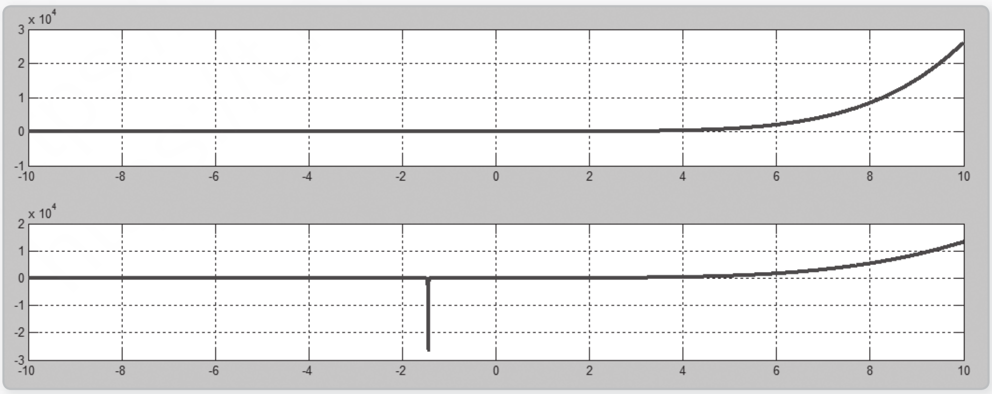


EJERCICIO 9

$$y = \left(3 + x^3\right)^{\arctan x}$$

Con MATLAB:

```
>> syms x y
>> y = (3 + x^3)^(atan(x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (3 + x.^3).^(atan(x));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (3 + x^3)^(atan(x));
>> diff(y)

ans =

(log(x^3 + 3)*(x^3 + 3)^atan(x))/(x^2 + 1) + 3*x^2*atan(x)*(x^3 + 3)^(atan(x) - 1)

>> x = -10:0.01:10;
>> y =(log(x.^3 + 3).*(x.^3 + 3).^atan(x))./(x.^2 + 1) + 3.*x.^2.*atan(x).*(x.^3
+ 3).^(atan(x) - 1);

>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

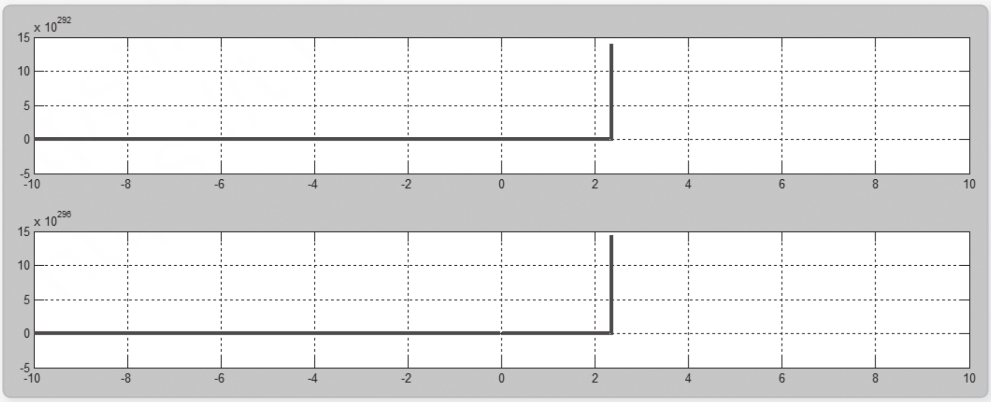


Ejercicio 10

$$y = e^{x^{x^x}}$$

Con MATLAB:

```
>> syms x y
>> y = exp(x^(x^x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = exp(x.^(x.^x));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = exp(x^(x^x));
>> diff(y)

ans =

exp(x^(x^x))*(x^x*x^(x^x - 1) + x^(x^x)*log(x)*(x*x^(x - 1) + x^x*log(x)))

>> x = -10:0.01:10;
>> y = exp(x.^(x.^x)).*(x.^x.*x.^(x.^x - 1) + x.^(x.^x).*log(x).*(x.*x.^(x - 1)
+ x.^x.*log(x)));
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

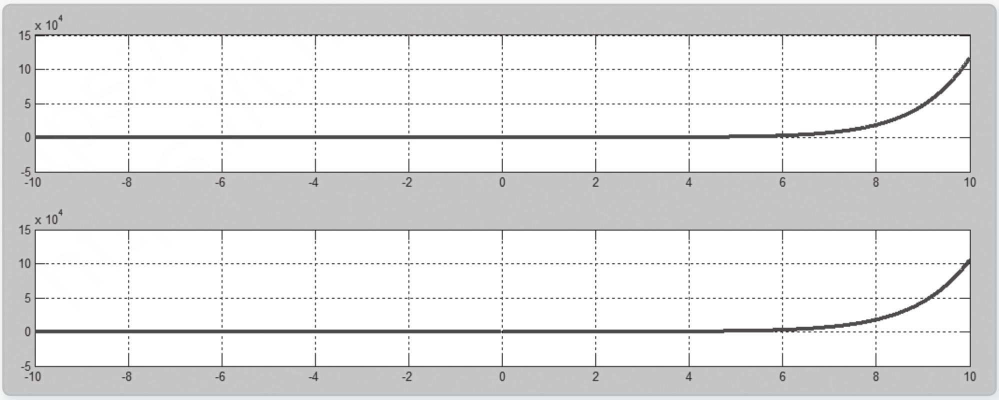


EJERCICIO 11

$$y = 4x^{\sqrt{x}}$$

Con MATLAB:

```
>> syms x y
>> y = (4*x)^(sqrt(x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (4.*x).^(sqrt(x));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (4*x)^(sqrt(x));
>> diff(y)

ans =

4*x^(1/2)*(4*x)^(x^(1/2) - 1) + (log(4*x)*(4*x)^(x^(1/2)))/(2*x^(1/2))

>> x = -10:0.01:10;
>> y = 4.*x.^(1./2).*(4.*x).^(x.^(1./2) - 1) + (log(4.*x).*(4.*x).^(x.^(1./2)))./
(2.*x.^(1./2));
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

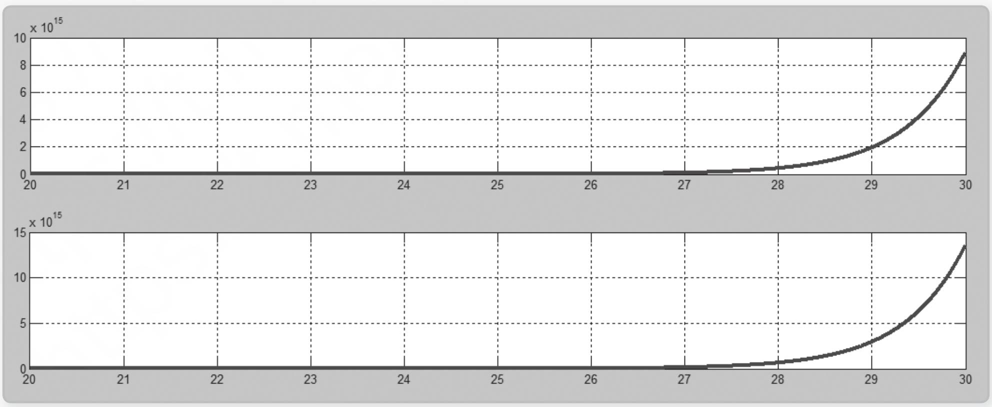


EJERCICIO 12

$$y = (\ln x)^x$$

Con MATLAB:

```
>> syms x y
>> y = (log(x))^x;
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = 20:0.01:30;
>> y = (log(x)).^x;
>> subplot(3,1,1)
>> plot(x,y)
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (log(x))^x;
>> diff(y)

ans =

log(x)^(x - 1) + log(log(x))*log(x)^x

>> x = 20:0.01:30;
>> y = log(x).^(x - 1) + log(log(x)).*log(x).^x;
>> subplot(3,1,2)
>> plot(x,y)
>> grid on
```

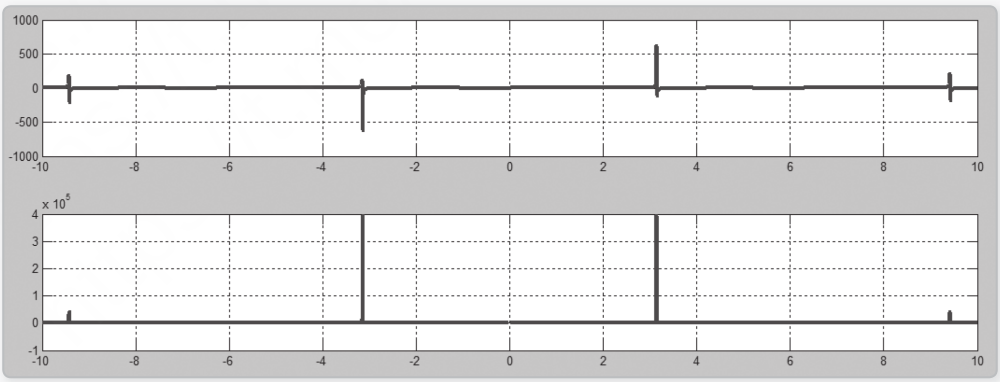


EJERCICIO 13

$$y = (\text{sen}x)^{\cos x}$$

Con MATLAB:

```
>> syms x y
>> y = (sin(x))^(cos(x));
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = (sin(x)).^(cos(x));
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = (sin(x))^(cos(x));
>> diff(y)

ans =

cos(x)^2*sin(x)^(cos(x) - 1) - log(sin(x))*sin(x)*sin(x)^cos(x)

>> x = -10:0.01:10;
>> y = cos(x).^2.*sin(x).^(cos(x) - 1) - log(sin(x)).*sin(x).*sin(x).^cos(x);
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

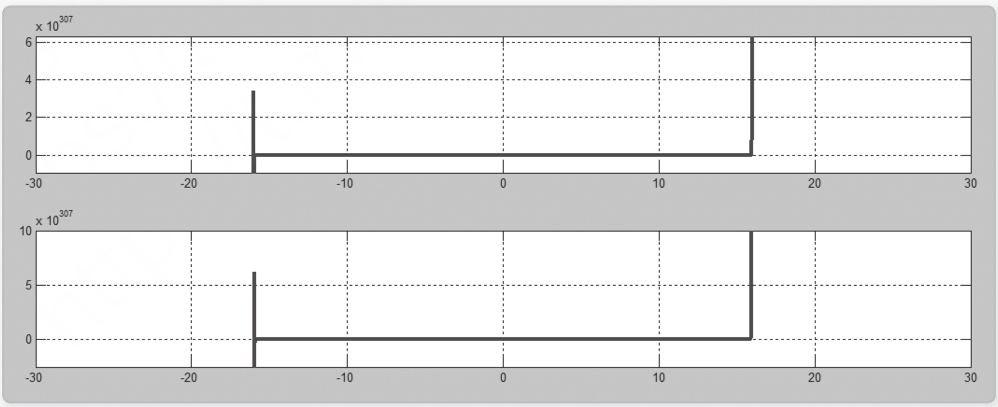


EJERCICIO 14

$$y = x^{x^2}$$

Con MATLAB:

```
>> syms x y
>> y = x^(x^2);
>> % HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -30:0.01:30;
>> y = x.^(x.^2);
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> % HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = x^(x^2);
>> diff(y)

ans =

x^(x^2 - 1)*x^2 + 2*x*x^(x^2)*log(x)

>> x = -30:0.01:30;
>> y = x.^(x.^2 - 1).*x.^2 + 2.*x.*x.^(x.^2).*log(x);
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```

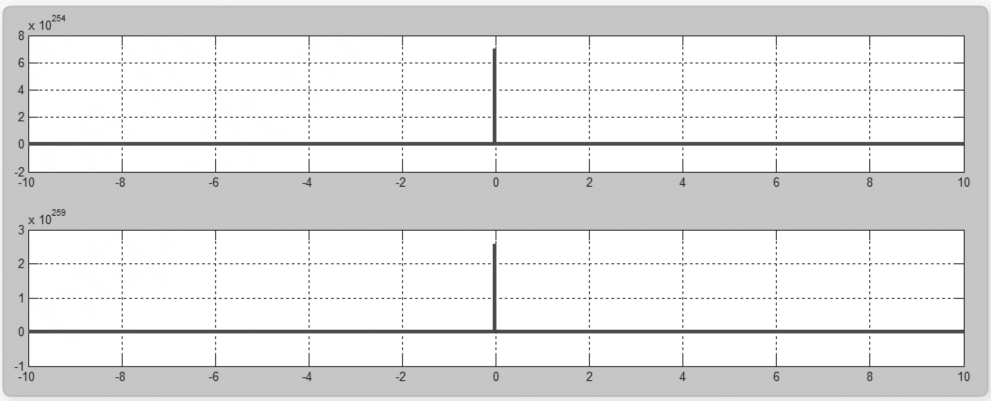


EJERCICIO 15

$$y = x^{3/x}$$

Con MATLAB:

```
>> syms x y
>> y = x^(3/x);
>> %  HALLANDO LA GRAFICA DE LA FUNCION y:
>> x = -10:0.01:10;
>> y = x.^(3./x);
>> subplot(3,1,1)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
>> %  HALLANDO LA GRAFICA DE LA DERIVADA DE LA FUNCION y:
>> syms x y
>> y = x^(3/x);
>> diff(y)

ans =

(3*x^(3/x - 1))/x - (3*x^(3/x)*log(x))/x^2

>> x = -30:0.01:30;
>> y =(3.*x.^(3./x - 1))./x - (3.*x.^(3./x).*log(x))./x.^2;
>> subplot(3,1,2)
>> plot(x,y)
Warning: Imaginary parts of complex X and/or Y arguments ignored
>> grid on
```



FUENTES BIBLIOGRÁFICAS

- Desmond J., MATLAB GUIDE, 2005, Editorial SIAM, USAA.
- Rudra Patrab, Getting Started with MATLAB: A Quick Introduction for Scientists and Engineers, Oxford University Press, 2009, USA.
- Stormy A. Matlab, Second Edition: A Practical Introduction to Programming and Problem Solving.
- Vera A. Apuntes de clase, Matlab aplicado a la Ingeniería, 2003.
- Mathworks notebooks, 2010, USA.

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